Mastering *the SAT Math

Brought to you by the experts at CliffsNotes®

An In-Depth Math Workout

Includes:
- Proven math strategies with important insights
- Analysis of all question types
- Focused review of each math area
- Sample problems following each area review
- Three practice math tests with complete explanations

Jerry Bobrow, Ph.D.

Mastering the *SAT *Math*

Jerry Bobrow, Ph.D.

Contributing Authors
Dale Johnson, M.A.
Joe Skinner, M.A.

BICENTENNIAL 1807 WILEY 2007 BICENTENNIAL

Wiley Publishing, Inc.

About the Author

Jerry Bobrow, Ph.D., is a national authority in the field of test preparation. As executive director of Bobrow Test Preparation Services, he has been administering the test preparation programs at more than 25 California institutions for the past 30 years. Dr. Bobrow has authored more than 30 national best-selling test preparation books, and his books and programs have assisted more than two million test-takers. Each year, Dr. Bobrow personally lectures to thousands of students on preparing for graduate, college, and teacher credentialing exams.

Author's Acknowledgments

I would like to thank Dr. Suzanne Snyder, of Wiley, for her assistance and careful attention to the production process. Finally, I would like to thank my wife, Susan; daughter, Jennifer; and sons, Adam and Jonathan, for their patience, moral support, and comic relief.

Publisher's Acknowledgments

Editorial

Project Editor: Suzanne Snyder

Acquisitions Editor: Greg Tubach

Copy Editor: Tere Stouffer

Production

Proofreader: Henry Lazarek

Wiley Publishing, Inc. Composition Services

Mastering the SAT®: Math

Published by:
Wiley Publishing, Inc.
111 River Street
Hoboken, NJ 07030-5774
www.wiley.com

Published by Wiley, Hoboken, NJ
Published simultaneously in Canada

Library of Congress Cataloging-in-Publication Data

Bobrow, Jerry.
Mastering the SAT : math / by Jerry Bobrow.
p. cm.
Includes bibliographical references and index.
ISBN-13: 978-0-470-03660-0 (alk. paper)
ISBN-10: 0-470-03660-5 (alk. paper)
1. Mathematics—Examinations—Study guides. 2. Mathematics—Examinations, questions, etc. 3. SAT (Educational test)—Study guides. 4. Universities and colleges—United States—Entrance examinations—Study guides. 5. Achievement tests—Study guides. I. Title.
QA43.B649 2007
510.76—dc22

2006029556

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

1O/SR/RR/QW/IN

WILEY

Table of Contents

PART III: MATH PRACTICE TESTS

Preface

The SAT I has changed, but your math scores can still really be improved with proper preparation!

And because of these facts, you can't afford to take a chance. Prepare with the best! Because better scores result from thorough preparation, your study time must be used most effectively. *Mastering the SAT: Math* has been designed by leading experts in the field of test preparation to be the most comprehensive guide to give you that extra boost in math. In keeping with the fine tradition of Cliffs Notes, this guide is written for the student. It is direct, precise, compact, easy to use, and thorough. The testing strategies, techniques, and materials have been researched, tested, and evaluated and are presently used in SAT I test preparation programs at many leading colleges and universities.

This guide combines introductory analysis sections with sample problems, diagnostic review tests, math area reviews, practice problems for each topic area (easy, average, and difficult problems), and three full-length practice math tests. The practice problems for each topic area and practice tests have complete answers and in-depth explanations. Analysis charts, and score range approximators are included following the practice tests to give you a thorough understanding of the *new* SAT I math sections.

Mastering the SAT: Math was written to give you the edge in doing your best by giving you maximum benefit in a reasonable amount of time and is meant to augment, not substitute for, formal or informal learning throughout junior high and high school.

Don't take a chance. Be prepared! Follow the SAT I Study Guide Checklist in this book and study regularly. You'll get the best test preparation possible.

SAT I Study Guide Checklist

❑ 1. Read the New SAT I information bulletin. Get information online at www.collegeboard.org.

❑ 2. Become familiar with the test format, page 1.

❑ 3. Familiarize yourself with the answers to Questions Commonly Asked about the New SAT I, page 3.

❑ 4. Learn the techniques of the Successful Overall Approaches, pages 5–7.

❑ 5. Carefully read Part I: Analysis and Strategies, beginning on page 11.

❑ 6. Start your math review on page 45 with the Arithmetic Diagnostic Test. Read the Arithmetic review as needed.

❑ 7. Next work the Numbers and Operations and Data Analysis, Statistics, and Probability SAT review problems at the end of the Arithmetic Review. Note there are easy, average, and difficult review problems.

❑ 8. Continue your math review on page 91 with the Algebra Diagnostic Test. Read the Algebra Review as needed.

❑ 9. Next work the Algebra and Functions SAT review problems at the end of the Algebra Review (easy, average, and difficult problems).

❑ 10. Continue your math review on page 145 with the Geometry Diagnostic Test. Read and review Geometry and Measurement as needed.

❑ 11. Next, work the Geometry and Measurement SAT review problems at the end of the Geometry Review (easy, average, and difficult problems).

❑ 12. Strictly observing time allotments, take Math Practice Test I, section by section (take Section 1 and then check your answers; take Section 2 and then check your answers, and so on), beginning on page 199.

❑ 13. Review the answers and explanations for each question on Math Practice Test I, beginning on page 216.

❑ 14. Analyze your Math Practice Test I answers by filling out the analysis charts, page 215.

❑ 15. Review your math skills as necessary.

❑ 16. Review or reread Part I: Analysis and Strategies, beginning on page 9, to see whether you applied some of the strategies.

❑ 17. Strictly observing time allotments, take Math Practice Test II, beginning on page 236. Take a very short break after each hour of testing.

❑ 18. Check your answers and use the Score Range Approximator (page 303) to get a very general score range.

❑ 19. Analyze your Math Practice Test II answers by filling out the analysis charts on page 252.

❑ 20. While referring to each item of Math Practice Test II, study all of the explanations that begin on page 253.

❑ 21. Selectively review some basic skills as necessary.

❑ 22. Strictly observing time allotments, take Math Practice Test III, beginning on page 272. Take a very short break after each hour of testing.

❑ 23. Check your answers and use the Score Range Approximator (page 303) to get a very general score range.

❑ 24. Analyze your Math Practice Test III answers by filling out the analysis charts on page 287.

❑ 25. While referring to each item of Math Practice Test III, study all of the explanations that begin on page 288.

❑ 26. Again, selectively review Part I: Analysis and Strategies, beginning on page 9, and any other basic skills or exam areas you feel are necessary.

Introduction

General Format of the New SAT I		
Section 1	***Writing Skills—Essay***	***1 Question***
25 Minutes	Writing Skills—Essay	1 Essay Question
Section 2	***Critical Reading***	***24–28 Questions***
25 Minutes	Sentence Completions	8–10 Questions
	Short Reading Passages	4–6 Questions
	Long Reading Passages	12–14 Questions
Section 3	***Mathematics***	***20 Questions***
25 Minutes	Multiple Choice	20 Questions
Section 4	***Writing Skills—Multiple Choice***	***35 Questions***
25 Minutes	Identifying Sentence Errors	15–19 Questions
	Improving Sentences	7–13 Questions
	Improving Paragraphs	6–10 Questions
Section 5	***Critical Reading***	***24–27 Questions***
25 Minutes	Sentence Completions	5–9 Questions
	Short Reading Passages	4–6 Questions
	Long Reading Passages	10–12 Questions
Section 6	***Mathematics***	***20 Questions***
25 Minutes	Multiple Choice	10 Questions
	Grid-ins	10 Questions
Section 7	***Critical Reading***	***15–20 Questions***
20 Minutes	Reading Passages (possible Sentence Completions)	13–15 Questions (4–6)
Section 8	***Mathematics***	***14–15 Questions***
20 Minutes	Multiple Choice	15 Questions
Section 9 25 Minutes	***Mathematics/Critical Reading/ or Writing Skills***	***20–35 Questions***
Section 10	***Writing Skills—Multiple Choice***	***14–15 Questions***
10 Minutes	Improving Sentences	15 Questions
Total Testing Time **225 Minutes = 3 Hours, 45 Minutes**		**Approximately** **202–212 Questions**

Note: The **order** in which the sections appear, the **question types** within a section**,** and the **number of questions** may vary, and there may be many forms of the test. Only three of the critical reading sections (two 25-minute sections and one 20-minute section), three of the math sections (two 25-minute sections and one 20-minute section), the writing essay (25 minutes), and the multiple-choice sections (one 25-minute section and one 10-minute section) actually count toward your new SAT I score.

One 25-minute section is a *pretest,* or experimental, section that does not count toward your score. The pretest or experimental section can be a critical reading, math, or writing multiple-choice section and can appear anywhere on your exam. It does not have to be Section 9. You should work all of the sections as though they count toward your score.

General Description

The new SAT I is used along with your high school record and other information to assess your competence for college work. The test lasts 3 hours and 45 minutes and consists of mostly multiple-choice type questions, with some grid-in questions, and an essay. The critical reading sections test your ability to read critically, to comprehend what you read, and to understand words in context. The math sections test your ability to solve problems using mathematical reasoning and your skills in arithmetic, algebra I and II, and geometry. The writing ability sections test your ability to write a clear, precise essay and to find grammar and usage errors, to correct sentence errors, and to improve paragraphs.

A Close Look at the New SAT I

Question Type	*Approximate Number of Questions*
Critical Reading	
Sentence Completions	19
Passage-Based Questions	48
Total Critical Reading Questions	67
Mathematics	
Multiple Choice	44
Grid-Ins	10
Total Mathematics Questions	54
Writing	
Multiple Choice	
Improving Sentences	25
Identifying Sentence Errors	18
Improving Paragraphs	6
Total Writing Multiple Choice	49
Essay Question	One Question

The problems in the math sections (multiple-choice and grid-ins) and the sentence completions section of the new SAT I are slightly graduated in difficulty. Many students make simple mistakes because they rush through the easy questions to get to the difficult ones. *Keep in mind that each question within a section is of equal value, so getting an easy question right is worth the same as getting a difficult question right.*

Special Notes for the New SAT I

- Verbal analogies are no longer on the exam.
- Quantitative comparison math questions are no longer on the exam.
- Critical reading sections now also include paragraph-length passages.
- Math sections have been enhanced, including some algebra II problems.
- The first part of the test will always be an essay.
- Writing multiple-choice sections have been added.

Questions Commonly Asked about the New SAT I

Q: WHO ADMINISTERS THE NEW SAT I?

A: The new SAT I is part of the entire Admissions Testing Program (ATP), which is administered by the College Entrance Examination Board in conjunction with Educational Testing Service of Princeton, New Jersey.

Q: HOW IS THE NEW SAT I SCORED?

A: The scoring will be as follows:

Critical Reading: 200–800

Mathematics: 200–800

Writing: 200–800 (subscores essay 2–12, multiple choice 20–80)

Total possible score: 600–2,400

Q: WILL THE NEW SAT I BE MORE DIFFICULT?

A: No. The new SAT has been designed so that a student who could score a 500 on the math section of the old SAT I could score a 500 on the math section of the New SAT I. This is the same for the Critical Reading, formerly called Verbal Reasoning.

Q: IS THERE A DIFFERENCE BETWEEN THE NEW SAT I AND THE SAT II?

A: Yes. The new SAT I assesses general critical reading, mathematical reasoning, writing, and editing abilities that you have developed over your lifetime. The SAT II measures your proficiency in specific subject areas. The SAT II tests how well you have mastered a variety of high school subjects.

Q: CAN I TAKE THE NEW SAT I MORE THAN ONCE?

A: Yes. On past score reports, scores up to five years old were also included on the report. It is not uncommon for students to take the test more than once.

Q: WHAT MATERIALS MAY I BRING TO THE NEW SAT I?

A: Bring your registration form, positive identification, a watch, three or four sharpened no. 2 pencils, a good eraser, and an approved calculator. You may not bring scratch paper or books. You may do your figuring in the margins of the test booklet or in the space provided.

Q: IF NECESSARY, MAY I CANCEL MY SCORE?

A: Yes. You may cancel your score on the day of the test by telling the test center supervisor, or you may write, fax, or e-mail a cancellation to College Board ATP. See specific instructions for canceling your score in the *Student Bulletin.* Your score report will record your cancellation, along with any completed test scores.

Q: SHOULD I GUESS ON THE NEW SAT I?

A: If you can eliminate one or more of the multiple-choice answers to a question, it is to your advantage to guess. Eliminating one or more answers increases your chance of choosing the right answer. To discourage wild guessing, a fraction of a point is subtracted for every wrong answer, but no points are subtracted if you leave the answer blank. On the grid-in questions, there is no penalty for filling in a wrong answer.

Q: HOW SHOULD I PREPARE FOR THE NEW TEST?

A: Understanding and practicing test-taking strategies helps a great deal, especially on the critical reading sections. Subject-matter review is particularly useful for the math section, and a review of basic grammar and usage will be helpful on the writing sections. Reviewing the writing process and practicing timed essay writing will also be helpful. The College Board offers additional practice online.

Q: HOW OFTEN ARE THE TESTS ADMINISTERED?

A: The new SAT I is usually scheduled to be administered nationwide seven times during the school year, in October, November, December, January, March, May, and June. Some special administrations are given in limited locations.

Q: WHERE IS THE SAT I ADMINISTERED?

A: Your local college testing or placement office will have information about local administrations; ask for the *Student Bulletin.* The SAT I is administered at hundreds of schools in and out of the United States.

Q: HOW AND WHEN SHOULD I REGISTER?

A: A registration packet, complete with return envelope, is attached to the *Student Bulletin.* Mailing in these forms, plus the appropriate fees, completes the registration process. You can also register online at www.collegeboard.org. You should register about six weeks prior to the exam date.

Q: IS WALK-IN REGISTRATION PROVIDED?

A: Yes, on a limited basis. If you are unable to meet regular registration deadlines, you may attempt to register on the day of the test. (An additional fee is required.) You will be admitted only if space remains after preregistered students have been seated.

Q: CAN I GET MORE INFORMATION?

A: Yes. If you require information that is not available in this book, you can check online at www.collegeboard.org. You can also write or call one of these College Board regional offices.

Middle States: 3440 Market Street, Suite 410, Philadelphia, Pennsylvania 19104-3338. (215) 387-7600. Fax (215) 387-5805.
or
126 South Swan Street, Albany, New York 12210-1715. (518) 472-1515. Fax (518) 472-1544.

Midwest: 1560 Sherman Avenue, Suite 1001, Evanston, Illinois 60201-4805. (847) 866-1700. Fax (847) 866-9280.

New England: 470 Totten Pond Road, Waltham, Massachusetts 02451-1982. (781) 890-9150. Fax (781) 890-0693.

South: 100 Crescent Centre Parkway, Suite 340, Tucker, Georgia 30084-7039. (770) 908-9737. Fax (770) 934-4885.
or
HighPoint Center, Suite 900, 106 East College Avenue, Tallahassee, Florida 32301-7732. (850) 222-7999. Fax (850) 224-3077.

Southwest: 4330 South MoPac Expressway, Suite 200, Austin, Texas 78735. (512) 891-8400. Fax (512) 891-8404.

West: 2099 Gateway Place, Suite 480, San Jose, California 95110-1017. (408) 452-1400. Fax (408) 453-7396.
or
Capitol Place, 915 L Street, Suite 1200, Sacramento, California 95814. (916) 444-6262. Fax (916) 444-2868.

Taking the New SAT I: Successful Overall Approaches for Multiple-Choice Questions

I. The "Plus-Minus" System

Many who take the New SAT I won't get their best possible score because they spend too much time on difficult questions, leaving insufficient time to answer the easy questions. Don't let this happen to you. Because every question within each section is worth the same amount, use the following system, marking on your answer sheet:

1. Answer easy questions immediately.
2. Place a "+" next to any problem that seems solvable but is too time-consuming.
3. Place a "–" next to any problem that seems impossible.

Act quickly; don't waste time deciding whether a problem is a "+" or a "–."

After working all the problems you can do immediately, go back and work your "+" problems. If you finish them, try your "–" problems (sometimes when you come back to a problem that seemed impossible, you suddenly realize how to solve it).

Your answer sheet should look something like this after you finish working your easy questions:

1. Ⓐ ● Ⓒ Ⓓ Ⓔ
+2. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3. Ⓐ Ⓑ ● Ⓓ Ⓔ
–4. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
+5. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Make sure to erase your "+" and "–" marks before your time is up. The scoring machine may count extraneous marks as wrong answers.

II. The Elimination Strategy

Take advantage of being allowed to mark in your testing booklet. As you eliminate an answer choice from consideration, make sure to mark it out in your question booklet as follows:

~~(A)~~
? (B)
~~(C)~~
~~(D)~~
? (E)

Notice that some choices are marked with question marks, signifying that they may be possible answers. This technique helps you avoid reconsidering those choices you have already eliminated and helps you narrow down your possible answers. These marks in your testing booklet do not need to be erased.

III. The "Avoiding Misreads" Method

Sometimes a question may have different answers depending upon what is asked. For example,

If $3x + x = 20$, what is the value of $x + 4$?

Notice that this question doesn't ask for the value of x, but rather the value of $x + 4$.

Or If $8y + 3x = 14$, what is the value of y in terms of x?

The question may instead have asked, "What is the value of x in terms of y?"

Or All of the following statements are true EXCEPT . . .

Notice that the words EXCEPT and NOT change these questions significantly.

To avoid "misreading" a question (and, therefore, answering it incorrectly), simply circle what you must answer in the question. For example, do you have to find x or $x + 4$? Are you looking for what is true or the exception to what is true? To help you avoid misreads, mark the questions in your test booklet in this way:

If $3x + x = 20$, what is the value of $x + 4$?
If $8y + 3x = 14$, what is the value of y in terms of x?
All of the following statements are true EXCEPT . . .

And, once again, these circles in your question booklet do not have to be erased.

IV. The Multiple-Multiple-Choice Technique

Some math and verbal questions use a "multiple-multiple-choice" format. At first glance, these questions appear more confusing and more difficult than normal five-choice (**A, B, C, D, E**) multiple-choice problems. Actually, once you understand "multiple-multiple-choice" problem types and technique, they are often easier than a comparable standard multiple-choice question. For example,

If x is a positive integer, then which of the following must be true?

I. $x > 0$
II. $x = 0$
III. $x < 1$

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

Because x is a positive integer, it must be a counting number. Note that possible values of x could be 1, or 2, or 3, or 4, and so on. Therefore, statement I, $x > 0$, is always true. So next to I on your question booklet, place a *T* for *true.*

T I. $x > 0$
II. $x = 0$
III. $x < 1$

Now realize that the correct final answer choice (**A, B, C, D,** or **E**) *must* contain *true statement I*. This eliminates **B** and **C** as possible correct answer choices, because they do not contain true statement I. You should cross out **B** and **C** on your question booklet.

Statement II is *incorrect.* If x is positive, x cannot equal zero. Thus, next to II, you should place an *F* for *false.*

T I. $x > 0$
F II. $x = 0$
III. $x < 1$

Knowing that II is false allows you to eliminate any answer choices that contain *false statement II.* Therefore, you should cross out **D**, because it contains false statement II. Only **A** and **E** are left as possible correct answers. Finally, you realize that statement III is also false, as x must be 1 or greater. So you place an F next to III, thus eliminating Choice **E** and leaving **A**, I only. This technique often saves some precious time and allows you to take a better educated guess should you not be able to complete all parts (I, II, III) of a multiple-multiple-choice question.

A Summary of General Strategies

- ❑ *Set a goal.* Remember that an average score is about 50 percent right.
- ❑ *Know the directions.*
- ❑ Go into each section *looking for the questions you can do and should get right.*
- ❑ *Don't get stuck* on any one question.
- ❑ Be sure to *mark your answers in the right place.*
- ❑ Be careful. *Watch out for careless mistakes.*
- ❑ *Don't make simple mistakes by rushing through the easy questions in math* to get to the difficult ones.
- ❑ *Know when to skip a question.*
- ❑ *Guess only if you can eliminate* one or more answers.
- ❑ Don't be afraid to *fill in your answer or guess on grid-ins.*
- ❑ Practice using the *"Plus-Minus" System,* the *Elimination Strategy,* the *"Avoiding Misreads" Method,* and the *Multiple-Multiple-Choice Technique.*
- ❑ *Remember to erase* any extra marks on your answer sheet.

PART I

ANALYSIS AND STRATEGIES

Introduction to the Mathematics Section

The Mathematics sections of the SAT consist of two basic types of questions: regular multiple-choice questions and student-produced responses also known as grid-ins.

Two Mathematics sections are 25 minutes in length and one math section is 20 minutes in length. Since one section of the test is experimental (although you don't know which one), you could have an additional 25-minute Math section.

Although the order of the sections and the number of questions may change, at this time, the three sections total about 52 to 56 math questions that count toward your score. These three sections generate a scaled math score that ranges from 200 to 800. About 50% right should generate an average score.

The Mathematics sections are slightly graduated in difficulty. That is, the easiest questions are basically at the beginning and the more difficult ones at the end. If a section has two types of questions, usually each type starts with easier problems. For example, a section starts with easy multiple-choice questions, and the last few multiple-choice questions are more difficult before you start the grid-ins; the grid-ins start with easy questions and move toward the more difficult ones at the end.

You will be given reference information preceding each Mathematics section. You should be familiar with this information.

You may use an approved calculator on the SAT I. Bring a calculator with which you are familiar.

Using Your Calculator

The new SAT I allows the use of approved calculators, and the College Board (the people who sponsor the exam) recommends that each test taker take a calculator to the test. Even though no question will require the use of a calculator—that is, each question can be answered without a calculator—in some instances, using a calculator will save you valuable time.

You should

- Bring your own calculator, because you can't borrow one during the exam.
- Bring a calculator even if you don't think you'll use it. Make sure that you are familiar with the use of your calculator.
- Make sure that your calculator has new, fresh batteries and is in good working order.
- Practice using your calculator on some of the problems to see when and where it will be helpful.
- Check for a shortcut to any problem that seems to involve much computation. But use your calculator if it will be time effective. If there appears to be too much computation or the problem seems impossible without the calculator, you're probably doing something wrong.
- Before doing an operation, check the number that you keyed on the display to make sure that you keyed in the right number. You may want to check each number as you key it in.

Be careful that you

- Don't rush out and buy a sophisticated calculator for the test.
- Don't bring a calculator that you're unfamiliar with.
- Don't bring a pocket organizer, handheld minicomputer, laptop computer, or calculator with a typewriter-type keypad or paper tape.
- Don't bring a calculator that requires an outlet or any other external power source.
- Don't bring a calculator that makes noise.

- Don't try to share a calculator.
- Don't try to use a calculator on every problem.
- Don't become dependent on your calculator.

Following is the Calculator Policy for the New SAT I as given by the College Board:

"The following are **not** permitted:

- Powerbooks and portable/handheld computers
- Electronic writing pads or pen-input/stylus-driven (e.g., Palm, PDA's, Casio ClassPad 300)
- Pocket organizers
- Models with QWERTY (i.e., typewriter) keyboards (e.g., TI-92 Plus, Voyage 200)
- Models with paper tapes
- Models that make noise or 'talk'
- Models that require an electrical outlet
- Cell phone calculators"

Take advantage of using a calculator on the test. Learn to use a calculator efficiently by practicing. As you approach a problem, first focus on how to solve that problem and then decide whether the calculator will be helpful. Remember, a calculator can save you time on some problems, but also remember that each problem can be solved without a calculator. Also remember that a calculator will not solve a problem for you. You must understand the problem first.

Basic Skills and Concepts That You Should Know

Number and Operations

- Operations with fractions
- Applying addition, subtraction, multiplication, and division to problem solving
- Arithmetic mean (average), mode, and median
- Ratio and proportion
- Number properties: positive and negative integers, odd and even numbers, prime numbers, factors and multiples, divisibility
- Word problems, solving for: percents, averages, rate, time, distance, interest, price per item
- Number line: order, consecutive numbers, fractions, betweenness
- Sequences involving exponential growth
- Sets (union, intersection, elements)

Algebra and Functions

- Operations with signed numbers
- Substitution for variables
- Absolute value
- Working with algebraic expressions
- Manipulating integer and rational exponents
- Solving rational equations and inequalities
- Working with linear functions—graphs and equations
- Solving radical equations

- Basic factoring
- Direct and inverse variation
- Function notation and evaluation
- Concepts of range and domain
- Working with positive roots
- Solving quadratic equations
- Working with quadratic functions and graphs

Geometry and Measurement

- Vertical angles
- Angles in figures
- Perpendicular and parallel lines
- Perimeter, area, angle measure of polygons
- Circumference, area, radius, diameter
- Triangles: right, isosceles, equilateral, angle measure, similarity
- Special triangles: 30°–60°–90°, 45°–45°–90°
- Pythagorean theorem
- Volume and surface area of solids
- Coordinate geometry: coordinates, slope
- Geometric notation for length, segments, lines, rays, and congruence
- Properties of tangent lines
- Problems in which trigonometry could be used as an alternate solution method
- Qualitative behavior of graphs and functions
- Transformations and their effect on graphs and functions

Data Analysis, Statistics, and Probability

- Interpreting graphs, charts, and tables
- Scatterplots
- Probability
- Geometric probability
- Basic statistics (mean, mode, median, range)

Multiple-Choice Questions

You should have a total of about 42 to 46 multiple-choice questions spread throughout the three Mathematics sections that count toward your score.

Ability Tested

The Mathematics multiple-choice questions test your ability to solve mathematical problems involving arithmetic, algebra I and II, geometry, data interpretation, basic statistics and probability, and word problems by using problem-solving insight, logic, and the application of basic skills.

Basic Skills Necessary

The basic skills necessary to do well on this section include high school algebra I and II and intuitive or informal geometry. No calculus is necessary. Logical insight into problem-solving situations is also necessary.

Directions

Solve each problem in this section by using the information given and your own mathematical calculations, insights, and problem-solving skills. Then select the one correct answer of the five choices given and mark the corresponding circle on your answer sheet. Use the available space on the page for your scratch work.

Notes

- All numbers used are real numbers.
- Calculators may be used.
- Some problems may be accompanied by figures or diagrams. These figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that a figure is not drawn to scale. The figures and diagrams are meant to provide information useful in solving the problem or problems. Unless otherwise stated, all figures and diagrams lie in a plane.
- A list of data that may be used for reference is included.

Analysis of Directions

1. All scratch work is to be done in the test booklet; get used to doing this because no scratch paper is allowed into the testing area.
2. You are looking for the one correct answer; therefore, although other answers may be close, there is never more than one right answer.

Suggested Approach with Samples

Circle or Underline

Take advantage of being allowed to mark on the test booklet by always underlining or circling what you are looking for. This will ensure that you are answering the right question.

Samples

1. If $x + 8 = 10$, then $4x + 1 =$

   A. 2
   B. 8
   C. 9
   D. 10
   E. 46

You should first circle or underline $4x + 1$ because this is what you are solving for. Solving for x leaves $x = 2$, then substituting into $4x + 1$ gives $4(2) + 1$, or 9. The most common mistake is to solve for x, which is 2, and *mistakenly choose* **A** as your answer. But remember, you are solving for $4x + 1$, not just x. You should also notice that most of the other choices would all be possible answers if you made common or simple mistakes. The correct answer is **C.** *Make sure that you are answering the right question.*

2. If $x^2 - y^2 = 8$ and $x^2 + y^2 = 4$, then what is the value of $x^4 - y^4$?

A. 4
B. 8
C. 12
D. 24
E. 32

First circle or underline $x^4 - y^4$.

Now factor this difference of two squares.

$$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$$

Next substitute in 8 for $x^2 - y^2$ and 4 for $x^2 + y^2$

$$= (8)(4)$$
$$= 32$$

The correct answer is **E.**

Pull Out Information

"Pulling" information out of the word problem structure can often give you a better look at what you are working with; therefore, you gain additional insight into the problem.

Samples

1. If the ratio of boys to girls in a drama class is 3 to 2, then which of the following is a possible number of students in the drama class?

A. 16
B. 18
C. 20
D. 24
E. 28

You should first circle or underline "possible number of students." Now pulling out information gives you the following.

$$b : g = 3 : 2$$

Since the ratio of boys to girls is 3 : 2, then the possible total number of students in the class must be a multiple of 3 + 2 (boys plus girls), or 5. The multiples of 5 are 5, 10, 15, 20, 25, ... Only Choice **C** is a multiple of 5. The correct answer is **C.**

2. Tom purchased a hat and coat for a total of $125. The coat costs $25 more than the hat. What is the cost of the coat?

A. $25
B. $50
C. $75
D. $100
E. $125

The key words here are *cost of the coat,* so circle those words. To solve algebraically,

$$x = \text{hat}$$
$$x + \$25 = \text{coat (cost \$25 more than the hat)}$$

Together they cost $125.

$$(x + 25) + x = 125$$
$$2x + 25 = 125$$
$$2x = 100$$
$$x = 50$$

But this is the cost of the hat. Notice that $50 is one of the answer choices, **B.** Since $x = 50$, then $x + 25 = 75$. Therefore, the coat costs $75, which is Choice **C.** *Always answer the question that is being asked.* Circling the key word or words will help you do that.

Work Forward

If you quickly see the method to solve the problem, then do the work. Work forward.

Samples

1. Which of the following numbers is between $\frac{2}{5}$ and $\frac{5}{8}$?

- **A.** .52
- **B.** .63
- **C.** .65
- **D.** .72
- **E.** .80

You should first underline or circle *between* $\frac{2}{5}$ and $\frac{5}{8}$? If you know that $\frac{2}{5}$ is .40 and $\frac{5}{8}$ is .625, you might have insight into the problem and should simply work forward. Or you could use your calculator and quickly find the decimals. Since .52 is the only number between, .40 and .625, the correct answer is **A.** By the way, a quick peek at the answer choices would tip you off that you should work in decimals.

2. If $3x - 6 < 3$, what are the possible values of x?

- **A.** $x < 4$
- **B.** $x < 2$
- **C.** $x > 0$
- **D.** $x > 3$
- **E.** $x < 3$

First circle or underline "possible values of *x*." Now solve the problem as follows:

	$3x - 6 < 3$
Add 6 to each side,	$\underline{+6 \quad +6}$
This gives	$3x < 9$
Now divide by 3,	$\frac{3x}{3} < \frac{9}{3}$
So	$x < 3$

The correct answer is **E.**

Work Backward

In some instances, it will be easier to work from the answers. Do not disregard this method, because it will at least eliminate some of the choices and could give you the correct answer.

Samples

1. If $\frac{x}{2} + \frac{3}{4} = \frac{5}{4}$, what is the value of x?

A. –2
B. –1
C. 0
D. 1
E. 2

If you cannot solve this algebraically, you may use the *work up from your choices* strategy. But start with **C**, 0. What if $x = 0$?

$$\frac{x}{2} + \frac{3}{4} = \frac{5}{4}$$
$$\frac{0}{2} + \frac{3}{4} \neq \frac{5}{4}$$

Since this answer is too small, try Choice **D,** a larger number.

Plugging in 1 gives.

$$\frac{x}{2} + \frac{3}{4} = \frac{5}{4}$$
$$\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$$

Change $\frac{1}{2}$ to $\frac{2}{4}$

$$\frac{2}{4} + \frac{3}{4} = \frac{5}{4}$$

The correct answer is **D.** *Working from the answers is a valuable technique.*

2. What is the greatest common factor of the numbers 18, 24, and 30?

A. 2
B. 3
C. 4
D. 6
E. 12

The largest number which divides evenly into 18, 24, and 30 is 6. You could have worked from the answers. But here you should start with the largest answer choice, since you're looking for the *greatest* common factor. The correct answer is **D.**

Use Your Calculator

Some questions will need to be completely worked out. If you don't see a fast method but do know that you could compute the answer, use your calculator.

1. What is the final cost of a television that sells for \$478.00 if the sales tax is 8%?

A. \$478.08
B. \$478.80
C. \$512.80
D. \$516.24
E. \$561.00

Since the sales tax is 8% of \$478.00,

$$8\% \text{ of } \$478.00 = (.08)(\$478.00) = \$38.24$$

The total cost of the television is therefore

$$\$478.00 + \$38.24 = \$516.24$$

The correct answer is **D.**

Your calculator would have helped with these calculations.

Substitute Simple Numbers

Substituting numbers for variables can often be an aid to understanding a problem. Remember to substitute simple numbers, since you have to do the work.

Samples

1. If x represents an even integer, then an odd integer is represented by which of the following?

A. $3x$
B. $2x + 1$
C. $3x + 2$
D. $4x - 2$
E. $5x - 4$

Since the question says that "x represents an even integer," substitute 2 for x. You should remember to circle "an odd integer" because that is what you are looking for. So as you plug 2 into each choice, you can stop when you get an odd integer.

$3x = 3(2) = 6$

$2x + 1 = 2(2) + 1 = 4 + 1 = 5$

Since 5 is an odd integer, the correct answer is **B.**

2. If $f(x) = 4^x$, which of the following CANNOT be a value of $f(x)$?

A. -4
B. $\frac{1}{4}$
C. 1
D. 4
E. 16

Notice that you are looking for "CANNOT be a value of $f(x)$." Try some simple numbers—0, 1, –1—and you will see that regardless of the values used for x, 4^x cannot be a negative number (or zero).

$$f(0) = 4^0 = 1$$
$$f(1) = 4^1 = 4$$
$$f(-1) = 4^{-1} = \frac{1}{4}$$

So $f(x) = 4^x > 0$, $f(x) \neq -4$

The correct answer is **A.**

Use 10 or 100

Some problems may deal with percent or percent change. If you don't see a simple method for working the problem, try using the values of 10 or 100 and see what you get.

1. If 40% of the students in a class have blue eyes, and 20% of those students with blue eyes have brown hair, then what percent of the original total number of students have brown hair and blue eyes?

A. 4%
B. 8%
C. 16%
D. 20%
E. 32%

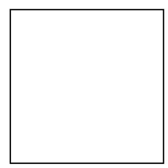
First circle or underline "percent of the original total number . . . brown hair . . . blue eyes." In this problem, if you don't spot a simple method, try starting with 100 students in the class. Since 40% of them have blue eyes, then 40 students have blue eyes. Now, the problem says 20% of those students with blue eyes have brown hair. Since 20% of 40 is 8, then 8 out of 100 (you started with 100 students), 8%, have blue eyes and brown hair. The correct answer is **B.**

2. Tom is building a square wooden framework to pour cement. His first frame is too small, so he increases each side by 20 percent. After careful measurement, he realizes this frame is too large, so he decreases each side by 10 percent. The area contained by his final wooden frame is what percent greater than the original wooden frame?

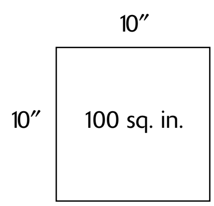
A. 10%
B. 10.8%
C. 16.64%
D. 20%
E. 40.44%

First circle or underline what you are looking for, in this case—*area . . . percent greater . . . than original.*

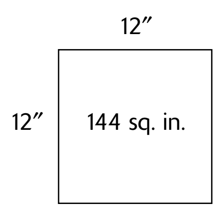
Next, draw the diagram.



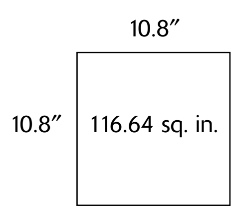
Now try some simple numbers. In this case 10":



Increasing this measurement by 20 percent gives a side of 12":



Decreasing this measurement by 10 percent gives 12 – 1.2 = 10.8. (10% × 12 = 1.2)



The area of the original was 100 sq. in. The area of the new figure is 116.64 sq. in. So the percent greater than the original would be 116.64 – 100 = 16.64 compared to the original 100 gives 16.64%. The correct answer is **C.** Your calculator could have been helpful in this problem.

Be Reasonable

Sometimes you will immediately recognize a simple method to solve a problem. If this is not the case, try a reasonable approach and then check the answers to see which one is most reasonable.

1. Will can complete a job in 30 minutes. Eli can complete the same job in 60 minutes. If they work together, approximately how many minutes will it take them to complete the job?

A. 90 minutes
B. 60 minutes
C. 45 minutes
D. 30 minutes
E. 20 minutes

First circle or underline "work together, approximately how many minutes." In a reasonable approach, you would reason that since Will can complete the job alone in 30 minutes, then if he received any help, that job should take less than 30 minutes. He's receiving a fair amount of help, so the answer must be well below 30 minutes. The only choice less than 30 is Choice **E,** 20.

Sketch a Diagram

Sketching diagrams or simple pictures can also be very helpful in problem solving because the diagram may tip off either a simple solution or a method for solving the problem.

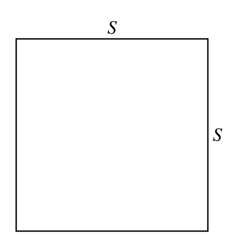
Samples

1. If all sides of a square are doubled, the area of that square

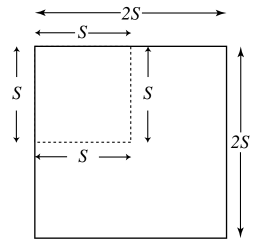
A. is doubled.
B. is tripled.
C. is multiplied by 4.
D. is multiplied by 8
E. remains the same.

One way to solve this problem is to draw a square and then double all its sides. Then compare the two areas.

Your first diagram



Doubling every side



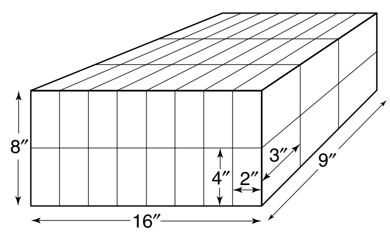
You can see that the total area of the new square will now be four times the original square. The correct answer is **C.**

2. What is the maximum number of milk cartons, each 2" wide by 3" long by 4" tall, that can be fit into a cardboard box with inside dimensions of 16" wide by 9" long by 8" tall?

A. 12
B. 18
C. 20
D. 24
E. 48

Drawing a diagram, as shown below, may be helpful in envisioning the process of fitting the cartons into the box. Notice that 8 cartons will fit across the box, 3 cartons deep, and two "stacks" high:

$$8 \times 3 \times 2 = 48 \text{ cartons}$$

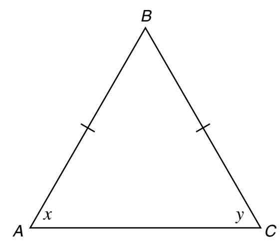


The correct answer is **E.**

Mark in Diagrams

Marking in or labeling diagrams as you read the questions can save you valuable time. Marking can also give you insight into how to solve a problem because you will have the complete picture clearly in front of you.

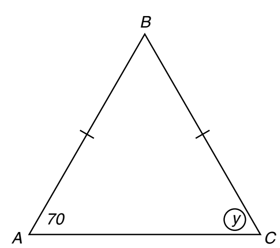
Samples



1. In the isosceles triangle above, $AB = BC$ and $\angle x$ is 70°. What is the measurement of angle y in degrees?

A. 50°
B. 60°
C. 70°
D. 100°
E. 130°

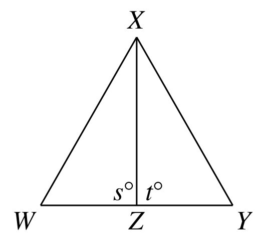
First, circle or underline what you are looking for. "$\angle y$." Next, in the isosceles triangle, immediately mark in that AB and BC are equal. Then mark in $\angle x$ as 70°. Since $AB = BC$, then $\angle x = \angle y$ (angles opposite equal sides are equal). After you marked in the information, your diagram should look like this.



The correct answer is **C. 70°.** Always *mark in diagrams as you read descriptions and information about them. This includes what you are looking for.*

Watch for Diagrams Not Drawn to Scale

Diagrams are drawn as accurately as possible, unless a diagram is labeled "not drawn to scale." That label is the tip-off that the diagram could be drawn differently or is out of proportion. In this case, mark the diagram and/or quickly redraw it differently. Marking and/or redrawing will give you insight into what information you really have about the diagram.

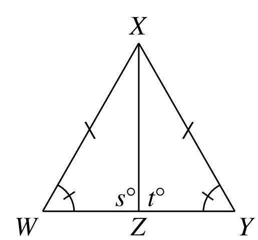


Note: Figure not drawn to scale.

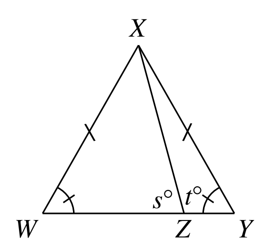
1. In △*WXY* above, *WX* = *XY*. Which of the following must be true?

A. *WZ* = *ZY*
B. *s* = *t*
C. perimeter of △*WXZ* = perimeter of △*ZXY*
D. area of △*WXZ* = area of △*ZXY*
E. ∠*XWZ* = ∠*XYZ*

Before doing anything else, underline or circle *must be true.* Now mark the diagram as follows.



Next, since the figure is *not* drawn to scale, quickly redraw it another way that still conforms to the given information.



Notice that by looking at the way the figure is initially drawn, you might think that *WZ* = *ZY* because they appear to be equal. But after you redraw the figure, you can see that *WZ* and *ZY* don't have to be equal, eliminating Choice **A.**

The same can be noticed of *s* and *t.* They don't have to be equal, eliminating **B.**

A quick look at the redrawn figure will help you eliminate Choice **C** as well, since it's evident that triangles *WXZ* and *ZXY* don't necessarily have equal perimeters.

You can also eliminate Choice **D** because even though the heights of triangles *WXZ* and *ZXY* are equal, their bases could be different, so the areas could be different. This fact is also evident from your redrawing of the diagram.

Choice **E** is the correct answer because in any triangle, equal angles are across from equal sides. Your markings in the figure remind you that this is true.

Glance at the Choices

Some problems may not ask you to solve for a numerical answer or even an answer including variables. Rather, you may be asked to set up the equation or expression without doing any solving. A quick glance at the answer choices will help you know what is expected.

Samples

Harold's age is 3 years less than half Sue's age.
If Harold is 9 years old, how old is Sue?

1. Suppose S represents Sue's age. Which of the following equations can be used to find Sue's age?

A. $9 = \frac{1}{2}(S) - 3$

B. $9 - 3 = \frac{1}{2}(S)$

C. $9 = 3 - \frac{1}{2}(S)$

D. $3 - 9 = \frac{1}{2}(S)$

E. $\frac{1}{2}(9) = S - 3$

Changing the word sentence into a number sentence (equation):

Harold's age is 3 years less than half Sue's age.

$$9 \quad = \quad \frac{1}{2}S - 3$$

The correct answer is **A.**

Grid-In Questions

The New SAT I grid-in question type is very similar to the familiar multiple-choice question except that you will now solve the problem and enter your answer by carefully marking the circles on a special grid. You will not be selecting from a group of possible answers.

Since you will not be selecting from a group of possible answers, you should be extra careful in checking and double-checking your answer. Your calculator can be useful in checking answers. Also, keep in mind that answers to grid-in questions are given either full credit or no credit. There is no partial credit. No points are deducted for incorrect answers in this section. That is, there is no penalty for guessing or attempting a grid-in, so at least take a guess.

Ability Tested

The grid-in questions test your ability to solve mathematical problems involving arithmetic, algebra I and II, geometry, data interpretation, basic statistics and probability, and word problems by using problem-solving insight, logic, and application of basic skills.

Basic Skills Necessary

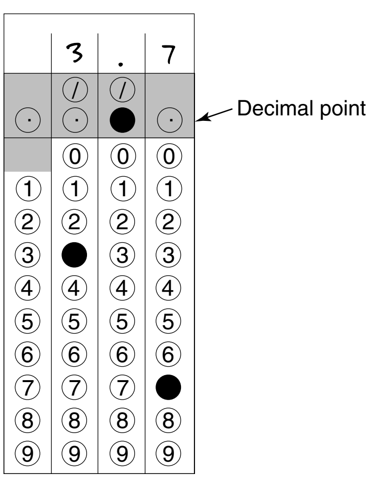
The basic skills necessary to do well on this question type include high school algebra I and II and intuitive or informal geometry. No calculus is necessary. Skills in arithmetic and basic algebra I and II, along with some logical insight into problem-solving situations, are also necessary to do well on this question type. Understanding the rules and procedures for gridding in answers is important.

Before you begin working grid-in questions, it is important that you become familiar with the grid-in rules and procedures and learn to grid accurately. Let's start explaining the rules and procedures by analyzing the directions.

Directions with Analysis

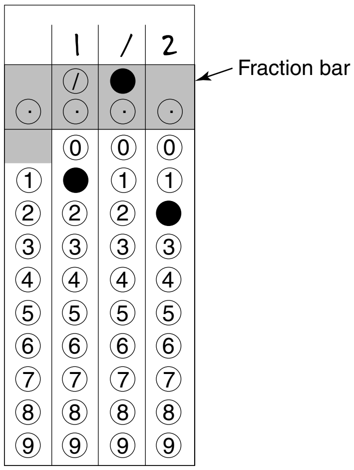
The following questions require you to solve the problem and enter your answer by carefully marking the circles on the special grid. Examples of the appropriate way to mark the grid follow. (Comments in parentheses have been added to help you understand how to grid properly.)

Answer: 3.7



(Notice that the decimal point is located in the shaded row, just above the numbers. Also notice that the answer has been written in above the gridding. You should always write in your answer, but the filled-in circles are most important because they are the ones scored.)

Answer: $\mathbf{\frac{1}{2}}$

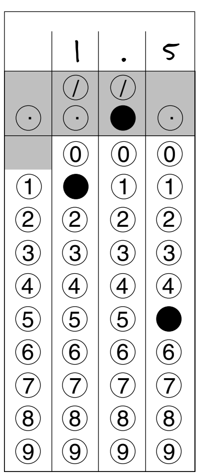


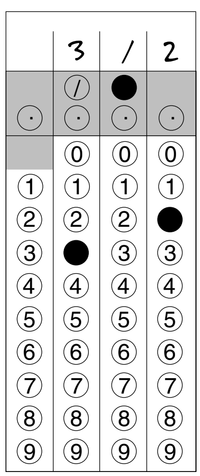
(Notice that the slash mark (/) indicates a fraction bar. This fraction bar is located in the shaded row and just above the decimal point in the two middle columns. Obviously, a fraction bar cannot be in the first or last column.)

Answer: $\mathbf{1\frac{1}{2}}$

Do not grid-in mixed numbers in the form of mixed numbers. **Always** change mixed numbers to improper fractions or decimals.

Change to 1.5 or **Change to** $\mathbf{\frac{3}{2}}$

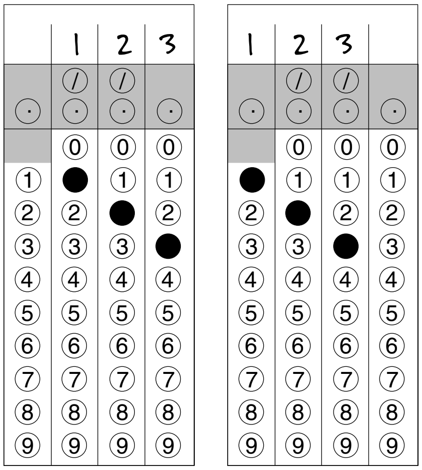




(Either an improper fraction or a decimal is acceptable. Never grid-in a mixed number because it will always be misread. For example, $1\frac{1}{2}$ will be read by the computer doing the scoring as $\frac{11}{2}$.)

Answer: 123

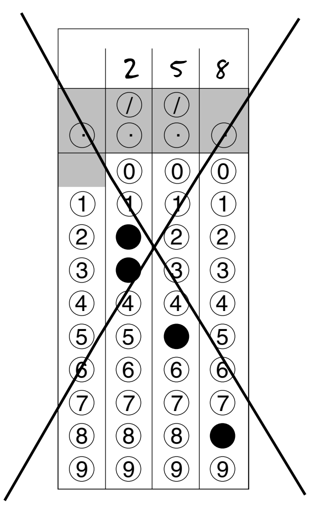
Space permitting, answers may start in any column. Each grid-answer below is correct.



(You should try to grid your answers from right to left, learning to be consistent as you practice. But space permitting, you may start in any column.)

Note: Circles must be filled in correctly to receive credit. Mark only one circle in each column. No credit will be given if more than one circle in a column is marked. Example:

Answer: 258 No credit!!!!

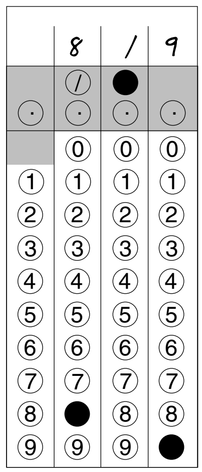


(Filling in more than one circle in a column is equivalent to selecting more than one answer in multiple choice. This type of answer fill-in will never receive any credit. Be careful to avoid this mistake.)

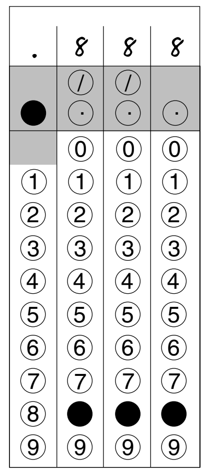
Answer: $\mathbf{\frac{8}{9}}$

Accuracy of decimals: Always enter the most accurate decimal value that the grid will accommodate. For example: An answer such as .8888 . . . can be gridded as .888 or .889. Gridding this value as .8, .88, or .89 is considered inaccurate and therefore not acceptable. The acceptable grid-ins of $\frac{8}{9}$ are

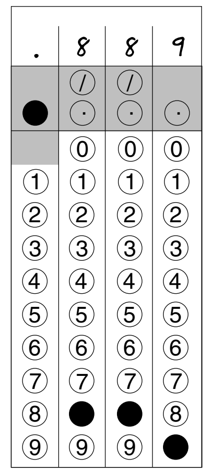
$\mathbf{\frac{8}{9}}$



.888



.889



(Review “accuracy of decimals” a second time. Notice that you must be as accurate as the grid allows.)

Be sure to write your answers in the boxes at the top of the circles before doing your gridding. Although writing out the answers above the columns is not required, it is very important to ensure accuracy. Even though some problems may have more than one correct answer, grid only one answer. Grid-in questions contain no negative answers.

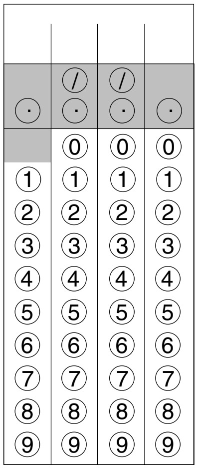
(Fractions can be reduced to lowest terms, but it is not required as long as they will fit in the grid. You are not required to grid a zero before a fraction. For example, either .2 or 0.2 is acceptable. If your answer is zero, you are required only to grid a zero in one column. Important: If you decide to change an answer, be sure to erase the old gridded answer completely.)

Practice Grid-Ins

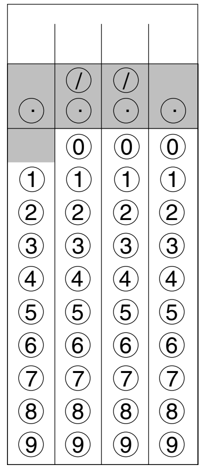
The following practice exercises will help you become familiar with the gridding process. Properly filled-in grids are given following each exercise. Hand write and grid-in the answers given.

Exercise

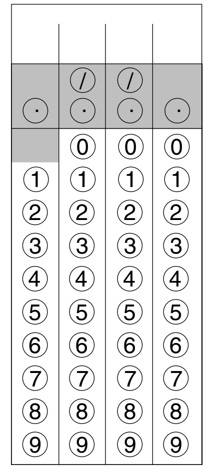
Answer: 2.4



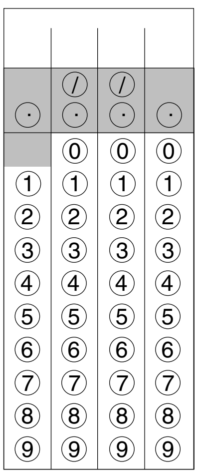
Answer: .8



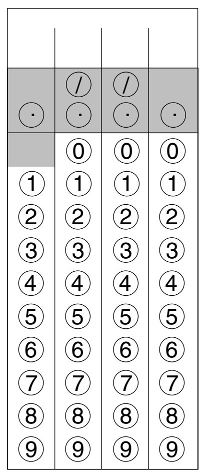
Answer: 20.5



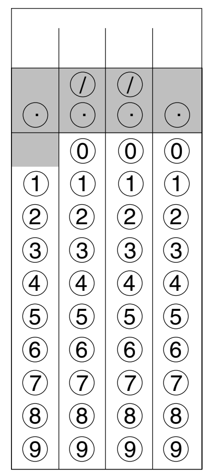
Answer: $\frac{2}{3}$



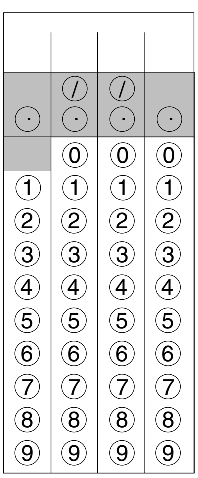
Answer: $\frac{4}{5}$



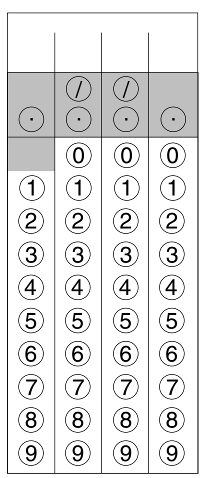
Answer: $\frac{9}{2}$



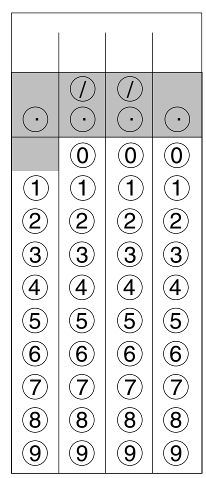
Answer: $5\frac{1}{4}$



Answer: $7\frac{1}{2}$

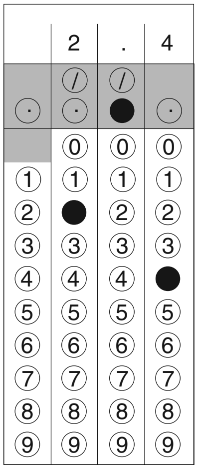


Answer: $3\frac{1}{7}$

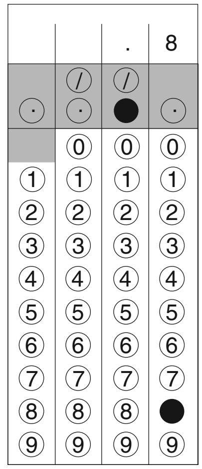


Answers to Exercise

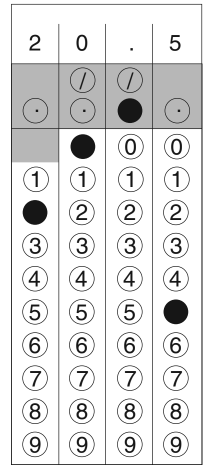
Answer: 2.4



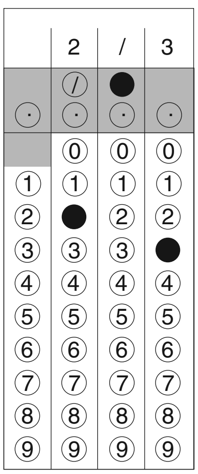
Answer: .8



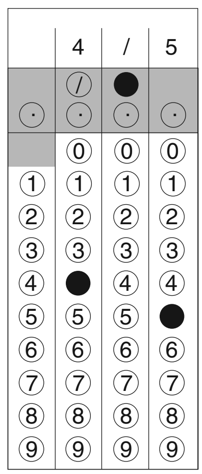
Answer: 20.5



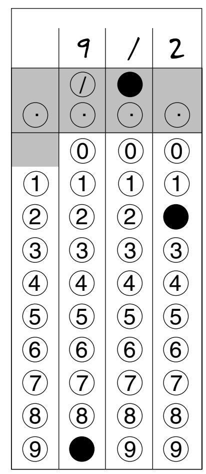
Answer: $\frac{2}{3}$



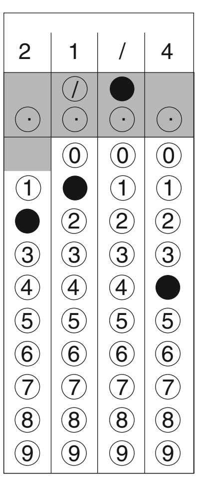
Answer: $\frac{4}{5}$



Answer: $\frac{9}{2}$

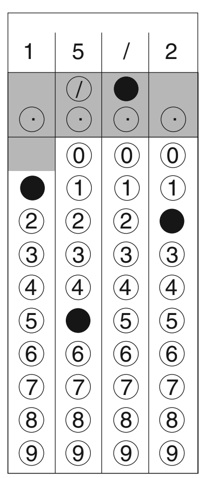


Answer: $5\frac{1}{4}$



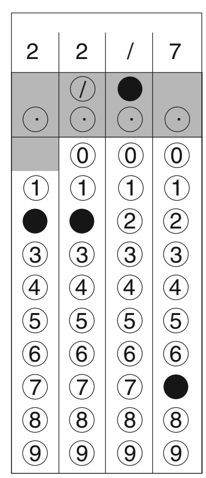
$5\frac{1}{4}$ must be changed to $\frac{21}{4}$ or 5.25

Answer: $7\frac{1}{2}$



$7\frac{1}{2}$ must be changed to $\frac{15}{2}$ or 7.5

Answer: $3\frac{1}{7}$



$3\frac{1}{7}$ must be changed to $\frac{22}{7}$ or 3.14

Suggested Approaches with Samples

Most of the following strategies, described and suggested in the multiple-choice section, will also work on grid-in questions.

- Circle or underline what you are looking for.
- Substitute numbers for variables to understand a problem.
- Try simple numbers to solve a problem.
- Pull information out of word problems.
- Draw/sketch diagrams or simple figures.
- Mark in diagrams.
- Use your calculator when appropriate to solve a problem.

You should also

- Make sure that your answer is reasonable.
- Jot down your scratch work or calculations in the space provided in your test booklet.
- Approximate or use your calculator to check your answers if time permits.

There are some specific items and strategies that should be noted for grid-in questions:

- There is no penalty for guessing on grid-in questions. Although it may be difficult to get an answer correct by simply writing in a wild guess, you should not be afraid to fill in your answer—even if you think it's wrong.
- Make sure to answer what is being asked. If the question asks for percent and you get an answer of 57%, grid-in 57, not .57. Or if a question asks for dollars and you get an answer of 75 cents, remember that 75 cents is .75 dollar. Grid-in .75 *not* 75.
- In some questions, more than one answer is possible. Grid-in only one answer. If you work out a question and get the answer $x > 5$, grid in 6, or 7, or 8 but *not more than one* of them.
- Some questions will have a note in parentheses ( ) that says, *Disregard the % sign when gridding your answer,* or *Disregard the $ sign when gridding your answer.* Follow the directions. If your answer is 68%, grid in 68, or if it's $95, grid in 95.
- Answers that are mixed numbers such as $3\frac{1}{2}$ or $7\frac{1}{4}$ must be changed to improper fractions $\left(3\frac{1}{2} = \frac{7}{2},\ 7\frac{1}{4} = \frac{29}{4}\right)$ or decimals $\left(3\frac{1}{2} = 3.5,\ 7\frac{1}{4} = 7.25\right)$ before being gridded. Improper fractions or decimals can be gridded. Mixed numbers cannot. The scoring system cannot distinguish between $3\frac{1}{2}$ and $\frac{31}{2}$.
- Since you cannot work from answer choices that are given or eliminate given choices, you will have to actually work out each answer. The use of your calculator on some problems in this section could enhance the speed and accuracy of your work.
- Writing in your answer in the space provided at the top of the grid is for your benefit only. It's wise to always write in your answer, but remember, the grid-in answer is the only one scored. Be sure to grid accurately and properly.

Following are some sample grid-in questions. Consider for each how you would grid-in the answer in the proper places on the answer sheet. Also consider when it would be appropriate to use a calculator to help you work out problems and to check the accuracy of your work.

Acceptable Answers

Fraction and decimal forms are acceptable, but not mixed numbers.

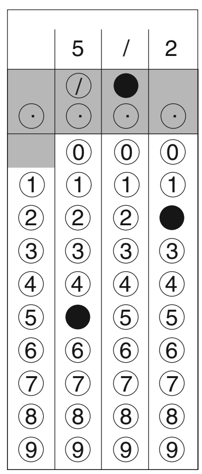
Sample

1. Let m # n be defined $m = -\left(\frac{1}{2}\right)n$. What is the value of 4 # 3?

This problem involves a made-up operation, or "false operation." Notice how it is defined.

$$\begin{aligned} m \;\#\; n = -\left(\frac{1}{2}\right)n &= 4 - \left(\frac{1}{2}\right)(3) \\ &= 4 - 1\frac{1}{2} \\ &= 2\frac{1}{2} \end{aligned}$$

But you can't grid in $2\frac{1}{2}$, so you must change it to either $\frac{5}{2}$ or 2.5 Now you can fill in the top and grid in the answer. Your grid-in would look like this.



The correct answer is $\frac{5}{2}$ or 2.5. (Either answer will receive full credit.)

Grid Carefully

Your gridding is what counts. Your written-in answer is for your benefit. Be accurate.

Sample

1. Acme Taxi lists the following rates on its door:

$1.20 for the first $\frac{1}{4}$ mile
$0.90 for each additional $\frac{1}{4}$ mile
$6.00 per hour for waiting

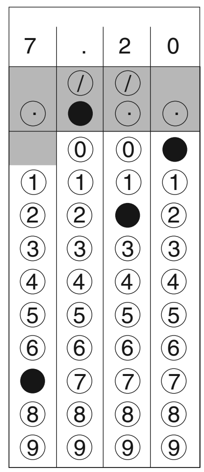
At these rates, if there was a 15-minute wait at the bank, how much will a 1.5-mile taxi trip cost? (Disregard the $ sign when gridding your answer.)

You should solve the problem as follows:

At \$6.00 per hour, a 15-minute ($\frac{1}{4}$ hour) wait will cost \$1.50. The first $\frac{1}{4}$ mile will cost \$1.20. The remaining $1\frac{1}{4}$, or $\frac{5}{4}$ miles will cost 5(.90) = \$4.50. The total bill will be the sum.

$$\$1.50 + \$1.20 + \$4.50 = \$7.20$$

The correct answer is \$7.20, gridded as 7.20. (You would **not** get credit for gridding 7.00, 7, 72, or 720, even if you had written the right answer above the grid circles.)



Answer the Question

Make sure that you answer the question that is being asked. Your answer must be in the units asked for.

Sample

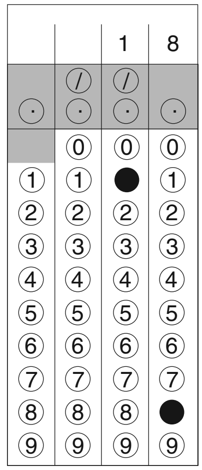
Lawns in Hill Crest Village		
Street Name	***Number of Lawns***	***Mowing Time Per Lawns***
Pine	3	25 minutes
Tamarind	7	30 minutes
Randall	9	35 minutes
Palmetto	12	40 minutes

2. How many hours will it take to mow all the lawns listed in the chart above?

First, underline or circle "how many hours" and "all the lawns." Next set up the information as follows and multiply (use your calculator if you wish.)

$$3 \times 25 = 75$$
$$7 \times 30 = 210$$
$$9 \times 35 = 315$$
$$12 \times 40 = 480$$

Now total the minutes and you get 1,080 minutes. But the question asks for "hours," so divide 1,080 by 60 and you get 18 hours. (Sometimes the units will be underlined in the question.) Your grid-in could look like the one that follows.



Grid Only One Answer

Even if more than one answer is possible, grid in only one answer.

Sample

3. If x and y are positive integers and $x + y > 10$, what is a possible value of x if $y > 5$?

Since $y > 5$, try plugging in 6 for y, and see what you get.

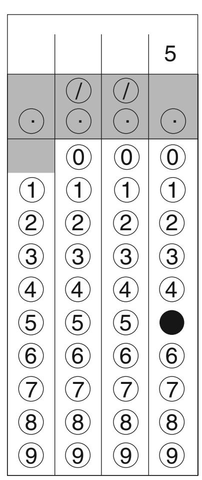
$$x + 6 > 10$$

Subtracting 6 from each side leaves

$$\begin{array}{r} x + 6 > 10 \\ \underline{-6 \quad -6} \\ x > 4 \end{array}$$

so, in this case, x is any integer greater than 4—that is 5, 6, 7, 8, . . . (if you assign a larger number to y, you'll get other possibilities for x.)

Now simply write in one answer, say 5, and then carefully mark it in the grid. Don't put in more than one possibility. Your grid could look like this.



PART II

REVIEW WITH SAMPLE PROBLEMS

A Quick Review of Mathematics

The following pages are designed to give you a quick review of some of the basic skills used on the math sections of the New SAT I. Before beginning the diagnostic review tests, it is wise to become familiar with basic mathematics terminology, formulas, and general mathematical information. These topics are covered in this chapter. Then proceed to the Arithmetic Diagnostic Test, which you should take to spot your weak areas. Then use the Arithmetic Review that follows to strengthen those areas.

Each review is followed by sample SAT-type questions with complete explanations for each topic area. The first three problems for each topic are multiple-choice and range (in order) from easy to difficult. The final problem is a sample grid-in problem. Carefully work through these problems.

After completing and reviewing the SAT-type questions for each topic, take the Algebra Diagnostic Test and again use the review that follows to strengthen your weak areas. Again, carefully work through the SAT-type questions that follow.

Next, take the Measurement and Geometry Diagnostic Test and carefully read the complete Measurement and Geometry Review. Again, carefully work through the SAT-type questions that follow.

Even if you are strong in the general topics—Numbers and Operations, Algebra and Functions, Geometry and Measurement, and Data Analysis, Statistics, and Probability—you might wish to skim the specific topic headings in each area to refresh your memory about important concepts. If you are weak in math, you should read through the complete review carefully.

Symbols, Terminology, Formulas, and General Mathematical Information

Common Math Symbols and Terms

Symbol References:	
$=$	is equal to
$\neq$	is not equal to
$>$	is greater than
$<$	is less than
$\geq$	is greater than or equal to
$\leq$	is less than or equal to
$\parallel$	is parallel to
$\perp$	is perpendicular to
Terms:	
Natural numbers	The counting numbers: 1, 2, 3, . . .
Whole numbers	The counting numbers beginning with zero: 0, 1, 2, 3, . . .
Integers	Positive and negative whole numbers and zero: . . . –3, –2, –1, 0, 1, 2, . . .
Odd number	Number not divisible by 2: 1, 3, 5, 7, . . .

(continued)

Even number	Number divisible by 2: 0, 2, 4, 6, . . .
Prime number	Number divisible by only 1 and itself: 2, 3, 5, 7, 11, 13, . . .
Composite number	Number divisible by more than just 1 and itself: 4, 6, 8, 9, 10, 12, 14, 15, . . . (0 and 1 are neither prime nor composite)
Square	The results when a number is multiplied by itself: $2 \times 2 = 4$; $3 \times 3 = 9$. Examples of squares are 1, 4, 9, 16, 25, 36, . . .
Cube	The results when a number is multiplied by itself twice: $2 \times 2 \times 2 = 8$; $3 \times 3 \times 3 = 27$. Examples of cubes are 1, 8, 27, 64 . . .

Math Formulas

Triangle	Perimeter = $s_1 + s_2 + s_3$
	Area = $\frac{1}{2}bh$
Square	Perimeter = $4s$
	Area = $s \cdot s$, or s^2
Rectangle	Perimeter = $2(b + h)$, or $2b + 2h$
	Area = bh, or lw
Parallelogram	Perimeter = $2(l + w)$, or $2l + 2w$
	Area = bh
Trapezoid	Perimeter = $b_1 + b_2 + s_1 + s_2$
	Area = $\frac{1}{2}h\left(b_1 + b_2\right)$, or $h\left(\frac{b_1 + b_2}{2}\right)$
Circle	Circumference = $2\pi r$, or πd
	Area = πr^2
Cube	Volume = $s \cdot s \cdot s = s^3$
	Surface area = $s \cdot s \cdot 6$
Rectangular Prism	Volume = $l \cdot w \cdot h$
	Surface area = $2(lw) + 2(lh) + 2(wh)$

Pythagorean theorem ($a^2 + b^2 = c^2$): The sum of the square of the legs of a right triangle equals the square of the hypotenuse.

Important Equivalents

$$\frac{1}{100} = 0.1 = 1\%$$

$$\frac{1}{10} = .1 = .10 = 10\%$$

$$\frac{1}{5} = \frac{2}{10} = .2 = .20 = 20\%$$

$\frac{3}{10} = .3 = .30 = 30\%$

$\frac{2}{5} = \frac{4}{10} = .4 = .40 = 40\%$

$\frac{1}{2} = \frac{5}{10} = .5 = .50 = 50\%$

$\frac{3}{5} = \frac{6}{10} = .6 = .60 = 60\%$

$\frac{7}{10} = .7 = .70 = 70\%$

$\frac{4}{5} = \frac{8}{10} = .8 = .80 = 80\%$

$\frac{9}{10} = .9 = .90 = 90\%$

$\frac{1}{4} = \frac{25}{100} = .25 = 25\%$

$\frac{3}{4} = \frac{75}{100} = .75 = 75\%$

$\frac{1}{3} = .33\frac{1}{3} = 33\frac{1}{3}\%$

$\frac{2}{3} = .66\frac{2}{3} = 66\frac{2}{3}\%$

$\frac{1}{8} = .125 = .12\frac{1}{2} = 12\frac{1}{2}\%$

$\frac{3}{8} = .375 = .37\frac{1}{2} = 37\frac{1}{2}\%$

$\frac{5}{8} = .625 = .62\frac{1}{2} = 62\frac{1}{2}\%$

$\frac{7}{8} = .875 = .87\frac{1}{2} = 87\frac{1}{2}\%$

$\frac{1}{6} = .16\frac{2}{3} = 16\frac{2}{3}\%$

$\frac{5}{6} = .83\frac{1}{3} = 83\frac{1}{3}\%$

$1 = 1.00 = 100\%$

$2 = 2.00 = 200\%$

$3\frac{1}{2} = 3.5 = 3.50 = 350\%$

Math Words and Phrases

Words that signal an operation:

Addition

- Sum
- Total
- Plus
- Increase
- More than
- Greater than

Multiplication

- Of
- Product
- Times
- At (sometimes)
- Total (sometimes)

Subtraction

- Difference
- Less
- Decreased
- Reduced
- Fewer
- Have left

Division

- Quotient
- Divisor
- Dividend
- Ratio
- Parts

Mathematical Properties

Some Properties (Axioms) of Addition

Commutative **means that the *order* does not make any difference.**

$2 + 3 = 3 + 2$

$a + b = b + a$

Note: The commutative property does *not* hold for subtraction.

$3 - 1 \neq 1 - 3$

$a - b \neq b - a$

Associative **means that the *grouping* does not make any difference.**

$(2 + 3) + 4 = 2 + (3 + 4)$

$(a + b) + c = a + (b + c)$

The grouping has changed (parentheses moved), but the sides are still equal.

Note: The associative property does *not* hold for subtraction.

$4 - (3 - 1) \neq (4 - 3) - 1$

$a - (b - c) \neq (a - b) - c$

The *identity element* for addition is 0. Any number added to 0 gives the original number.

$3 + 0 = 3$
$a + 0 = a$

The *additive inverse* is the opposite (negative) of the number. Any number plus its additive inverse equals 0 (the identity).

$3 + (-3) = 0$; therefore, 3 and -3 are inverses.
$-2 + 2 = 0$; therefore, -2 and 2 are inverses.
$a + (-a) = 0$; therefore, a and $-a$ are inverses.

Some Properties (Axioms) of Multiplication

***Commutative* means that the *order* does not make any difference.**

$2 \times 3 = 3 \times 2$
$a \times b = b \times a$

Note: The commutative property does *not* hold for division.

$2 \div 4 \neq 4 \div 2$

***Associative* means that the *grouping* does not make any difference.**

$(2 \times 3) \times 4 = 2 \times (3 \times 4)$
$(a \times b) \times c = a \times (b \times c)$

The grouping has changed (parentheses moved), but the sides are still equal.

Note: The associative property does *not* hold for division.

$(8 \div 4) \div 2 \neq 8 \div (4 \div 2)$

The *identity element* for multiplication is 1. Any number multiplied by 1 gives the original number.

$3 \times 1 = 3$
$a \times 1 = a$

The *multiplicative inverse* is the reciprocal of the number. Any number multiplied by its reciprocal equals 1.

$$2 \times \frac{1}{2} = 1; \text{ therefore, } 2 \text{ and } \frac{1}{2} \text{ are inverses.}$$
$$a \times \frac{1}{a} = 1; \text{ therefore, } a \text{ and } \frac{1}{a} \text{ are inverses.}$$

A Property of Two Operations

The *distributive property* is the process of distributing the number on the outside of a set of parentheses to each number on the inside.

$2(3 + 4) = 2(3) + 2(4)$

$a(b + c) = a(b) + a(c)$

Note: You cannot use the distributive property with only one operation.

$3(4 \times 5 \times 6) \neq 3(4) \times 3(5) \times 3(6)$

$a(bcd) \neq a(b) \times a(c) \times a(d)$ or $(ab)(ac)(ad)$

Numbers and Operations, Data Analysis, Statistics, and Probability

Arithmetic Diagnostic Test (Including Numbers and Operations, Data Analysis, Statistics, and Probability)

Questions

1. 28×39 is approximately:

2. $6 = \frac{?}{4}$

3. Change $5\frac{3}{4}$ to an improper fraction.

4. Change $\frac{32}{6}$ to a whole number or mixed number in lowest terms.

5. $\frac{2}{5} + \frac{3}{5} =$

6. $1\frac{3}{8} + 2\frac{5}{6} =$

7. $\frac{7}{9} - \frac{5}{9} =$

8. $11 - \frac{2}{3} =$

9. $6\frac{1}{4} - 3\frac{3}{4} =$

10. $\frac{1}{6} \times \frac{1}{6} =$

11. $2\frac{3}{8} \times 1\frac{5}{6} =$

12. $\frac{1}{4} \div \frac{3}{2} =$

13. $2\frac{3}{7} \div 1\frac{1}{4} =$

14. $.07 + 1.2 + .471 =$

15. $.45 - .003 =$

16. $\$78.24 - \$31.68 =$

17. $.5 \times .5 =$

18. $8.001 \times 2.3 =$

19. $.7\overline{).147} =$

20. $.002\overline{)12} =$

21. $\frac{1}{3}$ of $7.20 =

22. Circle the larger number: 7.9 or 4.35.

23. 39 out of 100 means:

24. Change 4% to a decimal.

25. 46% of 58 =

26. Change .009 to a percent.

27. Change 12.5% to a fraction.

28. Change $\frac{3}{8}$ to a percent.

29. Is 93 prime?

30. What is the percent increase in a rise in temperature from 80° to 100°?

31. $-6 + 8 =$

32. $-7 \times -9 =$

33. $|-9| =$

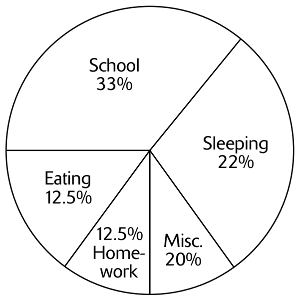
34. $8^2 =$

35. $3^2 \times 3^5 =$

36. The square root of 30 is approximately equal to:

37. What is the median of the numbers 4, 3, 5, 7, 9, and 1?

38. Based on the following circle graph, how much time does Timmy spend doing homework?



How Timmy Spends His 24-Hour Day

Answers

1. 1,200

2. 24

3. $\frac{23}{4}$

4. $5\frac{2}{6}$ or $5\frac{1}{3}$

5. $\frac{5}{5}$ or 1

6. $4\frac{5}{24}$

7. $\frac{2}{9}$

8. $10\frac{1}{3}$

9. $2\frac{2}{4}$ or $2\frac{1}{2}$

10. $\frac{1}{36}$

11. $\frac{209}{48}$ or $4\frac{17}{48}$

12. $\frac{1}{6}$

13. $\frac{68}{35}$ or $1\frac{33}{35}$

14. 1.741

15. .447

16. \$46.56

17. .25

18. 18.4023

19. .21

20. 6,000

21. \$2.40

22. 7.9

23. 39% or $\frac{39}{100}$

24. .04

25. 26.68

26. .9% or $\frac{9}{10}$%

27. $\frac{125}{1000}$ or $\frac{1}{8}$

28. 37.5% or $37\frac{1}{2}$%

29. No.

30. 25%

31. 2

32. 63

33. 9

34. 64

35. 3^7

36. 5.5 or $5\frac{1}{2}$

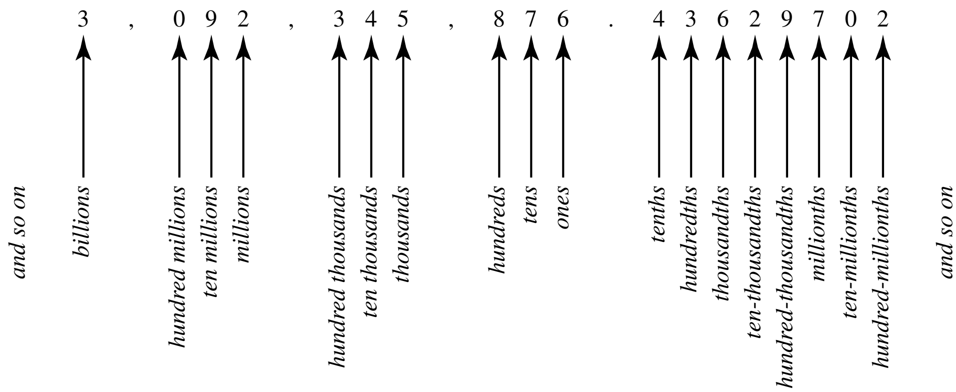
37. $4\frac{1}{2}$ or 4.5

38. 3 hours

Arithmetic Review

Place Value

Each position in any number has *place value.* For instance, in the number 485, 4 is in the hundreds place, 8 is in the tens place, and 5 is in the ones place. Thus, place value is as follows:



Rounding Off

To round off any number:

1. Underline the place value to which you're rounding off.
2. Look to the immediate right (one place) of your underlined place value.
3. Identify the number (the one to the right). If it is 5 or higher, round your underlined place value up 1. If the number (the one to the right) is 4 or less, leave your underlined place value as it is and change all the other numbers to its right to zeros. For example:

Round to the nearest thousand:

> $34\underline{5},678$ becomes 346,000
>
> $92\underline{8},499$ becomes 928,000

This works with decimals as well. Round to the nearest hundredth:

> $3.4\underline{6}78$ becomes 3.47
>
> $298,435.0\underline{8}3$ becomes 298,435.08

Estimating Sums, Differences, Products, and Quotients

Knowing how to approximate or estimate not only saves you time but can also help you check your answer to see whether it is reasonable.

Estimating Sums

Use rounded numbers to estimate sums. For example, give an estimate for the sum 3,741 + 5,021 rounded to the nearest thousand.

$$\begin{array}{ll} & 3{,}741 + 5{,}021 \\ & \quad\downarrow \qquad\ \downarrow \\ & 4{,}000 + 5{,}000 = 9{,}000 \\ \text{So} & 3{,}741 + 5{,}021 \approx 9{,}000 \end{array}$$

Note: The symbol ≈ means is approximately equal to.

Estimating Differences

Use rounded numbers to estimate differences. For example, give an estimate for the difference 317,753 – 115,522 rounded to the nearest hundred thousand.

$$\begin{array}{ll} & 317{,}753 - 115{,}522 \\ & \quad\downarrow \qquad\quad \downarrow \\ & 300{,}000 - 100{,}000 = 200{,}000 \\ \text{So} & 317{,}753 - 115{,}522 \approx 200{,}000 \end{array}$$

Estimating Products

Use rounded numbers to estimate products. For example, estimate the product of 722 × 489 by rounding to the nearest hundred.

$$\begin{array}{ll} & 722 \times 489 \\ & \ \downarrow \qquad \downarrow \\ & 700 \times 500 = 350{,}000 \\ \text{So} & 722 \times 489 \approx 350{,}000 \end{array}$$

If both multipliers end in 50 or are halfway numbers, then rounding one number up and one number down gives a better estimate of the product. For example, estimate the product of 650 × 350 by rounding to the nearest hundred.

$$\begin{array}{lrl} & & 650 \times 350 \\ & & \ \downarrow \qquad \downarrow \\ & \text{Round one number up and one down.} & 700 \times 300 = 210{,}000 \\ \text{So} & & 650 \times 300 \approx 210{,}000 \end{array}$$

You can also round the first number down and the second number up and get this estimate:

$$\begin{array}{ll} & 650 \times 350 \\ & \ \downarrow \qquad \downarrow \\ & 600 \times 400 = 240{,}000 \\ \text{So} & 650 \times 350 \approx 240{,}000 \end{array}$$

In either case, this approximation is closer than if you round both numbers up, which is the standard rule.

Estimating Quotients

Use rounded numbers to estimate quotients. For example, estimate the quotient of 891 ÷ 288 by rounding to the nearest hundred.

$$\begin{array}{ll} & 891 \div 288 \\ & \ \downarrow \qquad \downarrow \\ & 900 \div 300 = 3 \\ \text{So} & 891 \div 288 \approx 3 \end{array}$$

Fractions

Fractions consist of two numbers: a *numerator* (which is above the line) and a *denominator* (which is below the line).

$$\frac{1}{2}\frac{\text{numerator}}{\text{denominator}} \text{ or numerator } 1/2 \text{ denominator}$$

The denominator indicates the number of equal parts into which something is divided. The numerator indicates how many of these equal parts are contained in the fraction. Thus, if the fraction is $\frac{3}{5}$ of a pie, then the denominator, 5, indicates that the pie has been divided into 5 equal parts, of which 3 (the numerator) are in the fraction.

Sometimes it helps to think of the dividing line (in the middle of a fraction) as meaning *out of.* In other words, $\frac{3}{5}$ also means 3 *out of* 5 equal pieces from the whole pie.

Common Fractions and Improper Fractions

A fraction like $\frac{3}{5}$, where the numerator is smaller than the denominator, is less than one. This kind of fraction is called a *common fraction.*

Sometimes a fraction represents more than one. This is when the numerator is larger than the denominator. Thus, $\frac{12}{7}$ is more than one. This is called an *improper fraction.*

Mixed Numbers

When a term contains both a whole number (such as 3, 8, or 25) and a fraction (such as $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{4}$), it is called a *mixed number.* For example, $5\frac{1}{4}$ and $290\frac{3}{4}$ are both mixed numbers.

To change an improper fraction to a mixed number, divide the denominator into the numerator. For example:

$$\frac{18}{5} = 3\frac{3}{5} \qquad \begin{array}{r} 3 \\ 5\overline{)18} \\ \underline{15} \\ 3 \end{array}$$

To change a mixed number to an improper fraction, multiply the denominator by the whole number, add the numerator and put the total over the original denominator. For example:

$$4\frac{1}{2} = \frac{9}{2} \qquad 2 \times 4 + 1 = 9$$

Reducing Fractions

A fraction must be reduced to *lowest terms.* This is done by dividing both the numerator and the denominator by the largest number that divides evenly into both. For example, $\frac{14}{16}$ is reduced by dividing both terms by 2, giving $\frac{7}{8}$. Likewise, $\frac{20}{25}$ is reduced to $\frac{4}{5}$ by dividing both the numerator and denominator by 5.

Adding Fractions

To add fractions, first change all denominators to their *lowest common denominator* (LCD)—the lowest number that can be divided evenly by all the denominators in the problem. When all the denominators are the same, add fractions by simply adding the numerators (the denominator remains the same). For example:

$$\begin{array}{rl} \frac{3}{8} = \frac{3}{8} & \\ +\frac{1}{2} = \frac{4}{8} & \longleftarrow \left\{\text{one-half is changed to four-eighths}\right. \\ \hline \frac{7}{8} & \end{array} \qquad \begin{array}{rl} \frac{1}{4} = \frac{3}{12} & \\ +\frac{1}{3} = \frac{4}{12} & \longleftarrow \left\{\text{change both fractions to LCD of 12}\right. \\ \hline \frac{7}{12} & \end{array}$$

In the first example, you have to change the $\frac{1}{2}$ to $\frac{4}{8}$ because 8 is the LCD. Then you need to add the numerators 3 and 4 to get $\frac{7}{8}$. In the second example, you must change both fractions to get the LCD of 12, and then add the numerators to get $\frac{7}{12}$. Of course, if the denominators are already the same, just add the numerators. For example:

$$\frac{6}{11} + \frac{3}{11} = \frac{9}{11}$$

Adding Mixed Numbers

To add mixed numbers, the same rule (find the LCD) applies, but always add the whole number to get your final answer. For example:

$$\begin{array}{rl} 2\tfrac{1}{2} = 2\tfrac{2}{4} & \longleftarrow \left\{\text{change one-half to two-fourths}\right. \\ +\,3\tfrac{1}{4} = 3\tfrac{1}{4} & \\ \hline 5\tfrac{3}{4} & \longleftarrow \left\{\text{remember to add the whole numbers}\right. \end{array}$$

Subtracting Fractions

To subtract fractions, the same rule (find the LCD) applies, except subtract the numerators. For example:

$$\begin{array}{r} \frac{7}{8} = \frac{7}{8} \\ -\frac{1}{4} = \frac{2}{8} \\ \hline \frac{5}{8} \end{array} \qquad \begin{array}{r} \frac{3}{4} = \frac{9}{12} \\ -\frac{1}{3} = \frac{4}{12} \\ \hline \frac{5}{12} \end{array}$$

Subtracting Mixed Numbers

When you subtract mixed numbers, sometimes you have to borrow from the whole number, just like you sometimes borrow from the next column when subtracting ordinary numbers. For example:

$$\begin{array}{r} 6\overset{4}{\cancel{5}}\overset{11}{1} \\ -\,129 \\ \hline 522 \end{array} \qquad \begin{array}{r} \overset{3\frac{7}{6}}{\cancel{4}\tfrac{1}{6}} \\ -\,2\tfrac{5}{6} \\ \hline 1\tfrac{2}{6} = 1\tfrac{1}{3} \end{array}$$

you borrowed 1 from the tens column

you borrowed one in the form $\frac{6}{6}$ from the ones column

To subtract a mixed number from a whole number, you have to borrow from the whole number. For example:

$$\begin{array}{rl} 6 & = 5\tfrac{5}{5} \\ -3\tfrac{1}{5} & = 3\tfrac{1}{5} \\ \hline & \quad 2\tfrac{4}{5} \end{array}$$

borrow one in the form of 5⁄5 from the 6

remember to subtract the remaining whole numbers

Multiplying Fractions

Simply multiply the numerators, and then multiply the denominators. Reduce to lowest terms if necessary. For example:

$$\frac{2}{3} \times \frac{5}{12} = \frac{10}{36} \qquad \text{reduce } \frac{10}{36} \text{ to } \frac{5}{18}$$

This answer had to be reduced because it wasn't in lowest terms.

Canceling When Multiplying Fractions

You could have *canceled* first. Canceling eliminates the need to reduce your answer. To cancel, find a number that divides evenly into one numerator and one denominator. In this case, 2 divides evenly into 2 in the numerator (it goes in one time) and 12 in the denominator (it goes in 6 times). Thus:

$$\frac{\cancel{2}^{1}}{3} \times \frac{5}{\cancel{12}_{6}} =$$

Now that you've canceled, you can multiply as before.

$$\frac{\cancel{2}^{1}}{3} \times \frac{5}{\cancel{12}_{6}} = \frac{5}{18}$$

You can cancel only when *multiplying* fractions.

Multiplying Mixed Numbers

To multiply mixed numbers, first change any mixed number to an improper fraction. Then multiply as previously shown. To change mixed numbers to improper fractions:

1. Multiply the whole number by the denominator of the fraction.
2. Add this to the numerator of the fraction.
3. This is now your numerator.
4. The denominator remains the same.

$$3\frac{1}{3} \times 2\frac{1}{4} = \frac{10}{3} \times \frac{9}{4} = \frac{90}{12} = 7\frac{6}{12} = 7\frac{1}{2}$$

Then, change the answer, if it is in improper form, back to a mixed number and reduce if necessary.

Dividing Fractions

To divide fractions, invert (turn upside down) the second fraction and multiply. Then, reduce if necessary. For example:

$$\frac{1}{6} \div \frac{1}{5} = \frac{1}{6} \times \frac{5}{1} = \frac{5}{6} \qquad \frac{1}{6} \div \frac{1}{3} = \frac{1}{6} \times \frac{3}{1} = \frac{1}{2}$$

Simplifying Fractions

If either numerator or denominator consists of several numbers, these numbers must be combined into one number. Then, reduce if necessary. For example:

$$\frac{28+14}{26+17}=\frac{42}{43} \text{ or}$$

$$\frac{\frac{1}{4}+\frac{1}{2}}{\frac{1}{3}+\frac{1}{4}}=\frac{\frac{1}{4}+\frac{2}{4}}{\frac{4}{12}+\frac{3}{12}}=\frac{\frac{3}{4}}{\frac{7}{12}}=\frac{3}{4}\times\frac{12}{7}=\frac{36}{28}=\frac{9}{7}=1\frac{2}{7}$$

Decimals

Fractions can also be written in decimal form by using a symbol called a decimal point. All numbers to the left of the decimal point are whole numbers. All numbers to the right of the decimal point are fractions with denominators of only 10, 100, 1,000, 10,000, and so on, as follows:

$$.6=\frac{6}{10}=\frac{3}{5}$$

$$.7=\frac{7}{10}$$

$$.07=\frac{7}{100}$$

$$.007=\frac{7}{1000}$$

$$.0007=\frac{7}{10000}$$

$$.25=\frac{25}{100}=\frac{1}{4}$$

Adding and Subtracting Decimals

To add or subtract decimals, just line up the decimal points and then add or subtract in the same manner as when adding or subtracting regular numbers. For example:

$$23.6+1.75+300.002= \qquad \begin{array}{r} 23.6 \\ 1.75 \\ \underline{300.002} \\ 325.352 \end{array}$$

Adding zeros can make the problem easier to work:

$$\begin{array}{r} 23.600 \\ 1.750 \\ \underline{300.002} \\ 325.352 \end{array}$$

and

$$54.26-1.1= \qquad \begin{array}{r} 54.26 \\ \underline{-1.10} \\ 53.16 \end{array}$$

and

$$78.9-37.43= \qquad \begin{array}{r} 78.\overset{8}{\cancel{9}}\,{}^{1}0 \\ \underline{-37.43} \\ 41.47 \end{array}$$

Whole numbers can have decimal points to their right. For example:

$$17 - 8.43 = \begin{array}{r} 1\overset{6}{\cancel{7}}.\overset{9}{\cancel{0}}{}^{1}0 \\ -8.43 \\ \hline 8.57 \end{array}$$

Multiplying Decimals

To multiply decimals, multiply as usual. Then, count the total number of digits above the line that are to the right of all decimal points. Place the decimal point in the answer so that the number of digits to the right of the decimal is the same as it is above the line. For example:

$$\begin{array}{rl} 40.012 & \longleftarrow \text{3 digits} \\ \times \quad 3.1 & \longleftarrow \text{1 digit} \\ \hline 40012 & \\ 120036 & \\ \hline 124.0372 & \longleftarrow \text{4 digits} \end{array} \quad \begin{array}{l} \left\{ \begin{array}{l} \text{total of 4 digits above the line that} \\ \text{are to the right of the decimal point} \end{array} \right. \\ \\ \left\{ \begin{array}{l} \text{decimal point placed so there is} \\ \text{same number of digits to the right} \\ \text{of the decimal point} \end{array} \right. \end{array}$$

Dividing Decimals

Dividing decimals is the same as dividing other numbers, except that when the divisor (the number you're dividing by) has a decimal, move it to the right as many places as necessary until it is a whole number. Then move the decimal point in the dividend (the number being divided into) the same number of places. Sometimes you have to add zeros to the dividend (the number inside the division sign).

$$1.25\overline{)5.} = 125\overset{\displaystyle 4.}{\overline{)500.}}$$

or

$$0.002\overline{)26.} = 2\overset{\displaystyle 13000.}{\overline{)26000.}}$$

Conversions

Changing Decimals to Percents

To change decimals to percents:

1. Move the decimal point two places to the right.
2. Insert a percent sign.

$$.75 = 75\%$$
$$.05 = 5\%$$

Changing Percents to Decimals

To change percents to decimals:

1. Eliminate the percent sign.
2. Move the decimal point two places to the left. (Sometimes adding zeros is necessary.)

$$75\% = .75$$
$$5\% = .05$$
$$23\% = .23$$
$$.2\% = .002$$

Changing Fractions to Percents

To change a fraction to a percent:

1. Multiply by 100.
2. Insert a percent sign.

$$\frac{1}{2} = \left(\frac{1}{2}\right) \times 100 = \frac{100}{2} = 50\%$$

$$\frac{2}{5} = \left(\frac{2}{5}\right) \times 100 = \frac{200}{5} = 40\%$$

Changing Percents to Fractions

To change percents to fractions:

1. Divide the percent by 100.
2. Eliminate the percent sign.
3. Reduce if necessary.

$$60\% = \frac{60}{100} = \frac{3}{5} \qquad 13\% = \frac{13}{100}$$

Changing Fractions to Decimals

To change a fraction to a decimal, simply do what the operation says. In other words, $\frac{13}{20}$ means 13 divided by 20. So, do just that. (Insert decimal points and zeros accordingly.)

$$20\overset{\Large .65}{\overline{)13.00}} = .65 \qquad \frac{5}{8} = \qquad 8\overset{\Large .625}{\overline{)5.000}} = .625$$

Changing Decimals to Fractions

To change a decimal to a fraction:

1. Move the decimal point two places to the right.
2. Put that number over 100.
3. Reduce if necessary.

$$.65 = \frac{65}{100} = \frac{13}{20}$$

$$.05 = \frac{5}{100} = \frac{1}{20}$$

$$.75 = \frac{75}{100} = \frac{3}{4}$$

Read it: .8

Write it: $\frac{8}{10}$

Reduce it: $\frac{4}{5}$

Using Percents

Finding the Percent of a Number

To determine the percent of a number, change the percent to a fraction or decimal (whichever is easier for you) and multiply. The word *of* means multiply.

For example:

1. What is 20% of 80?

$$\left(\frac{20}{100}\right) \times 80 = \frac{1600}{100} = 16 \text{ or } .20 \times 80 = 16.00 = 16$$

2. What is 12% of 50?

$$\left(\frac{12}{100}\right) \times 50 = \frac{600}{100} = 6 \text{ or } .12 \times 50 = 6.00 = 6$$

3. What is $\frac{1}{2}$% of 18?

$$\frac{\frac{1}{2}}{100} \times 18 = \left(\frac{1}{200}\right) \times 18 = \frac{18}{200} = \frac{9}{100} \text{ or } .005 \times 18 = .09$$

Other Applications of Percent

Turn the question (word for word) into an equation. For *what* substitute the letter *x;* for *is* substitute an *equal sign;* for *of* substitute a *multiplication sign.* Change percents to decimals or fractions, whichever you find easier. Then solve the equation.

For example:

1. 18 is what percent of 90?

$$\begin{aligned} 18 &= x(90) \\ \frac{18}{90} &= x \\ \frac{1}{5} &= x \\ 20\% &= x \end{aligned}$$

2. 10 is 50% of what number?

$$\begin{aligned} 10 &= .50(x) \\ \frac{10}{.50} &= x \\ 20 &= x \end{aligned}$$

3. What is 15% of 60?

$$x = \left(\frac{15}{100}\right) \times 60 = \frac{900}{100} = 9$$

$$\text{or } .15(60) = 9$$

Percentage Increase or Decrease

To find the *percentage change* (increase or decrease), use this formula:

$$\frac{\text{change}}{\text{starting point}} \times 100 = \text{percentage change}$$

For example:

1. What is the percentage decrease of a $500 item on sale for $400?

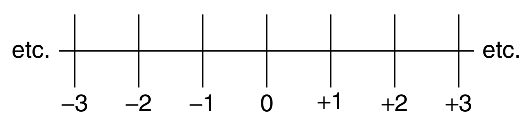
$$\frac{\text{change}}{\text{starting point}} \times 100 = \frac{100}{500} \times 100 = \frac{1}{5} \times 100 = 20\% \text{ decrease}$$

2. What is the percentage increase of Jon's salary if it goes from $150 a month to $200 a month?

$$\frac{\text{change}}{\text{starting point}} \times 100 = \frac{50}{150} \times 100 = \frac{1}{3} \times 100 = 33\frac{1}{3}\% \text{ increase}$$

Signed Numbers (Positive Numbers and Negative Numbers)

On a number line, numbers to the right of 0 are positive. Numbers to the left of 0 are negative, as follows:



Given any two numbers on a number line, the one on the right is always larger, regardless of its sign (positive or negative).

Adding Signed Numbers

When adding two numbers with the same sign (either both positive or both negative), add the numbers and keep the same sign. For example:

$$\begin{array}{r} +5 \\ \underline{+\,+7} \\ +12 \end{array} \qquad \begin{array}{r} -8 \\ \underline{+\,-3} \\ -11 \end{array}$$

When adding two numbers with different signs (one positive and one negative), subtract the numbers and keep the sign from the larger one. For example:

$$\begin{array}{r} +5 \\ \underline{+\,-7} \\ -2 \end{array} \qquad \begin{array}{r} -59 \\ \underline{+\,+72} \\ +13 \end{array}$$

Subtracting Signed Numbers

To subtract positive and/or negative numbers, just change the sign of the number being subtracted and add. For example:

$$\begin{array}{r} +12 \\ \underline{-\,+4} \\ \end{array} \quad \begin{array}{r} +12 \\ \underline{+\,-4} \\ +8 \end{array} \qquad \begin{array}{r} -19 \\ \underline{-\,+6} \\ \end{array} \quad \begin{array}{r} -19 \\ \underline{+\,-6} \\ -25 \end{array}$$

$$\begin{array}{r} -14 \\ \underline{-\,-4} \\ \end{array} \quad \begin{array}{r} -14 \\ \underline{+\,+4} \\ -10 \end{array} \qquad \begin{array}{r} +20 \\ \underline{-\,-3} \\ \end{array} \quad \begin{array}{r} +20 \\ \underline{+\,+3} \\ +23 \end{array}$$

Multiplying and Dividing Signed Numbers

To multiply or divide signed numbers, treat them just like regular numbers but remember this rule: An odd number of negative signs produces a negative answer; an even number of negative signs produces a positive answer. For example:

$$(-3)(+8)(-5)(-1)(-2) = +240$$

Absolute Value

The numerical value when direction or sign is not considered is called the *absolute value.* The value of a number is written $|3| = 3$ and $|-4| = 4$. The absolute value of a number is always positive except when the number is 0. For example:

$$|-8| = 8$$
$$|3-9| = |-6| = 6$$
$$3 - |-6| = 3 - 6 = -3$$

Note: Absolute values must be taken first, or the work must be done first within the absolute value signs.

Powers and Exponents

An *exponent* is a positive or negative number placed above and to the right of a quantity. It expresses the power to which the quantity is to be raised or lowered. In 4^3, 3 is the exponent. It shows that 4 is to be used as a factor three times. $4 \times 4 \times 4$ (multiplied by itself twice). 4^3 is read *four to the third power* (or *four cubed*). For example:

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$
$$3^2 = 3 \times 3 = 9$$

Remember that $x^1 = x$ and $x^0 = 1$ when x is any number (other than 0). For example:

$$2^1 = 2$$
$$2^0 = 1$$
$$3^1 = 3$$
$$3^0 = 1$$

Negative Exponents

If an exponent is negative, such as 3^{-2}, then the number and exponent can be dropped under the number 1 in a fraction to remove the negative sign. The number can be simplified as follows:

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Operations with Powers and Exponents

To multiply two numbers with exponents, if the base numbers are the same, simply keep the base number and add the exponents. For example:

$$2^3 \times 2^5 = 2^8 \qquad [(2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) = 2^8] \qquad [2^{(3+5)} = 2^8]$$
$$7^2 \times 7^4 = 7^6$$

To divide two numbers with exponents, if the base numbers are the same, simply keep the base number and subtract the second exponent from the first. For example:

$$3^4 \div 3^2 = 3^2 \qquad \left[3^{(4-2)} = 3^2\right]$$
$$\frac{9^6}{9^2} = 9^6 \div 9^2 = 9^4 \qquad \left[9^{(6-2)} = 9^4\right]$$

Three notes:

- If the base numbers are different in multiplication or division, simplify each number with an exponent first, and then perform the operation.
- To add or subtract numbers with exponents, whether the base is the same or different, simplify each number with an exponent first, and then perform the indicated operation.
- If a number with an exponent is taken to another power $(4^2)^3$, simply keep the original base number and multiply the exponents. For example:

$$(4^2)^3 = 4^6 \qquad [4^{(2\times 3)} = 4^6]$$
$$(3^4)^2 = 3^8$$

Squares and Square Roots

To *square* a number, just multiply it by itself. For example, 6 squared (written 6^2) is 6×6, or 36. Thirty-six is called a perfect square (the square of a whole number).

Following is a list of some perfect squares:

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144 \ldots$

Square roots of nonperfect squares can be approximated. Two approximations to remember are:

$$\sqrt{2} \approx 1.4$$
$$\sqrt{3} \approx 1.7$$

To find the *square root* of a number, find some number that when multiplied by itself gives you the original number. In other words, to find the square root of 25, find the number that when multiplied by itself gives you 25. The square root of 25, then, is 5. The symbol for square root is $\sqrt{\ }$. Following is a list of perfect (whole number) square roots:

$$\sqrt{1} = 1$$
$$\sqrt{4} = 2$$
$$\sqrt{9} = 3$$
$$\sqrt{16} = 4$$
$$\sqrt{25} = 5$$
$$\sqrt{36} = 6$$
$$\sqrt{49} = 7$$

$$\sqrt{64} = 8$$
$$\sqrt{81} = 9$$
$$\sqrt{100} = 10$$

Square Root Rules

Two numbers multiplied under a radical (square root) sign equal the product of the two square roots. For example:

$$\sqrt{(4)(25)} = \sqrt{4} \times \sqrt{25} = 2 \times 5 = 10 \text{ or } \sqrt{100} = 10$$

Likewise with division:

$$\sqrt{\frac{64}{4}} = \frac{\sqrt{64}}{\sqrt{4}} = \frac{8}{2} = 4 \text{ or } \sqrt{16} = 4$$

Addition and subtraction, however, are different. The numbers must be combined under the radical before any computation of square roots is done. For example:

$$\sqrt{10+6} = \sqrt{16} = 4 \qquad \sqrt{10+6} \text{ does } \textit{not} \text{ equal } [\neq]\sqrt{10} + \sqrt{6}$$
$$\sqrt{93-12} = \sqrt{81} = 9$$

Approximating Square Roots

To find a square root that is not a whole number, you should approximate. For example:

Approximate $\sqrt{57}$.

Since $\sqrt{57}$ is between $\sqrt{49}$ and $\sqrt{64}$, it falls somewhere between 7 and 8. And because 57 is just about halfway between 49 and 64, $\sqrt{57}$ is approximately $7\frac{1}{2}$.

Approximate $\sqrt{83}$.

$$\overset{9}{\sqrt{81}} < \sqrt{83} < \overset{10}{\sqrt{100}}$$

Since $\sqrt{83}$ is slightly more than $\sqrt{81}$ (whose square root is 9), $\sqrt{83}$ is a little more than 9. Because 83 is only two steps up from the nearest perfect square (81) and 17 steps to the next perfect square (100), 83 is $\frac{2}{19}$ of the way to 100.

$$\frac{2}{19} \approx \frac{2}{20} \approx \frac{1}{10} = .1$$

Therefore, $\sqrt{83} \approx 9.1$.

Simplifying Square Roots

To simplify numbers under a radical (square root sign):

1. Factor the number to two numbers, one (or more) of which is a perfect square.
2. Take the square root of the perfect square(s).
3. Leave the other factors under the $\sqrt{\ }$.

Simplify $\sqrt{75}$.

$$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$$

Simplify $\sqrt{200}$.

$$\sqrt{200} = \sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$$

Simplify $\sqrt{900}$.

$$\sqrt{900} = \sqrt{100 \times 9} = \sqrt{100} \times \sqrt{9} = 10 \times 3 = 30$$

Parentheses

Parentheses are used to group numbers. Everything inside a set of parentheses must be done before any other operations. For example:

$$\begin{aligned}&6 - (-3 + a - 2b + c) = \\ &6 + (+3 - a + 2b - c) = \\ &\quad 6 + 3 - a + 2b - c = 9 - a + 2b - c\end{aligned}$$

Order of Operations

If addition, multiplication, division, powers, parentheses, and so on are all contained in one problem, the order of operations is as follows:

1. parentheses
2. (exponents)

3. multiplication
4. division

$\left.\begin{array}{l}\text{3. multiplication}\\ \text{4. division}\end{array}\right\}$ whichever comes first, left to right

$\left.\begin{array}{l}\text{5. addition}\\ \text{6. subtraction}\end{array}\right\}$ whichever comes first, left to right

For example:

$$\begin{aligned}10 - 3 \times 6 + 10^2 + (6 + 1) \times 4 &= \\ 10 - 3 \times 6 + 10^2 + (7) \times 4 &= \text{(parentheses first)}\\ 10 - 3 \times 6 + 100 + (7) \times 4 &= \text{(exponents next)}\\ 10 - 18 + 100 + 28 &= \text{(multiplication)}\\ -8 + 100 + 28 &= \text{(addition/subtraction, left to right)}\\ 92 + 28 &= 120\end{aligned}$$

An easy way to remember the order of operations after parentheses is: Please Excuse My Dear Aunt Sally, or PEMDAS (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction).

Basic Set Theory

A *set* is a group of objects, numbers, etc. $\{1, 2, 3\}$

An *element* is a member of a set. $3 \in \{1, 2, 3\}$

3 is an element of the set of 1, 2, 3.

Special Sets

A subset is a set within a set. $\{2,3\} \subset \{1,2,3\}$

The set of 2, 3 is a subset of the set of 1, 2, 3.

The universal set is the general category set, or the set of all those elements under consideration.

The empty set, or null set, is a set with no members. $\emptyset$ or { }.

Describing Sets

Rule is a method of naming a set by describing its elements.

$$\{x|x > 3, \text{ is a whole number}\}$$

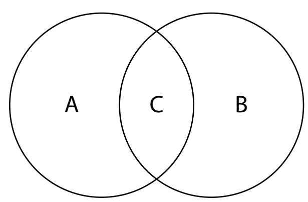
{all students in the class with blue eyes}

Roster is a method of naming a set by listing its members.

{4, 5, 6 . . . }

{Fred, Tom, Bob}

Venn Diagrams (and Euler Circles) are ways of pictorially describing sets.



Types of Sets

Finite sets are countable. They stop. {1, 2, 3, 4}

Infinite sets are uncountable; they continue forever. {1, 2, 3, . . .}

Comparing Sets

Equal sets are those that have the exact same members.

{1, 2, 3} = {3, 2, 1}

Equivalent sets are sets that have the same number of members.

$$\{1, 2, 3\} \sim \{a, b, c\}$$

Operations with Sets

The union of two or more sets is all of the members in those sets.

$$\{1,2,3\} \cup \{3,4,5\} = \{1,2,3,4,5\}$$

The union of sets with members 1, 2, 3 and 3, 4, 5 is the set with members 1, 2, 3, 4, 5.

The intersection of two or more sets is where they intersect, or overlap.

$$\{1,2,3\} \cap \{3,4,5\} = \{3\}$$

The intersection of a set with members 1, 2, 3 and a set with members 3, 4, 5 is a set with only member 3.

Some Basic Probability and Statistics

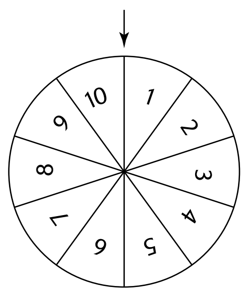
Probability

Probability is the numerical measure of the chance of an outcome or event occurring. When all outcomes are equally likely to occur, the probability of the occurrence of a given outcome can be found by using the following formula:

$$\text{probability} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

For example:

1. Using the spinner shown below, what is the probability of spinning a 6 in one spin?



Since there is only one 6 on the spinner out of ten numbers and all the numbers are equally spaced, the probability is 1 out of 10 or $\frac{1}{10}$.

2. Again, using the spinner shown above, what is the probability of spinning a 3 or a 5 in one spin?

Since there are two favorable outcomes out of ten possible outcomes, the probability is 2 out of 10 or $\frac{2}{10}$, or $\frac{1}{5}$.

When two events are independent of each other, you need to multiply to find the favorable and/or possible outcomes.

3. What is the probability of tossing heads three consecutive times with a two-sided fair coin?

Because each toss is independent and the odds are $\frac{1}{2}$ for each toss, the probability is

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

4. Three green marbles, two blue marbles, and five yellow marbles are placed in a jar. What is the probability of selecting at random, a green marble on the first draw?

Since there are ten marbles (total possible outcomes) and three green marbles (favorable outcomes), the probability is 3 out of 10 or $\frac{3}{10}$.

Statistics

The study of numerical data and its distribution is called *statistics.*

The three basic measures indicating the center of a distribution are: mean, median, and mode.

Mean, Arithmetic Mean, or Average

To find the *average* of a group of numbers:

1. Add them up.
2. Divide by the number of items you added.

For example:

1. What is the average of 10, 20, 35, 40, and 45?

$$10 + 20 + 35 + 40 + 45 = 150$$
$$150 \div 5 = 30$$

The average is 30.

2. What is the average of 0, 12, 18, 20, 31, and 45?

$$0 + 12 + 18 + 20 + 31 + 45 = 126$$
$$126 \div 6 = 21$$

The average is 21.

3. What is the average of 25, 27, 27, and 27?

$$25 + 27 + 27 + 27 = 106$$
$$106 \div 4 = 26\frac{1}{2}$$

The average is $26\frac{1}{2}$.

Median

A *median* is simply the middle number in a list of numbers that have been written in numerical order.

For example, in the following list—3, 4, 6, 9, 21, 24, 56—the number 9 is the median. If the list contains an even number of items, average the two middle numbers to get the median.

For example, in the following list—5, 6, 7, 8, 9, 10—the median is $7\frac{1}{2}$. Because there is an even number of items, the average of the middle two, 7 and 8, gives the median. The list has to be in numerical order (or put in numerical order) first. The median is easy to calculate and is not influenced by extreme measures.

Mode

A *mode* is simply the number most frequently listed in a group of numbers.

For example, in the following group—5, 9, 7, 3, 9, 4, 6, 9, 7, 9, 2—the mode is 9 because it appears more often than any other number. There can be more than one mode. If there are two modes, the group is called bimodal.

Data Analysis: Graphs

Information can be displayed in many ways. The three basic types of graphs you should know about are bar graphs, line graphs, and circle graphs (or pie charts). You should also know the scatter plot, which is similar to a coordinate graph.

When answering questions related to a graph:

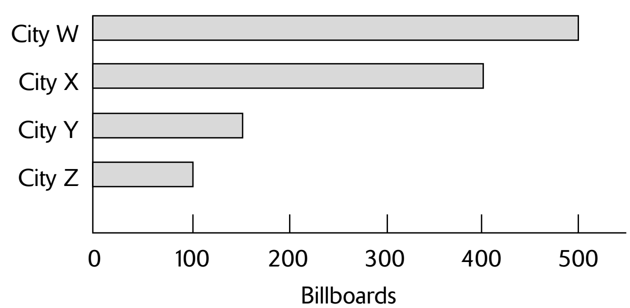
- Examine the entire graph—notice labels and headings.
- Focus on the information given.
- Look for major changes—high points, low points, trends.
- Do not memorize the graph; refer to it.
- Pay special attention to which part of the graph the question is referring to.
- Reread the headings and labels if you don't understand.

Bar Graphs

Bar graphs convert the information in a chart into separate bars or columns. Some graphs list numbers along one edge and places, dates, people, or things (individual categories) along the other edge. Always try to determine the relationship between the columns in a graph or chart.

For example:

1. The following bar graph indicates that City W has approximately how many more billboards than City Y?



The graph shows the number of billboards in each city, with the numbers given along the bottom of the graph in increases of 100. The names are listed along the left side. City W has approximately 500 billboards. The bar graph for City Y stops about halfway between 100 and 200. Consider that halfway between 100 and 200 is 150. So City W (500) has approximately 350 more billboards than City Y (150).

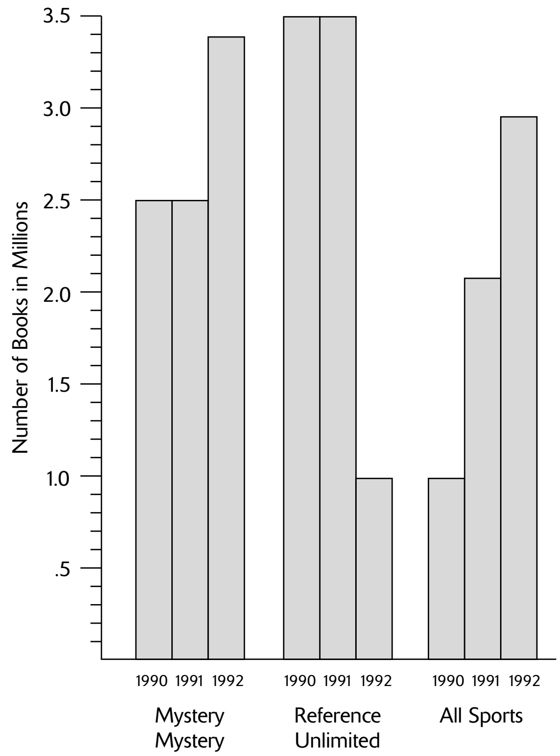
$$500 - 150 = 350$$

2. Based on the following bar graph, answer these questions:

A. The number of books sold by Mystery Mystery from 1990 to 1992 exceeded the number of those sold by All Sports by approximately how many?

B. From 1991 to 1992, the percent increase in number of books sold by All Sports exceeded the percent increase in number of books sold by Mystery Mystery by approximately how much?

C. What caused the 1992 decline in Reference Unlimited's number of books sold?



The graph contains multiple bars representing each publisher. Each single bar stands for the number of books sold in a single year. You might be tempted to write out the numbers as you do your arithmetic (3.5 million = 3,500,000). Writing out the numbers is unnecessary, as it often is with graphs that use large numbers. Since all measurements are in millions, adding zeros does not add precision to the numbers.

Answer to Question **A**: Regarding the Mystery Mystery bars, the number of books sold per year can be determined as follows:

1990 = 2.5

1991 = 2.5

1992 = 3.4

Use a piece of paper as a straightedge to determine this last number. Totaling the number of books sold for all three years gives 8.4.

Referring to the All Sports bars, the number of books sold per year is as follows:

1990 = 1

1991 = 2.1

1992 = 3

Again, use a piece of paper as a straightedge, but don't designate numbers beyond the nearest 10th because the graph numbers prescribe no greater accuracy than this. Totaling the number of books sold for all three years gives 6.1.

So, the number of books sold by Mystery Mystery exceeded the number of books sold by All Sports by 2.3 million.

Answer to Question **B.**: Graph and chart questions might ask you to calculate percent increase or percent decrease. The formula for figuring either of these is the same.

$$\frac{\text{change}}{\text{starting point}} = \text{percent change}$$

In this case, the percent increase in number of books sold by Mystery Mystery can be calculated first.

$$\begin{aligned} \text{Number of books sold in 1991} &= 2.5 \\ \text{Number of books sold in 1992} &= 3.4 \\ \text{Change} &= .9 \end{aligned}$$

The 1991 amount is the starting point, so:

$$\frac{\text{change}}{\text{starting point}} = \frac{.9}{2.5} = 36\%$$

The percent increase in number of books sold by All Sports can be calculated as follows:

$$\begin{aligned} \text{Number of books sold in 1991} &= 2.1 \\ \text{Number of books sold in 1992} &= 3 \\ \text{Change} &= .9 \end{aligned}$$

$$\frac{\text{change}}{\text{starting point}} = \frac{.9}{2.1} = 4.28 \approx 43\%$$

So the percent increase of All Sports exceeded that of Mystery Mystery by 7%:

$$43\% - 36\% = 7\%$$

Answer to Question **C.**: This question cannot be answered based on the information in the graph. Never assume information that is not given. In this case, the multiple factors that could cause a decline in the number of books sold are not represented by the graph.

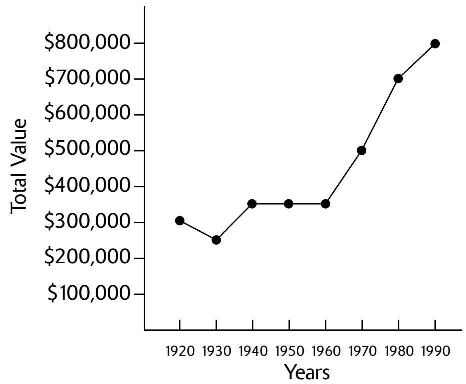
Line Graphs

Line graphs convert data into points on a grid. These points are then connected to show a relationship among items, dates, times, and so on. Notice the slopes of the lines connecting the points. These lines show increases and decreases—the sharper the slope *upward,* the greater the *increase;* the sharper the slope *downward,* the greater the *decrease.* Line graphs can show trends, or changes, in data over a period of time.

For example:

1. Based on the following line graph, answer these questions:

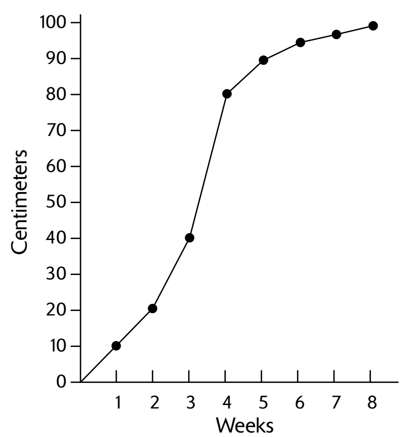
A. In what year was the property value of Moose Lake Resort about $500,000?
B. In which 10-year period was there the greatest decrease in the property value of Moose Lake Resort?



Answer to Question **A.**: The information along the left side of the graph shows the property value of Moose Lake Resort in increments of $100,000. The bottom of the graph shows the years from 1920 to 1990. In 1970, the property value was about $500,000. Using a sheet of paper as a straightedge helps to see that the dot in the 1970 column lines up with $500,000 on the left.

Answer to Question **B.**: Since the slope of the line goes down from 1920 to 1930, there must have been a decrease in property value. If you read the actual numbers, you notice a decrease from $300,000 to about $250,000.

2. According to the following line graph, the tomato plant grew the most between which two weeks?



The numbers at the bottom of the graph give the weeks of growth of the plant. The numbers on the left give the height of the plant in centimeters. The sharpest upward slope occurs between week 3 and week 4, when the plant grew from 40 centimeters to 80 centimeters, a total of 40 centimeters growth.

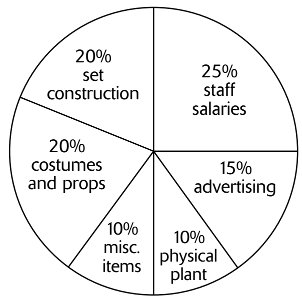
Circle Graphs or Pie Charts

A *circle graph,* or *pie chart,* shows the relationship between the whole circle (100%) and the various slices that represent portions of that 100%—the larger the slice, the higher the percentage.

For example:

1. Based on the following circle graph:

Smithville Community Theater Expenditures



A. If Smithville Community Theater has $1,000 to spend this month, how much is spent on set construction?
B. What is the ratio of the amount of money spent on advertising to the amount of money spent on set construction?

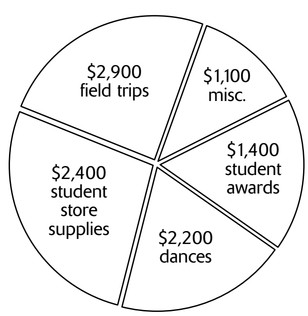
Answer to Question **A.**: The theater spends 20% of its money on set construction. Twenty percent of $1,000 is $200, so $200 is spent on set construction.

Answer to Question **B.**: To answer this question, you must use the information in the graph to make a ratio.

$$\frac{\text{advertising}}{\text{set construction}} = \frac{15\% \text{ of } 1000}{20\% \text{ of } 1000} = \frac{150}{200} = \frac{3}{4}$$

Notice that $\frac{15\%}{20\%}$ reduces to $\frac{3}{4}$.

2. Based on the following circle graph:



$10,000 Total Expenditures

A. If the Bell Canyon PTA spends the same percentage on dances every year, how much do they spend on dances in a year in which the total amount spent is $15,000?
B. The amount of money spent on field trips in 1995 was approximately what percent of the total amount spent?

Answer to Question **A.**: To answer this question, you must find a percent, and then apply this percent to a new total. In 1995, the PTA spent \$2,200 on dances. This can be calculated as 22% of the total spent in 1995 by the following method:

$$\frac{2{,}200}{10{,}000} = \frac{22}{100} = 22\%$$

Multiplying 22% by the *new* total amount spent (\$15,000) gives the right answer.

$$22\% = .22$$
$$.22 \times 15{,}000 = 3{,}300 \text{ or } \$3{,}300$$

You could use another common-sense method. If \$2,200 out of \$10,000 is spent for dances, \$1,100 out of every \$5,000 is spent for dances. Since \$15,000 is 3 × \$5,000, 3 × \$1,100 is \$3,300.

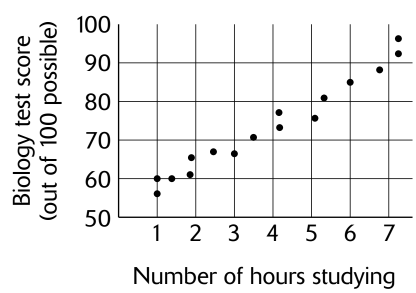
Answer to Question **B.**: By carefully reading the information in the graph, you find that \$2,900 was spent on field trips. The information describing the graph explains that the total expenditures were \$10,000. Because \$2,900 is approximately \$3,000, the approximate percentage is worked out as follows:

$$\frac{3{,}000}{10{,}000} = \frac{30}{100} = 30\%$$

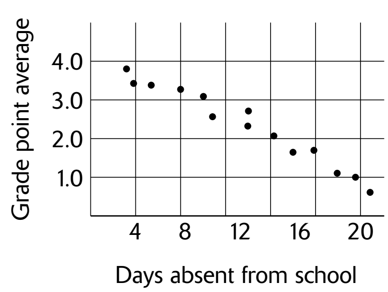
Scatter Plot

A *scatter plot* is a graph representing a set of data and showing a relationship or connection between the two quantities given. The graph is typically placed in one part of a coordinate plane (the upper right quarter called Quadrant I). When the data is placed on the scatter plot, usually a relationship can be seen. If the points appear to form a line, a linear relationship is suggested.

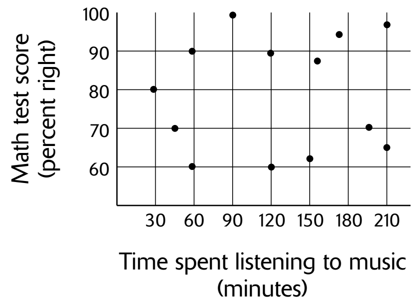
If the line goes up to the right, that is, one quantity increases as another increases, then the relationship is called *a positive correlation*. For example:



If the line goes down to the right, that is, one quantity decreases as another increases, then the relationship is called a *negative correlation*. For example:



If the data does not appear to show any line or any relationship between the quantities, the scatter plot is said to show no correlation. For example:



Sample SAT-Type Problems

Now you can try actual SAT-like problems in each topic area. *Note:* Remember that multiple-choice problems range from easy to average to difficult.

Number and Operations

Operations with Fractions

1. $2 \div \left(\frac{2}{3}\right) + 3 \div 4 \div \left(\frac{4}{5}\right) =$

A. $\frac{1}{5}$

B. $\frac{11}{15}$

C. $\frac{47}{60}$

D. $3\frac{13}{30}$

E. 12

2. $\dfrac{3\frac{1}{4} - 1\frac{2}{3}}{\frac{5}{6} - \frac{1}{2}} =$

A. $\frac{17}{12}$

B. $\frac{19}{12}$

C. $\frac{7}{2}$

D. $\frac{19}{4}$

E. $\frac{17}{2}$

3. If $\frac{2}{5}$ of $\frac{2}{3} \div \frac{5}{6}$ is divided by $\frac{3}{10}$, what is the resulting value?

A. $\frac{1}{15}$

B. $\frac{12}{25}$

C. $\frac{8}{15}$

D. $\frac{5}{3}$

E. $\frac{125}{27}$

4. The weight of coffee in a bag is $\frac{3}{4}$ pound. A coffee company packages 12 of these packages in a case. If the coffee company fills an order for $4\frac{1}{2}$ cases how many pounds of coffee will the company need to fill this order?

Explanations

1. E. $2 \div \left(\frac{2}{3}\right) + 3 \div \left(\frac{3}{4}\right) + 4 \div \left(\frac{4}{5}\right) = 2 \cdot \frac{3}{2} + 3 \cdot \frac{4}{3} + 4 \cdot \frac{5}{4} = 3 + 4 + 5 = 12$

In the second step, after inverting and multiplying, the denominators all cancel out.

2. D. $\dfrac{3\frac{1}{4} - 1\frac{2}{3}}{\frac{5}{6} - \frac{1}{2}} = \dfrac{\left(\frac{13}{4} - \frac{5}{3}\right)}{\left(\frac{5}{6} - \frac{1}{2}\right)} \cdot \frac{12}{12} = \frac{39 - 20}{10 - 6} = \frac{19}{4}$. To get rid of all the fractions in the second step, multiply the top and bottom of the complex fraction by 12, the LCD for all the fractions.

3. C. $\frac{2}{5}\left(\frac{2}{3} \div \frac{5}{6}\right) \div \frac{3}{10} = \frac{2}{5} \cdot \frac{2}{3} \cdot \frac{6}{5} \cdot \frac{10}{3} = \frac{8}{15}$, after canceling, where appropriate, in the fraction computation.

4. $\mathbf{\frac{81}{2}}$ **or 40.5.** Multiply $\frac{3}{4}$ by 12. The product is 9, which is the weight of coffee in one case. Next, multiply 9 by $4\frac{1}{2}$. This product is $(9)\left(\frac{9}{2}\right) = \frac{81}{2}$. The correct answer is $\frac{81}{2}$ or 40.5.

Applying Addition, Subtraction, Multiplication, and Division to Problem Solving

5. During a sale Arianna purchases a sweater listed at \$100. With 20% off and 7% sales tax, how much did she pay for the discounted sweater?

A. \$73.00
B. \$80.00
C. \$85.60
D. \$87.00
E. \$88.30

6. Franco decides to purchase a new washing machine that sells for \$1,100. After making a down payment of \$125, he will pay off the remainder in monthly payments of \$75. How many months will it take for Franco to pay off the new washing machine?

A. 7
B. 8
C. 9
D. 13
E. 15

7. An electronics store owner purchases DVDs from his supplier, paying \$270.80 for 20 DVDs. He then sells them to his customers during a special sale, charging \$53.01 for 3 DVDs. How much profit, per DVD, does the owner make during the special sale?

A. \$4.13
B. \$9.33
C. \$13.54
D. \$17.67
E. \$82.60

8. A grocery store sends an order to a local sandwich shop. The store charges the sandwich shop \$11.28 for lettuce, \$23.55 for bread, \$17.37 for tomatoes, and \$14.80 for avocados. If the sandwich shop owner paid with a \$100 bill, how much change should he receive in dollars?

Explanations

5. **C.** 20% of \$100 is just \$20. So the sale price is \$100 – \$20 = \$80.

7% tax on \$80 is (.07)\$80 = \$5.60. The total price is then \$80 + \$5.60 = \$85.60.

6. **D.** \$1,100 – \$125 = \$975. At \$75 per month, it will take \$975 ÷ \$75 = 13 months.

7. **A.** The store owner pays \$270.80 ÷ 20 = \$13.54 per DVD. He sells them for \$53.01 ÷ 3 = \$17.67. His profit per DVD is therefore \$17.67 – \$13.54 = \$4.13.

8. **33.** Add the following amounts: \$11.28 + \$23.55 + \$17.37 + \$14.80. The sum is \$67.00. Next, subtract \$67 from \$100. The difference is \$33, which is the correct answer.

Arithmetic Mean (Average), Mode, and Median

9. If 10 boys averaged 78% on a test for which 15 girls averaged 89%, what is the test average for all 25 students?

A. 76.4%
B. 83.5%
C. 84.6%
D. 86.2%
E. 87.8%

10. In a set of 15 different integers, which of the following changes would NOT effect the value of the median of these 15 integers?

A. Subtract 7 from each integer.
B. Add 11 to the largest integer.
C. Triple each integer.
D. Add 3 to the smallest integer.
E. Subtract 5 from the largest integer.

11. The shoes sizes of 10 men are 8, 10, 9, 12, 7, 10, 10, 11, 6, and 7. How much greater than the mode is the median?

A. 0
B. 0.5
C. 5
D. 9.5
E. 10

12. The average daily temperature in a city during the month of April was 8.0°C. The average temperature in May of that year was 9.5°C. What is the average temperature in this city for the months of April and May? (There are 30 days in April and 31 days in May.)

Explanations

9. **A.** The class average can be found as follows: $\frac{10(78\%) + 15(89\%)}{25} = \frac{780\% + 1335\%}{25} = 84.6\%$

10. **B.** Since the median is the middle of the list of the set of integers, adding 11 to the largest integer will not affect the median. The other option—which is not one the given choices—would be to subtract some positive integer from the smallest number in the set.

11. **B.** Listing the shoe sizes from small to large, you have: 6, 7, 7, 8, 9, 10, 10, 10, 11, 12

The mode (most frequent shoe size) is 10. The median (the middle of the list) would be the average of 9 and 10, which gives 9.5. Notice that since you have an even number (10) of shoe sizes, you had to find the average of the fifth and sixth shoe sizes (9 and 10) to compute the median.

12. **8.76.** Begin by determining the number of temperature points for the month of April: $30 \times 8 = 240$ temperature points for the month of April. Next, determine the number of temperature points for the month of May: $31 \times 9.5 = 294.5$ temperature points for the month of May. Since the average is the sum of these temperature points, divided by 61 (the number of days in April and May combined), the average is:

$$\frac{240 + 294.5}{61} = 8.7622°\text{C}$$

The answer must be 8.76, since this is the most exact number the grid field will accommodate.

Ratio and Proportion

13. At a party, the ratio of males to females is 3 : 4. Which of the following could NOT be the total number of people at the party?

A. 14
B. 35
C. 44
D. 63
E. 70

14. If $\frac{r}{m} = \frac{2}{3}$ and $\frac{m}{c} = \frac{4}{5}$, what is the value of $\frac{r}{c}$?

A. $\frac{3}{10}$
B. $\frac{2}{5}$
C. $\frac{8}{15}$
D. $\frac{5}{6}$
E. $\frac{15}{8}$

15. A special snack mix uses raisins, peanuts, and chocolate candies in the ratio 1 : 2 : 5, respectively. In a snack mix of 112 ounces, how many ounces of peanuts are used?

A. 80
B. 70
C. 50
D. 16
E. 14

16. A certain brand of paint consists of water and paint concentrate. Each gallon of paint contains $\frac{1}{4}$ part water. How much concentrate would be needed to produce 3 gallons of this paint?

Explanations

13. **C.** Since the ratio of males to females is 3 parts to 4 parts, the number of people at the party must be a multiple of 7 parts. Choice **C,** 44, is not a multiple of 7.

14. **C.** Notice that $\frac{r}{m} \times \frac{m}{c} = \frac{r}{c}$, which is the ratio you are trying to find. Substituting the numerical values for the ratios on the left in the equation above, you have $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

15. **E.** If you write your raisins to peanuts to candies ratio as $1x : 2x : 5x$, you will have

$$1x + 2x + 5x = 112$$
$$8x = 112$$
$$\text{so } x = 14.$$

Therefore the number of ounces of peanuts will be $5x = 5(14) = 70$.

16. $\mathbf{\frac{9}{4}}$ **or 2.25.** If the paint is $\frac{1}{4}$ water, it must be $\frac{3}{4}$ parts paint concentrate. If there are $\frac{3}{4}$ parts paint in one gallon of paint, then there are 3 times $\frac{3}{4}$ or $2\frac{1}{4}$ gallons of paint concentrate in 3 gallons of paint. See the proportion below:

$$\frac{\frac{3}{4}\ \textit{gallon}}{1\ \textit{gallon}} = \frac{x\ \textit{gallons}}{3\ \textit{gallons}}$$ Solving for x by cross multiplication, $x = (3)\left(\frac{3}{4}\right)$ *gallons.*

The correct answer is $x = \left(\frac{9}{4}\right)$ *gallons* or 2.25 *gallons*

Number Properties: Positive and Negative Integers, Odd and Even Numbers, Prime Numbers, Factors and Multiples, Divisibility

17. How many two-digit positive integers are multiples of both 2 and 5?

A. 5
B. 6
C. 9
D. 45
E. 48

18. If x and y are positive integers, and are not both even, which of the following must be even?

A. $x + y$
B. $x - y$
C. $(x + 3)(y - 4)$
D. $x + y - 6$
E. $4(x + y) - 2$

19. The prime factorization of 360 can be written in the form $a^m b^n c^r$. What is the value of $a + b + c - mnr$?

A. 4
B. 6
C. 10
D. 11
E. 24

20. A number is divisible by 2, 3, and 8. What is the smallest three-digit number that is divisible by this number?

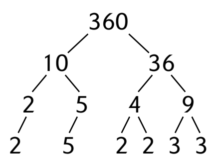
Explanations

17. **B.** If the positive integer is a multiple of both 3 and 5, it must also be a multiple of 15. So our two-digit multiples of 15 are: 15, 30, 45, 60, 75, and 90. So our correct answer is 6 two-digit multiples of 15.

18. **E.** Trying each of the choices one at a time, you have:

- **A.** $x + y$ = odd + even = odd, or $x + y$ = even + odd = odd
- **B.** $x - y$ = odd – even = odd, or $x - y$ = even – odd = odd
- **C.** $(x + 3)(y - 4)$ = (odd + odd)(even – even) = (even)(even) = even or $(x + 3)(y - 4)$ = (even + odd)(odd – even) = (odd)(odd) = odd. So Choice **C** does NOT have to be even.
- **D.** $x + y - 6$ = odd + even – even = odd or $x + y - 6$ = even + odd – even = odd
- **E.** By default, Choice **E** must therefore be correct. Just to double-check, you have: $4(x + y) - 2$ = 4(any number) – even = even – even = even

19. **A.** Making a "factor tree", you find the prime factorization of 360.



Therefore, $360 = 2^3 \times 3^2 \times 5^1$. You are comparing this to $a^m \times b^c \times c^r$. So the sum $(a + b + c)$ is just the sum of the bases, $2 + 3 + 5 = 10$. And the product $m \times n \times r$ is just the product of the exponents, $3 \times 2 \times 1 = 6$. Finally, the value of $(a + b + c) - m \times n \times r = 10 - 6 = 4$.

20. **120.** Begin by finding the least common multiple (LCM) of 2, 3, and 8. Look for the smallest number that is divisible by all three numbers. 24 is divisible by 2, 3, and 8. No smaller number is divisible by all three numbers. The correct number is 24. The smallest three-digit multiple of 24 is 120, the smallest three-digit multiple of 48 is 144, the smallest three-digit multiple of 72 is 144, and the smallest three-digit multiple of 96 is 192. Since you are looking for the smallest three-digit number, the answer is 120.

Word Problems: Solving for Percents, Average, Rate, Time, Distance, Interest, Price per Item

21. In a one-week period, the price of regular unleaded gasoline went from \$2.50/gallon to \$2.75/gallon. What was the percent increase in the price of a gallon of regular unleaded gasoline?

- **A.** 9%
- **B.** 10%
- **C.** 20%
- **D.** 25%
- **E.** 90%

22. Serena has \$500 in a savings account that pays 4% simple interest per year. If she does not deposit or withdraw from this savings account, how much money will be in Serena's savings account after 3 years?

- **A.** \$512.00
- **B.** \$512.37
- **C.** \$560.00
- **D.** \$562.43
- **E.** \$566.84

23. Ling rides his bicycle a distance of 40 miles in 5 hours and then returns the same distance in only 4 hours. What is his average rate, in miles/hour, for the entire trip?

A. $\frac{20}{9}$
B. $\frac{40}{9}$
C. $\frac{9}{2}$
D. $\frac{80}{9}$
E. 9

24. It took Esteban 1 minute and 20 seconds to travel one mile. If he then drives this one-mile distance in 55 seconds, how much faster is he driving in miles per hour?

Explanations

21. **B.** \$2.75 – \$2.50 = \$0.25. Then $\frac{\$0.25}{\$2.50} = \frac{1}{10}$ = 10% increase in price of gasoline

22. **C.** Simple interest = principal × rate × time

$I = 500 \times (.04) \times 3$

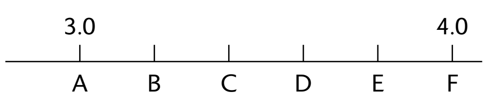
$I = 60$

$$\begin{array}{r} 60 \\ +500 \\ \hline 560 \end{array}$$

23. **B.** Average rate $= \frac{dist_{total}}{time_{total}} = \frac{40+40}{\frac{40}{5}+\frac{40}{4}} = \frac{80}{8+10} = \frac{80}{18} = \frac{40}{9}$. To compute the time going and returning, you need to use $time = \frac{dist}{rate}$.

24. **21.** If Esteban travels one mile in 1 minute and 20 seconds (80 seconds), he will travel 60/80 (3/4) miles in one minute. If he travels one mile in 55 seconds, he will travel 60/55 miles in one minute. Multiplying both fractions by 60 to convert to miles per hour, the speeds will be 45 mph in the first case and 66 mph in the second case. The difference between 66 and 45 is 21.

Number Line: Order, Consecutive Numbers, Fractions, Betweenness



25. In the diagram above, points *A, B, C, D, E,* and *F* are equally spaced on the number line. If *M* is the midpoint of $\overline{AB}$ and *N* is the midpoint of $\overline{DE}$, what is the distance from *M* to *N?*

A. 0.6
B. 0.75
C. 3.25
D. 3.6
E. 3.8

26. On a number line, points *A, B, C, D,* and *E* have respective coordinates *a*, *b*, *c*, *d*, and *e* but not necessarily in that order. If $c < d$, point *A* is between points *C* and *D,* point *D* is between points *A* and *E,* which of the following is a possible order of the points *A, B, C, D,* and *E?*

A. *A, C, B, D, E*
B. *C, A, B, E, D*
C. *D, C, B, A, E*
D. *C, A, D, E, B*
E. *C, D, A, B, E*

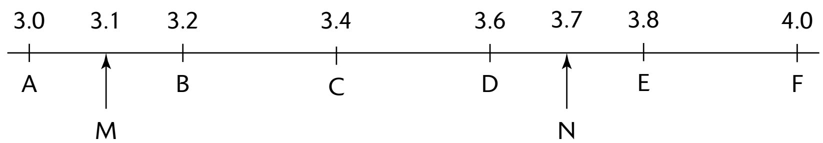
27. v, w, x, y, and z are consecutive odd integers, not necessarily in that order. If $x - 4 = z$, $y + 6 = x$, and $w > v$, which of the following is the correct order for the integers v, w, x, y, and z?

A. $y < z < v < x < w$
B. $v < z < y < x < w$
C. $z < v < x < w < y$
D. $z < y < x < v < w$
E. $v < w < z < y < x$

28. The sum of five consecutive even integers is 536. What is the middle number of the five numbers?

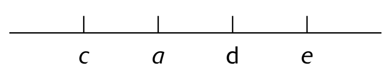
Explanations

25. **A.**



In the figure above, points B, C, D, and E have been labeled with their respective coordinates: 3.2, 3.4, 3.6, and 3.8. Since M is the midpoint of $\overline{AB}$, the coordinate of point M is 3.1. Since N is the midpoint of $\overline{DE}$, the coordinate of point E is 3.7. Therefore the distance from M to N is $3.7 - 3.1 = 0.6$

26. **D.** Using the given relationships between coordinates and points, you have the following order of the coordinates:



Point B, with coordinate b, can be located anywhere. Choice **D** is the only choice given a possible order.

27. **A.** From the first piece of data, $x - 4 = z$, you get $x = z + 4$; so z is less than x, and there is one other odd integer between z and x, as shown below:

z, __, x

From the second piece of data, $y + 6 = x$, you know there are two other odd integers between x and y; combined with the figure above, you have the series below:

y, z, __, x, __ Then since $w > v$, the final order is as follows: y, z, v, x, w, corresponding with Choice **A.**

28. **108.** Since the five numbers add to 540, the middle number must be 540 divided by 5 (the average), which is 108. The other numbers would be two greater, two less, four greater, and four less. Therefore, the numbers are 104, 106, 108, 110, and 112. The correct answer is 108.

Using an equation to solve the problem, let the numbers be: x, $x + 2$, $x + 4$, $x - 2$, $x - 4$.

Therefore, $x + (x + 2) + (x + 4) + (x - 2) + (x - 4) = 540$

$5x = 540$, and $x = \frac{540}{5} = 108$.

Sequence Involving Exponential Growth

29. The first four numbers in a patterned sequence of positive integers are 1, 2, 4, 8, ... What is the tenth integer in this sequence?

A. 20
B. 128
C. 512
D. 1,024
E. 2,048

30. Yuri deposits $100 into a new savings account that pays him 3% interest compounded annually. What is the accumulated value of this deposit 6 years after the day of the initial deposit?

A. $118.00
B. $119.41
C. $121.37
D. $125.92
E. $141.85

31. A culture of 207 bacteria triples every 12 minutes. What will the bacterial count 2 hours after the original culture of 207 was established?

A. 207×3^2
B. 207×3^6
C. 207×3^{10}
D. 207×3^{12}
E. 207×3^{13}

32. In a laboratory experiment a culture of bacteria contained a population of 25,000 when first measured. The bacteria population decreased such that each hour the population was $\frac{1}{5}$ that of the preceding hour. What is the change in population during the fifth hour?

Explanations

29. **C.** The integers double as you move from one term of the sequence to the next. Therefore, the first ten integers in the sequence are: 1, 2, 4, 8, 16, 32, 64, 128, 512, 1024.

30. **B.** Since the annual interest rate is 3%, the value of the account one year is 1.03 times the previous year's value. After 6 years, the account will be worth $\$100 \times (1.03)^6 = \$119.41$.

31. **C.** A 2-hour time period represents 120 minutes. This is 10 of the 12-minute time periods in which the bacteria triples. So after 120 minutes, the initial count of 207 will grow to 207×3^{10}.

32. **32.** Start by making a chart showing the population for each hour. The population at the end of the first hour is 5,000, at the end of the second hour is 1,000, and so forth.

$t(0) = 25{,}000$

$t(1) = 5{,}000$

$t(2) = 1{,}000$

$t(3) = 200$

$t(4) = 40$

The change in the fifth hour is the difference between 40 and 8.

$t(5) = 8$

32 is the correct answer.

Sets: Union, Intersection, Elements

33. If S is the set of perfect squares less than 100, and C is the set of perfect cubes less than 100, what is $S \cap C$?

A. {1, 4, 8, 9, 16, 25, 36, 49, 64, 81}
B. {1, 9, 25, 27, 48, 81}
C. {1}
D. {1, 64}
E. {64}

34. If A is the set of positive odd integers less than 21, and T is the set of positive multiples of 3 less than 21, what is $A \cup T$?

A. {3, 9, 15}
B. {6, 12, 18}
C. {1, 3, 5, 6, 7, 9, 11, 13, 15, 17, 18, 19}
D. {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}
E. { }

35. M is the set of integers that can be written as $\sqrt{n} - 3$, where n is a nonzero integer. Which of the following integers is a member of set M?

I. 4
II. 5
III. 7

A. I only
B. II only
C. III only
D. I and II only
E. I, II, and III

36. A = the set of prime numbers and B = the set of positive odd integers. What is the smallest three-digit member of the intersection of sets A and B?

Explanations

33. **D.** $S = \{1, 4, 9, 16, 25, 36, 49, 64, 81\}$ and $C = \{1, 8, 27, 64\}$. $S \cap C$ is the "intersection" of the two sets. The integers common to both sets are only 1 and 64. So the correct answer is the set {1, 64}.

34. **C.** $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$ and $T = \{3, 6, 9, 12, 15, 18\}$. $A \cup T$ is the "union" of the two sets. The integers belonging to either one of the sets are {1, 3, 5, 6, 7, 9, 11, 13, 15, 17, 18, 19}.

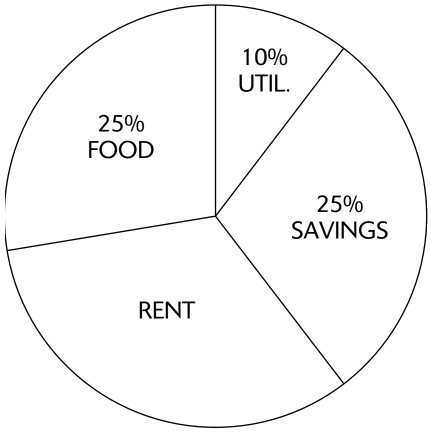
35. **E.** Checking the answer choices one at a time, you have:

I. 4: 4 can be written as $\sqrt{49} - 3 = 7 - 3 = 4$
II. 5: 5 can be written as $\sqrt{64} - 3 = 8 - 3 = 5$
III. 7: 7 can be written as $\sqrt{100} - 3 = 10 - 3 = 7$

Therefore, the correct answer is Choice **E**, because all three of the numbers are in set *A*.

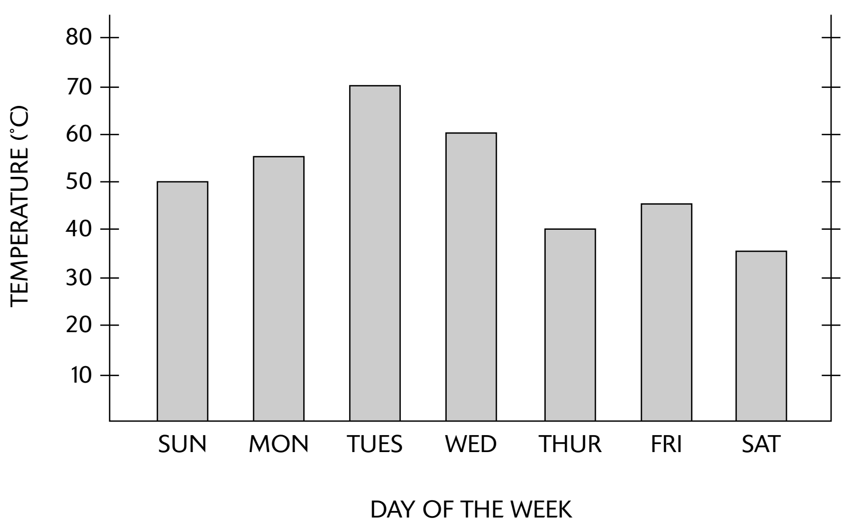
36. **101.** Find a few prime numbers that are greater than 99. These include 101, 107, etc. Since 101 is prime and 100 is not, 101 is the correct answer. You can use the divisibility rules for 2, 3, 4, 5, 6, 8, 9, 10 to help in finding the prime numbers. Remember that a prime number is only divisible exactly by one and itself.

Interpreting Graphs, Charts, and Tables



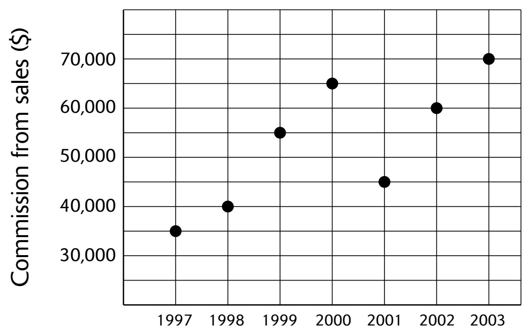
37. Nicole spends her monthly salary as shown in the graph above. If she earns $2,500 per month, how much does spend each month on rent?

A. $40
B. $800
C. $1,000
D. $1,040
E. $1,250



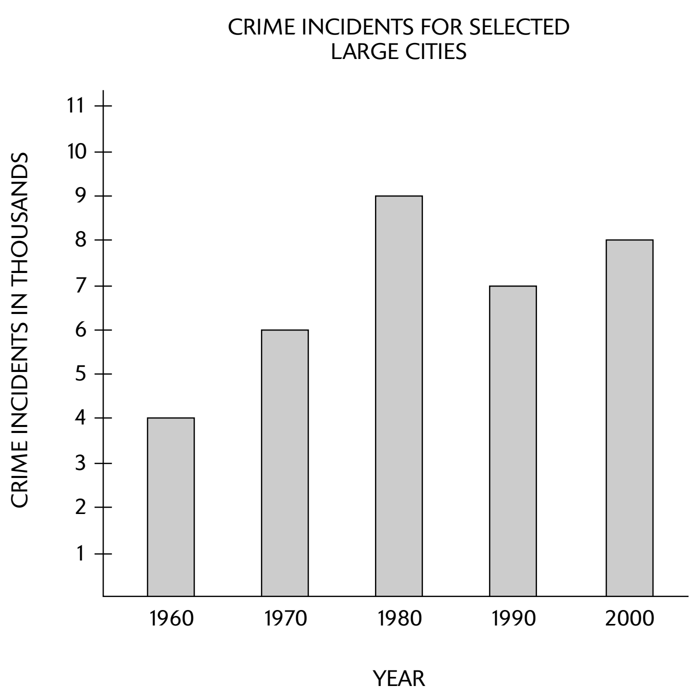
38. According to the graph above, between which pair of consecutive days did the greatest change in temperature occur?

A. Sunday and Monday
B. Monday and Tuesday
C. Tuesday and Wednesday
D. Wednesday and Thursday
E. Thursday and Friday



39. According to the graph above, the percent increase in commission from sales from 2001 to 2002 is approximately how much greater than the percent increase in commission from sales from 1998 to 1999?

A. 2
B. $12\frac{1}{2}$
C. 21
D. 25
E. $33\frac{1}{3}$



40. According to the bar graph (above), what was the percent decrease in crime between the years 1980 and 1990 in selected large cities?

Explanations

37. C. % spent on rent = 100% – (25% + 10% +25%) = 40%. Then money spent on rent = 40% of $2,500 = $1,000.

38. D. Since the bar graph is a "scale" drawing, visually the biggest changes occur between Monday and Tuesday, an increase of 15°, and between Wednesday and Thursday, a decrease of 20°. So the greatest change is Choice **D.**

39. C.

$$\begin{array}{r} 2002 \text{ comm.} = \$60{,}000 \\ \underline{2001 \text{ comm.} = \$45{,}000} \\ \text{inc. in comm.} = \$15{,}000 \end{array}$$

$$\begin{array}{r} 1999 \text{ comm.} = \$55{,}000 \\ \underline{1998 \text{ comm.} = \$40{,}000} \\ \text{inc. in comm.} = \$15{,}000 \end{array}$$

$$\%inc. = \left(\frac{amt.\ inc}{amt.\ org.}\right) \cdot 100$$

$$\%inc. = \left(\frac{amt.\ inc.}{amt.\ orig.}\right) \cdot 100$$

$$= \left(\frac{15{,}000}{45{,}000}\right) \cdot 100 = 33\frac{1}{3}\%$$

$$= \left(\frac{15{,}000}{40{,}000}\right) \cdot 100 = 12\frac{1}{2}\%$$

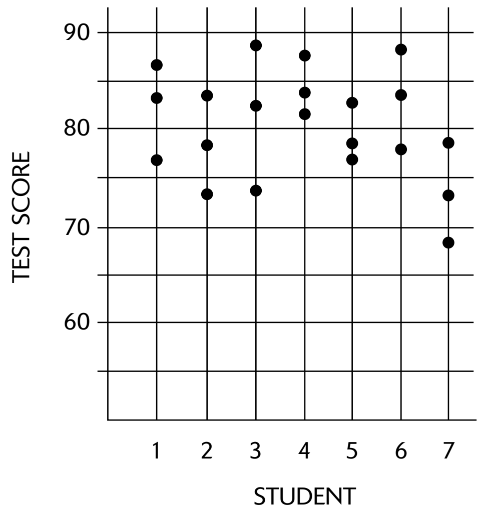
So the difference is $33\frac{1}{3} - 12\frac{1}{2} = 21$, approximately.

40. $\mathbf{\frac{2}{9}}$ **or 22.2** First, analyze the graph for important details. Note that the number of crime incidents for 1980 is 9,000 and for 1990 is 7,000. The vertical axis indicates that these figures are in thousands. The method to calculate percent increase or percent decrease is the difference in the two amounts divided by the original amount. In our figure, the expression for this method is:

$$\frac{9000 - 7000}{9000}$$

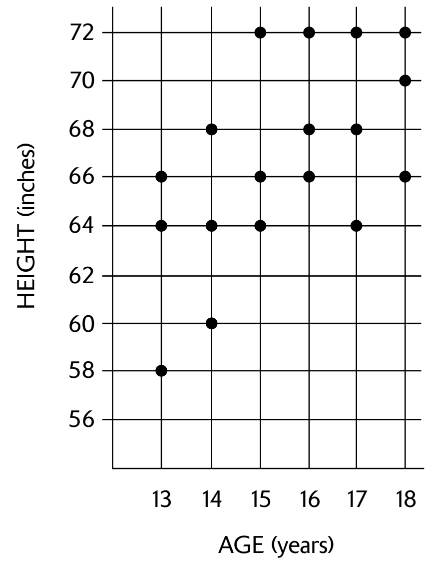
This is equal to $\frac{2}{9}$ or about 22.2 percent. The correct answer is $\frac{2}{9}$ or 22.2%. *Note:* The expression could have been $\frac{9-7}{9}$, and the same answer would have occurred.

Scatter Plots



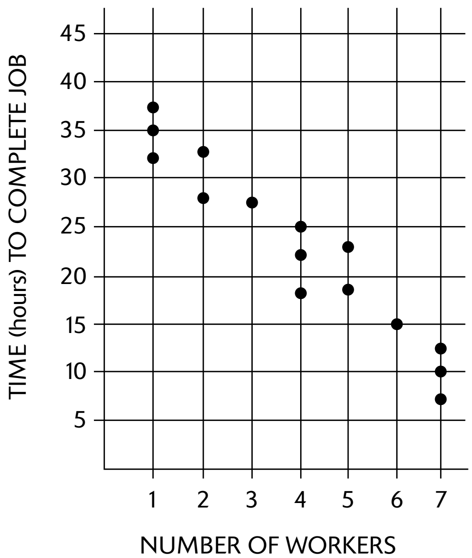
41. The scatter plot above shows the score on each of three tests for seven students. Which of the following could be the median score for all seven students?

A. 65
B. 71
C. 78
D. 81
E. 91



42. The scatter plot above shows the heights of three randomly selected students from each of six age groups. According to the data in this plot, which of the following statements is true?

A. There are only two students having heights 68 inches.
B. Of the students who are 72 inches tall, none is 13 or 14 years old.
C. More 18-year-old students than 15-year-old students have heights greater than 66 inches.
D. Half of the students have heights greater than 66 inches.
E. There are no 17- or 18-year-old students having heights less than 66 inches.



43. The scatter plot above shows the time it took one or more workers to complete a particular job. In some cases, the workers made just one attempt to complete the job; in other cases, the workers made multiple attempts to complete the job. Based on the data presented, which of the following functions best states the relationship between *t,* the time in hours to complete the job, and *w,* the number of workers?

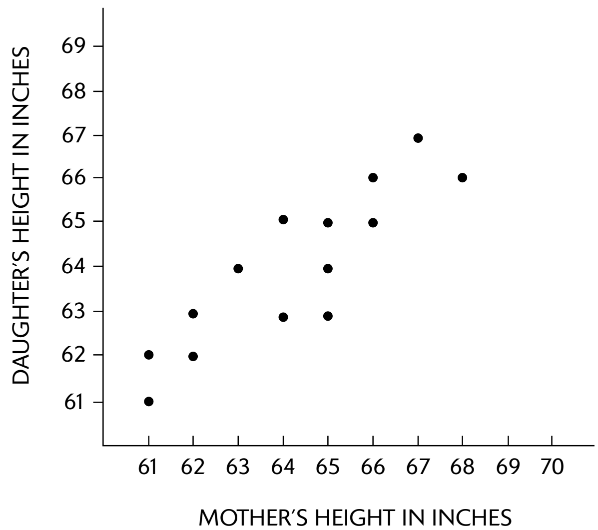
A. $t(w) = \frac{-4}{5}w + 40$

B. $t(w) = -4w + 40$

C. $t(w) = -w + 35$

D. $t(w) = \frac{3}{4}w + 40$

E. $t(w) = \frac{1}{2}w + 35$



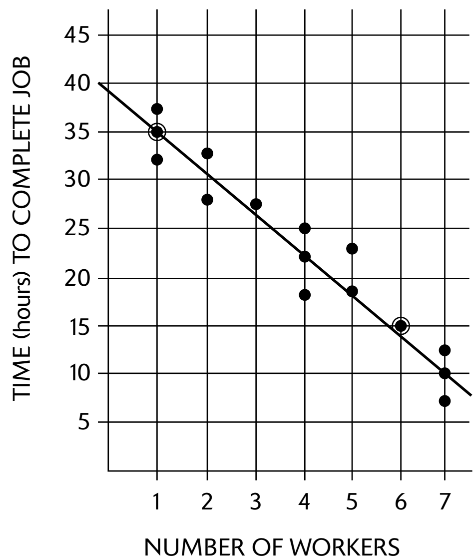
44. In the scatter plot above, mothers' heights are compared to their daughters' heights. This was done at a parent meeting, for 15 junior girls who were participating in the graduation ceremony. According to the graph, which height for mothers had no daughter's height given?

Explanations

41. **D.** The median test score is the one score above and below which half of the scores lie. Note that 10 scores are below 80, which means that 11 scores are above 80. So the median score should be a little above 80, as in Choice **D.**

42. **C.** Checking the answer choices one at a time, it will be noted that **C** is correct.

43. **B.**



A line has been sketched that seems to best fit the data presented. Since the line travels downhill to the right, our function, *t(w),* must have a negative slope, as in A, B, and C. Next, the *t*-intercept appears to be closer to 40 than to 35, as in A and B. Last, you must compute the slope of the best-fit line, using the two circled points. Going from left to right down hill, the vertical decrease is 4 squares (but each square represents 5 units) or –20; the horizontal change is 5 squares (here, each square is just 1 unit) or +5. So the slope = $\frac{\text{vert. change}}{\text{horiz. change}} = \frac{-20}{+5} = -4$. Therefore, the equation of our line of best fit is: $t(w) = -4w + 40$

44. **69 or 70.** Looking above the values for the mothers' heights that are expressed along the horizontal axis, the frequency of heights are 2, 2, 1, 2, 3, etc. There are no entries above a mother's height of 69 or 70 inches. Therefore, the correct answer is 69 or 70.

Probability

45. On a true-false quiz having just four questions, what is probability of guessing and getting all four questions correct?

A. $\frac{1}{16}$

B. $\frac{1}{8}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

E. 2

46. A marble is to be selected at random from a jar that contains marbles of five different colors. If the probability that a red marble will be selected is $\frac{2}{7}$, which of the following could NOT be the number of marbles in the jar?

A. 14
B. 35
C. 42
D. 65
E. 70

47. A jar contains three green, four red, and five blue marbles. In four successive draws, with no replacement of marbles that have been already drawn out, what is the probability of drawing a green, a red, another green, and then a blue marble in exactly that order?

A. $\frac{5}{576}$ (approximately 0.009)

B. $\frac{1}{99}$ (approximately 0.010)

C. $\frac{1}{66}$ (approximately 0.015)

D. $\frac{7}{6}$ (approximately 1.167)

E. $\frac{2711}{1980}$ (approximately 1.369)

48. Five students are chosen to attend a student nominating convention for their school election to represent their geometry class. In how many ways can the five students be arranged in the five chairs that are reserved for their seating?

Explanations

45. **A.** The probability of guessing a give question correct is $\frac{1}{2}$. So the probability of getting all four correct is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$.

46. **D.** If the probability of selecting a red marble is $\frac{2}{7}$, then the total number of marbles has to be multiple of 7, the denominator. Therefore, 65 cannot be the total number of marbles, since it is not a multiple of 7.

47. **B.** Probability of drawing a green first = $\frac{3}{12}$, 3 green out of 12 total marbles.

Probability of drawing a red second = $\frac{4}{11}$, 4 red out of 11 marbles left in jar

Probability of drawing a green third = $\frac{2}{10}$, 2 green remain out of 10 marbles

Probability of drawing a blue fourth = $\frac{5}{9}$, 5 blue out of 9 marbles left

Then the final probability is just the product of the 4 probabilities found above: $\frac{3}{12} \cdot \frac{4}{11} \cdot \frac{2}{10} \cdot \frac{5}{9} = \frac{1}{99}$, when completely reduced.

48. **120.** Since this is an arrangement problem, the solution is the number of permutations of five students arranged five at a time. If your calculator has a permutation option, this can be calculated on the calculator using the appropriate sequence of keystrokes. In a non-calculator method, you use the following:

Make five consecutive blanks in a line to represent the five seats. Next, there are 5 ways to pick the person for the first seat. Once that person is selected, there are 4 ways to pick the person for the second seat, then 3 ways to pick the third seat, then 2 ways, and then 1 way to pick the last seat. Since this is a permutation problem, the arithmetic will be:

— — — — —

$5 \times 4 \times 3 \times 2 \times 1 = 120$

Geometric Probability

49. The probability of a high school's girl's volleyball, basketball, and softball teams winning their league title this year is 0.90, 0.75, and 0.82, respectively. What is the approximate probability that both the volleyball and softball teams win their league titles, but basketball does not?

A. 0.005
B. 0.014
C. 0.185
D. 0.554
E. 1.970

50. Using the letters of the word "EXCLUSION," five letters are chosen at random to form a "word;" that is, any arrangement of the five letters. What is the probability that the "word" chosen begins with a vowel and ends with a consonant?

A. $\frac{27}{700}$ (approximately 0.039)

B. $\frac{9}{50}$ (approximately 0.180)

C. $\frac{6}{23}$ (approximately 0.261)

D. $\frac{5}{18}$ (approximately 0.278)

E. $\frac{17}{48}$ (approximately 0.354)

51. A deck of playing cards has 52 cards in 4 different suits—Clubs, Diamonds, Hearts, and Spades, with 13 cards in each suit. Within each suit, cards are labeled as 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, and Ace. If one card is randomly drawn from the deck in each of 4 successive draws—with no replacement of cards after each draw—which of the following could NOT be a possible probability of drawing an Ace, a Club, a King, and then another Club, in specifically that order?

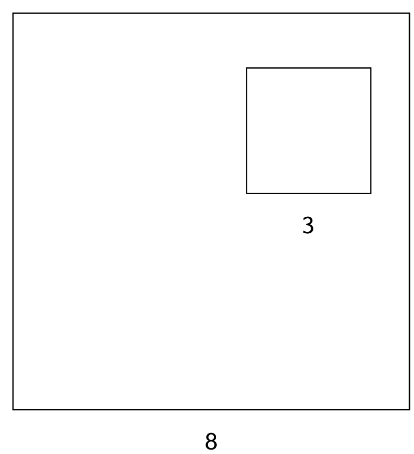
A. $\frac{4}{52} \times \frac{13}{51} \times \frac{4}{50} \times \frac{12}{49}$

B. $\frac{4}{52} \times \frac{13}{51} \times \frac{4}{50} \times \frac{11}{49}$

C. $\frac{4}{52} \times \frac{12}{51} \times \frac{4}{50} \times \frac{11}{49}$

D. $\frac{4}{52} \times \frac{12}{51} \times \frac{4}{50} \times \frac{10}{49}$

E. $\frac{45}{52} \times \frac{12}{51} \times \frac{4}{50} \times \frac{9}{49}$



52. A small square is located in the interior of a larger square as shown in the diagram above. The large square is 8 units on a side and the small square is 3 units on a side. If a point is randomly selected from the region of the large square, what is the probability that the point will be in the smaller square?

Explanations

49. **C.** Probability of the volleyball team winning = 0.90. Probability of the basketball team losing = 0.25; this is just 1.00 – 0.75. Probability of the softball team winning = 0.82. The requested probability is the product of the three probabilities above: (0.90)(0.25)(0.82) = 0.185

50. **D.** The word "EXCLUSION" has a total of nine letters—four of them vowels and five of them consonants. You want a five-letter "word" that begins with a vowel, but ends with a consonant.

Choice #1—Select a vowel: This can be done in four ways.

Choice #2—Select a consonant: This can be done in five ways.

Choice #3—Select the remaining three letters to complete the five-letter "word": This can be done in $7 \times 6 \times 5 = 210$ ways (after choosing a vowel and a consonant, you have only seven letters remaining, of which you have to select three).

Note: The number of five-letter "words" that can be chosen under the given conditions is the product of our three choices above: $4 \times 210 \times 5 = 4{,}200$

Choice #4—The number of five-letter "words" that can be chosen with no conditions on our choices is: $9 \times 8 \times 7 \times 6 \times 5 = 15{,}120$

Last, the requested probability is the ratio of $\frac{4200}{15120} = \frac{5}{18} = 0.278$ approximately.

To do this all in one step, you have the following (where #1 through #4 represent the choices you made above):

$$\frac{\overset{\#1}{4}\times\overset{\#3}{(7\times6\times5)}\times\overset{\#2}{5}}{\underset{\#4}{9\times8\times7\times6\times5}} = \frac{5}{18} \text{ after completely reducing.}$$

51. **E.** Note that the first and third fractions for each answer choice are the same. These just represent:

For the first fraction: 4 out of 52 cards are Aces.

For the third fraction: 4 out of the remaining 50 cards are Kings – 50 cards because 2 have already been drawn during the first 2 draws.

Also note that the denominators decrease by one as you move from the first to the second to the third and then to the fourth fractions since there is one card less in the deck after the preceding draw has been made.

So you only need to look at the second and fourth fractions to locate which could NOT be possible.

In **A:** If the Ace of Clubs is <u>not</u> chosen, there are still 13 Clubs left in the deck of 51 remaining cards. Therefore, the probability of drawing a Club is 13/51. If the King of Clubs is <u>not</u> chosen, there are 12 Clubs left – so the probability of drawing a Club on the fourth draw is 12/49.

In **B:** If the Ace of Clubs is <u>not</u> chosen, there are 13 clubs left – so the probability of drawing a Club on the second draw is 13/51. If the King of Clubs is chosen, there are 11 Clubs left – so the probability of drawing a Club on the fourth draw is 11/49.

In **C:** If the Ace of Clubs is chosen, there are 12 Clubs left – so the probability of drawing a Club on the second draw is 12/51. If the King of Clubs is <u>not</u> chosen, there are 11 Clubs left – so probability of drawing a Club on the fourth draw is 11/49.

In **D:** If the Ace of Clubs is chosen, there are 12 Clubs left – so the probability of drawing a Club on the second draw is 12/51. If the King of Clubs is chosen, there are 10 Clubs left – so the probability of drawing a Club on the fourth draw is 10/49.

So by default, Choice **E** is NOT possible.

52. $\frac{\mathbf{9}}{\mathbf{64}}$ **or 0.14** The area of the larger square is given by 8^2, which is equal to 64 square units. The area of the smaller square is 3^2, which is 9 square units. Probability is defined as the numbers of ways the event (a point in the small square) can occur divided by the total ways a point can be selected (sample space) for the larger area. Therefore, the probability is:

$P = \frac{9}{64}$ or gridded as approximately 0.14.

Basic Statistics (Mean, Mode, Median, Range)

53. Jahwad earned grades of 85%, 78%, 73%, and 81% on his last four tests. What score does he need to earn on his next test in order to average 80% on all five tests?

A. 75%
B. 79%
C. 80%
D. 83%
E. 84%

54. If 7 is added to each of 13 different numbers, which of the following statements would NOT be true?

A. The mode is increased by 7.
B. The average (arithmetic mean) of the 6 largest numbers is increased by 7.
C. The median of the 13 numbers is increased by 7.
D. The range of the 13 numbers is increased by 7.
E. The average (arithmetic mean) of the 5 smallest numbers is increased by 7.

55. The average (arithmetic mean) of a set of six two-digit even integers is 17. What is the largest possible even integer in this set?

A. 20
B. 22
C. 32
D. 34
E. 40

56. The following numbers were obtained in a statistical experiment, but one of the numbers was misplaced. If the misplaced number is *k,* the other numbers are 14, 11, 16, 17, 26, 16, 21, 11, and the mean is known to be 16, what is the value of *k?*

Explanations

53. **D.** Let T be the score on Jahwad's next test. Then to earn an 80% average on the five tests, you have:

$$\frac{85 + 78 + 73 + 81 + T}{5} = 80$$
$$\frac{371 + T}{5} = 80$$
$$5\left(\frac{317 + T}{5}\right) = 5(80)$$
$$317 + T = 400$$
$$T = 83$$

54. **D.** If 7 is added to each member of the set, the mode, median, and mean are all increased by 7. The range is the difference between the largest and smallest of the numbers in the set, so the new range would be found by: (largest number + 7) – (smallest number + 7)

= largest number + 7 – smallest number -7

= largest number – smallest number

= original range

Thus range is not affected.

55. C. If the average of the six positive even integers is 17, then the total of the six integers is: $6(17) = 102$. The smallest that the first five positive even two-digit integers could be: 10, 12, 14, 16, and 18. The sum of these five numbers is 70. Therefore, the sixth and largest possible even integer would be: $102 - 70 = 32$.

56. 12. The process in finding a mean requires finding the sum of all the numbers that are being added. This value must be 16×9 since there are nine numbers that figure into the mean. Therefore, $\frac{132 + k}{9} = 16$ and $132 + k = 16 \times 9 = 144$.

$k = 144 - 132$ and $k = 12$

The correct answer is 12.

Algebra and Functions

Algebra Diagnostic Test (Algebra and Functions)

Questions

1. The sum of a number and 9 can be written:

2. Evaluate: $3x^2 + 5y + 7$ if $x = -2$ and $y = 3$.

3. Solve for x: $x + 5 = 17$.

4. Solve for x: $4x + 9 = 21$.

5. Solve for x: $5x + 7 = 3x - 9$.

6. Solve for x: $|x| - 4 = 8$.

7. Solve for x: $mx - n = y$.

8. Solve for x: $\frac{r}{x} = \frac{s}{t}$.

9. Solve for y: $\frac{3}{7} = \frac{y}{8}$.

10. Simplify $8xy^2 + 3xy + 4xy^2 - 2xy$.

11. Simplify $6x^2(4x^3y)$.

12. $x^{-5} =$

13. Simplify $(5x + 2z) + (3x - 4z)$.

14. Simplify $(4x - 7z) - (3x - 4z)$.

15. Factor $ab + ac$.

16. Factor $x^2 - 5x - 14$.

17. Solve $x^2 + 7x = -10$.

18. Solve for x: $2x + 3 \leq 11$.

19. Solve for x: $3x + 4 \geq 5x - 8$.

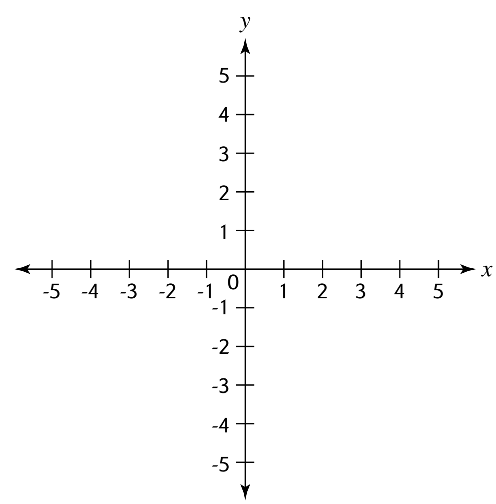
20. Solve for x: $|x - 3| < 6$.

21. Solve for x: $2|x| + 4 \geq 8$.

22. Solve this system for x and y:

$$8x + 2y = 7$$
$$3x - 4y = 5$$

23. Graph $x + y = 6$ on the following graph:



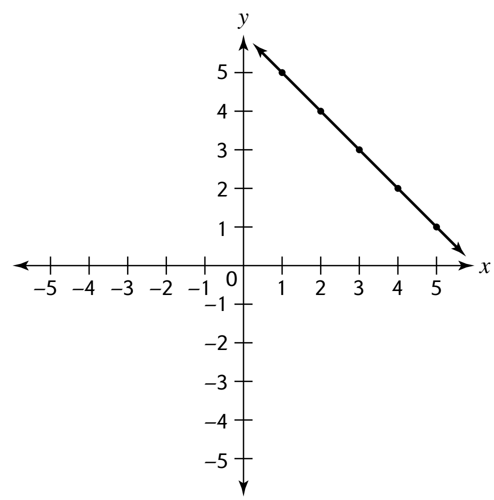
24. What is the slope and the y-intercept of the equation $y = 6x + 2$?

25. What is the equation of the line passing through the points (3,6) and (2,–3)?

26. If $x = 3$, then what is the value of $f(x)$ if $f(x) = x^2 + 3x + 7$?

Answers

1. $n + 9$ or $9 + n$
2. 34
3. $x = 12$
4. $x = 3$
5. $x = -8$
6. $x = 12$ or -12
7. $x = \frac{(y+n)}{m}$
8. $x = \frac{rt}{s}$
9. $y = \frac{24}{7}$ or $3\frac{3}{7}$
10. $12xy^2 + xy$
11. $24x^5y$
12. $\frac{1}{x^5}$
13. $8x - 2z$
14. $x - 3z$
15. $a(b + c)$
16. $(x - 7)(x + 2)$
17. $x = -2$ or $x = -5$
18. $x \leq 4$
19. $x \leq 6$
20. $-3 < x < 9$ could also be written $x > -3$ and $x < 9$
21. $x \geq 2$ or $x \leq -2$
22. $x = 1,\ y = -\frac{1}{2}$
23.



24. Slope = 6, y-intercept = 2
25. $y = 9x - 21$
26. 25

Algebra Review

Variables and Algebraic Expressions

A *variable* is a symbol used to denote any element of a given set—often a letter used to stand for a number. Variables are used to change verbal expressions into *algebraic expressions.* For example:

Verbal Expression	***Algebraic Expression***
(a) the sum of a number and 7	$n + 7$ or $7 + n$
(b) the number diminished by 10	$n - 10$
(c) seven times a number	$7n$
(d) x divided by 4	$x/4$
(e) five more than the product of 2 and n	$2n + 5$ or $5 + 2n$

These words can be helpful in making algebraic expressions.

Key Words Denoting Addition		
sum	larger than	enlarge
plus	gain	rise
more than	increase	grow
greater than		

Key Words Denoting Subtraction		
difference	smaller than	lower
minus	fewer than	diminish
lose	decrease	reduced
less than	drop	

Key Words Denoting Multiplication		
product	times	of
multiplied by	twice	

Key Words Denoting Division	
quotient	ratio
divided by	half

Evaluating Expressions

To *evaluate* an expression, insert the value for the unknowns and do the arithmetic.

For example:

Evaluate each of the following.

1. $ab + c$ if $a = 5$, $b = 4$ and $c = 3$

$$\begin{aligned} 5(4) + 3 &= \\ 20 + 3 &= 23 \end{aligned}$$

2. $2x^2 + 3y + 6$ if $x = 2$ and $y = 9$

$$\begin{aligned} 2(2^2) + 3(9) + 6 &= \\ 2(4) + 27 + 6 &= \\ 8 + 27 + 6 &= 41 \end{aligned}$$

3. If $x = -4$, then $|6 - x| - |x - 2| =$

$$\begin{aligned} |6 - (-4)| - |(-4) - 2| &= \\ |6 + 4| - |-4 - 2| &= \\ |10| - |-6| &= \\ 10 - 6 &= 4 \end{aligned}$$

Equations

An *equation* is a relationship between numbers and/or symbols. It helps to remember that an equation is like a balance scale, with the equal sign (=) being the fulcrum, or center. Thus, if you do the *same thing to both sides* of the equal sign (say, add 5 to each side), the equation is still balanced. To solve an equation, first you must get the variable you are looking for on one side of the equal sign and everything else on the other side.

For example:

1. Solve for x: $x - 5 = 23$

To solve the equation $x - 5 = 23$, get x by itself on one side; therefore, add 5 to both sides:

$$\begin{array}{rcl} x - 5 &=& 23 \\ +5 && +5 \\ \hline x &=& 28 \end{array}$$

In the same manner, subtract, multiply, or divide *both* sides of an equation by the same (nonzero) number, and the equation does not change. Sometimes you have to use more than one step to solve for an unknown.

For example:

2. Solve for x: $3x + 4 = 19$

Subtract 4 from both sides to get the $3x$ by itself on one side:

$$\begin{array}{rcl} 3x + 4 &=& 19 \\ -4 && -4 \\ \hline 3x &=& 15 \end{array}$$

Then, divide both sides by 3 to get x:

$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

3. Solve for x: $6x + 3 = 4x + 5$

Add –3 to each side:

$$\begin{array}{r} 6x + 3 = 4x + 5 \\ \underline{\quad -3 \qquad -3} \\ 6x \qquad = 4x + 2 \end{array}$$

Add –4x to each side:

$$\begin{array}{r} 6x = 4x + 2 \\ \underline{-4x \; -4x \qquad} \\ 2x = \qquad 2 \end{array}$$

Divide each side by 2:

$$\frac{2x}{2} = \frac{2}{2}$$
$$x = 1$$

Remember: Solving an equation requires using opposite operations until the variable is on one side by itself (for addition, subtract; for multiplication, divide; and so on).

Solving Equations Containing Absolute Value

To solve an equation containing absolute value, isolate the absolute value on one side of the equation. Then, set its contents equal to both + and – the other side of the equation and solve both equations.

For example:

1. Solve for x: $|x| + 2 = 5$

Isolate the absolute value

$$\begin{array}{r} |x| + 2 = \; 5 \\ \underline{\quad -2 \;\; -2} \\ |x| \qquad = \; 3 \end{array}$$
$$x = 3$$
$$x = -3$$

Set the contents of the absolute value portion equal to +3 and –3.

$$x = 3$$
$$x = -3$$

Answer: 3, –3

2. Solve for x: $3|x - 1| - 1 = 11$

Isolate the absolute value.

$$\begin{array}{lll} & 3|x-1|-1 & = 11 \\ & \quad\quad\quad +1 & +1 \\ \hline & 3|x-1| & = 12 \\ \text{Divide by 3} & \dfrac{3|x-1|}{3} & = \dfrac{12}{3} \\ & |x-1| & = 4 \end{array}$$

Set the contents of the absolute value portion equal to +4 and –4.

Solving for x,

$$\begin{array}{ll} x-1 = 4 & x-1 = -4 \\ \underline{\quad +1 \quad +1} & \underline{\quad +1 \quad +1} \\ x \quad\ = 5 & x \quad\ = -3 \end{array}$$

Answer: 5, –3

Understood Multiplying

When two or more variables, or a number and variables, are written next to each other, they are understood to be multiplied. Thus, $8x$ means 8 times x, ab means a times b, and $18ab$ means 18 times a times b.

Parentheses also represent multiplication. Thus, $(a)b$ means a times b. A raised dot also means multiplication. Thus, $6 \cdot 5$ means 6 times 5.

Literal Equations

Literal equations have no numbers, only symbols (variables). For example:

Solve for Q: $QP - X = Y$

First add X to both sides:

$$\begin{array}{ll} QP - X & = Y \\ \quad\ \ +X & \ \ +X \\ \hline QP & = Y + X \end{array}$$

Then, divide both sides by P:

$$\frac{QP}{P} = \frac{Y+X}{P}$$
$$Q = \frac{Y+X}{P}$$

Again opposite operations were used to isolate Q.

Cross Multiplying

Solve for x: $\frac{b}{x} = \frac{p}{q}$

To solve this equation quickly, cross multiply. To cross multiply:

1. Bring the denominators up next to the numerators on the opposite side.
2. Multiply.

$$\frac{b}{x} = \frac{p}{q}$$
$$bq = px$$

Then, divide both sides by p to get x alone:

$$\frac{bq}{p} = \frac{px}{p}$$
$$\frac{bq}{p} = x \text{ or } x = \frac{bq}{p}$$

Cross multiplying can be used only when the format is two fractions separated by an equal sign.

Proportions

Proportions are written as two fractions equal to each other.

Solve this proportion for x: $\frac{p}{q} = \frac{x}{y}$

This is read "p is to q as x is to y." Cross multiply and solve:

$$py = xq$$
$$\frac{py}{q} = \frac{xq}{q}$$
$$\frac{py}{q} = x \text{ or } x = \frac{py}{q}$$

Monomials and Polynomials

A *monomial* is an algebraic expression that consists of only one term. For instance, $9x$, $4a^2$, and $3mpxz^2$ are all monomials.

A *polynomial* consists of two or more terms; $x + y$, $y^2 - x^2$, and $x^2 + 3x + 5y^2$ are all polynomials.

Adding and Subtracting Monomials

To add or subtract monomials, follow the same rules as with regular signed numbers, provided that the terms are alike:

$$\begin{array}{r} 15x^2yz \\ \underline{-18x^2yz} \\ -3x^2yz \end{array} \qquad 3x + 2x = 5x$$

Multiplying and Dividing Monomials

To multiply monomials, add the exponents of the same terms:

$$(x^3)(x^4) = x^7$$
$$(x \cdot x \cdot x)(x \cdot x \cdot x \cdot x) = x^7$$

To divide monomials, subtract the exponents of like terms:

$$\frac{y^{15}}{y^4} = y^{11} \qquad \frac{x^5y^2}{x^3y} = x^2y \qquad \frac{36a^4b^6}{-9ab} = -4a^3b^5$$

Remember: x is the same as x^1.

Working with Negative Exponents

If an exponent is negative, such as x^{-3}, then the variable and exponent can be dropped under the number 1 in a fraction to remove the negative sign, as follows.

$$x^{-3} = \frac{1}{x^3}$$

Another example:

$$a^{-5} = \frac{1}{a^5}$$

A few examples including some with multiplication and division follow:

1. $a^{-2}b = \frac{b}{a^2}$
2. $\frac{a^{-3}}{b^4} = \frac{1}{a^3 b^4}$
3. $\left(a^2 b^{-3}\right)\left(a^{-1} b^4\right) = ab$
   $\begin{bmatrix} a^2 \bullet a^{-1} = a \\ b^{-3} \bullet b^4 = b \end{bmatrix}$

If the negative exponent belongs to a number or variable below the fraction bar, then simply bring the number or variable up and drop the negative sign.

4. $\frac{1}{x^{-2}} = x^2$
5. $\frac{x^6}{x^{-3}} = x^6 \bullet x^3 = x^9$
6. $\left(3x^{-2}\right)^{-2} = 3^{-2} \bullet x^{-2 \cdot -2} = 3^{-2} \bullet x^4 = \frac{1}{9} \bullet x^4 = \frac{x^4}{9}$
7. $\left(\frac{1}{3}\right)^{-2} = \frac{1}{\left(\frac{1}{3}\right)^2} = \frac{1}{\frac{1}{9}} = \frac{9}{1} = 9$

Or simply invert the fraction and drop the negative sign:

$$\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^2 = 9$$

If the exponent is a fraction, then the number above the bar is the power, and the number below the bar is the root

For example:

$$3^{\frac{1}{2}} = \sqrt[2]{3^1} = \sqrt{3}$$

(2 is understood in the square root)

A few more examples:

$$5^{\frac{1}{3}} = \sqrt[3]{5^1} = \sqrt[3]{5}$$
$$6^{\frac{2}{3}} = \sqrt[3]{6^2} = \sqrt[3]{36}$$
$$2^{\frac{3}{2}} = \sqrt[2]{2^3} = \sqrt{8} \text{ reduces to } \sqrt{4 \bullet 2} = 2\sqrt{2}$$
$$4^{\frac{1}{4}} = \sqrt[4]{4^1} = \sqrt[4]{4}$$

Adding and Subtracting Polynomials

To add or subtract polynomials, just arrange like terms in columns, and then add or subtract:

Add:

$$\begin{array}{rr} & a^2 + ab + b^2 \\ \text{Add:} & \underline{3a^2 + 4ab - 2b^2} \\ & 4a^2 + 5ab - b^2 \end{array}$$

Subtract:

$$\begin{array}{r} a^2+b^2 \\ \underline{(-)\,2a^2-b^2} \end{array} \rightarrow \begin{array}{r} a^2+\ \ b^2 \\ \underline{+-2a^2+\ \ b^2} \\ -a^2+2b^2 \end{array}$$

Multiplying Polynomials

To multiply polynomials, multiply each term in one polynomial by each term in the other polynomials. Then simplify if necessary:

$$(3x+a)(2x-2a)=$$

$$\begin{array}{r} 2x-2a \\ \times\underline{3x+a} \\ +2ax-2a^2 \\ \underline{6x^2-6ax\qquad} \\ 6x^2-4ax-2a^2 \end{array} \quad \text{similar to} \quad \begin{array}{r} 23 \\ \underline{\times 19} \\ 207 \\ \underline{230} \\ 427 \end{array}$$

Factoring

To *factor* means to find two or more quantities whose product equals the original quantity.

Factoring Out a Common Factor

Factor $2y^3 - 6y$.

1. Find the largest common monomial factor of each term.
2. Divide the original polynomial by this factor to obtain the second factor. (The second factor is a polynomial.)

$$2y^3 - 6y = 2y(y^2 - 3)$$

Another example: $x^5 - 4x^3 + x^2 = x^2(x^3 - 4x + 1)$

Factoring the Difference between Two Squares

Factor $x^2 - 144$.

1. Find the square root of the first term and the square root of the second term.
2. Express your answer as the product of the sum of the quantities from step 1 times the difference of those quantities.

$$x^2 - 144 = (x + 12)(x - 12)$$

Another example: $a^2 - b^2 = (a + b)(a - b)$

Another example: $9x^2 - 16y^2 = (3x + 4y)(3x - 4y)$

Note: $x^2 + 144$ is not factorable.

Factoring Polynomials That Have Three Terms: $Ax^2 + Bx + C$

To factor polynomials that have three terms, of the form $Ax^2 + Bx + C$:

1. Check to see if you can monomial factor (factor out common terms). Then, if $A = 1$ (that is, the first term is simply x^2), use double parentheses and factor the first term. Place these factors in the left sides of the parentheses. For example, $(x\ \ )(x\ \ )$.
2. Factor the last term, and place the factors in the right sides of the parentheses.

To decide on the signs of the numbers, do the following:

If the sign of the last term is *negative*:

1. Find two numbers whose product is the last term and whose *difference* is the *coefficient* (number in front) of the middle term.
2. Give the larger of these two numbers the sign of the middle term, and give the *opposite* sign to the other factor.

If the sign of the last term is *positive*:

1. Find two numbers whose product is the last term and whose sum is the coefficient of the middle term.
2. Give both factors the sign of the middle term.

For example,

1. Factor $x^2 - 3x - 10$.

   First check to see if you can *monomial factor* (factor out common terms). Because this is not possible, use double parentheses and factor the first terms as follows: $(x\quad)(x\quad)$.

   Next, factor the last term (10) into 2 times 5. (Using the preceding information, 5 must take the negative sign and 2 must take the positive sign because they then total the coefficient of the middle term, which is –3.) Add the proper signs, leaving:

   $$(x - 5)(x + 2)$$

   Multiply the means (inner terms) and extremes (outer terms) to check your work.

   $$\begin{array}{l} (x - 5)(x + 2) \\ \quad -5x \\ \quad \underline{+2x} \\ \quad -3x \text{ (which is the middle term)} \end{array}$$

   To completely check, multiply the factors together.

   $$\begin{array}{r} x - 5 \\ \underline{\times\ \ x + 2} \\ +2x - 10 \\ \underline{x^2 - 5x\qquad\ } \\ x^2 - 3x - 10 \end{array}$$

2. Factor $x^2 + 8x + 15$.

   $(x + 3)(x + 5)$

   Notice that $3 \times 5 = 15$ and $3 + 5 = 8$, the coefficient of the middle term. Also, the signs of both factors are +, the sign of the middle term. To check your work:

   $$\begin{array}{l} (x + 3)(x + 5) \\ \quad +3x \\ \quad \underline{+5x} \\ \quad +8x \text{ (the middle term)} \end{array}$$

   If, however, $A \neq 1$ (that is, the first term has a coefficient—for example, $4x^2 + 5x + 1$), then additional trial and error is necessary.

3. Factor $4x^2 + 5x + 1$.

$(2x + \ )(2x + \ )$ might work for the first term. But when 1's are used as factors to get the last term—$(2x + 1)(2x + 1)$—the middle term comes out as $4x$ instead of $5x$.

$$\begin{array}{c} (2x + 1)(2x + 1) \\ +2x \\ \underline{+2x} \\ +4x \end{array}$$

Therefore, try $(4x + \ )(x + \ )$. This time, using 1's as factors to get the last terms gives $(4x + 1)(x + 1)$. Checking for the middle term:

$$\begin{array}{c} (4x + 1)(x + 1) \\ +1x \\ \underline{+4x} \\ +5x \end{array}$$

Therefore, $4x^2 + 5x + 1 = (4x + 1)(x + 1)$.

4. Factor $5x^3 + 6x^2 + x$.

Factoring out an x leaves $x(5x^2 + 6x + 1)$.

Now, factor as usual giving $x(5x + 1)(x + 1)$.

To check your work:

$$\begin{array}{c} (5x + 1)(x + 1) \\ +1x \\ \underline{+5x} \\ +6x \end{array}$$

(the middle term after x was factored out)

Solving Quadratic Equations

A *quadratic equation* is an equation that could be written in the form $Ax^2 + Bx + C = 0$. To solve a quadratic equation:

1. Put all terms on one side of the equal sign, leaving zero on the other side.
2. Factor.
3. Set each factor equal to zero.
4. Solve each of these equations.
5. Check by inserting your answer in the original equation.

For example,

1. Solve $x^2 - 6x = 16$.

Following the steps, $x^2 - 6x = 16$ becomes $x^2 - 6x - 16 = 0$.

Factoring, $(x - 8)(x + 2) = 0$

$$\begin{array}{lcl} x - 8 = 0 & \text{or} & x + 2 = 0 \\ x \quad = 8 & & x \quad = -2 \end{array}$$

To check:

$$
\begin{aligned}
8^2 - 6(8) &= 16 \quad \text{or} \quad (-2)^2 - 6(-2) = 16 \\
64 - 48 &= 16 \quad \text{or} \quad 4 + 12 = 16 \\
16 &= 16 \quad \text{or} \quad 16 = 16
\end{aligned}
$$

Both values 8 and –2 are solutions to the original equation.

2. Solve $y^2 = -6y - 5$.
Setting all terms equal to zero:

$$y^2 + 6y + 5 = 0$$

Factoring, $(y + 5)(y + 1) = 0$
Setting each factor to 0:

$$
\begin{aligned}
y + 5 &= 0 \quad \text{or} \quad y + 1 = 0 \\
y &= -5 \quad \text{or} \quad y = -1
\end{aligned}
$$

To check:

$$
\begin{aligned}
(-5)^2 &= -6(-5) - 5 \quad \text{or} \quad (-1)^2 = -6(-1) - 5 \\
25 &= 30 - 5 \quad \text{or} \quad 1 = 6 - 5 \\
25 &= 25 \quad \text{or} \quad 1 = 1
\end{aligned}
$$

A quadratic equation with a term missing is called an *incomplete quadratic equation*.

3. Solve $x^2 - 16 = 0$.
Factoring, $(x + 4)(x - 4) = 0$:

$$
\begin{aligned}
x + 4 &= 0 \quad \text{or} \quad x - 4 = 0 \\
x &= -4 \quad \text{or} \quad x = 4
\end{aligned}
$$

To check:

$$
\begin{aligned}
(-4)^2 - 16 &= 0 \quad \text{or} \quad (4)^2 - 16 = 0 \\
16 - 16 &= 0 \quad \text{or} \quad 16 - 16 = 0 \\
0 &= 0 \quad \text{or} \quad 0 = 0
\end{aligned}
$$

4. Solve $x^2 + 6x = 0$.
Factoring, $x(x + 6) = 0$:

$$
\begin{aligned}
x &= 0 \quad \text{or} \quad x + 6 = 0 \\
x &= 0 \quad \text{or} \quad x = -6
\end{aligned}
$$

To check:

$$
\begin{aligned}
(0)^2 + 6(0) &= 0 \quad \text{or} \quad (-6)^2 + 6(-6) = 0 \\
0 + 0 &= 0 \quad \text{or} \quad 36 + -36 = 0 \\
0 &= 0 \quad \text{or} \quad 0 = 0
\end{aligned}
$$

Algebraic fractions are fractions using a variable in the numerator or denominator, such as $\frac{3}{x}$. Since division by 0 is impossible, variables in the denominator have certain restrictions. The denominator can never equal 0. Therefore in the fractions

$\frac{5}{x}$ x cannot equal 0 $(x \neq 0)$

$\frac{2}{x-3}$ x cannot equal 3 $(x \neq 3)$

$\frac{3}{a-b}$ $a-b$ cannot equal 0 $(a-b \neq 0)$
so a cannot equal b $(a \neq b)$

$\frac{4}{a^2b}$ a cannot equal 0 and b cannot equal 0
$(a \neq 0$ and $b \neq 0)$

Be aware of these types of restrictions.

Operations with Algebraic Fractions

Reducing Algebraic Fractions

- To *reduce an algebraic fraction* to lowest terms, first factor the numerator and the denominator; then cancel (or divide out) common factors.

Examples:

1. Reduce $\frac{4x^3}{8x^2}$

$$\frac{\overset{1}{\cancel{4}}\overset{1}{\cancel{x^3}}}{\underset{2}{\cancel{8}}\cancel{x^2}} = \frac{1}{2}x$$

2. Reduce $\frac{3x-3}{4x-4}$

$$\frac{3x-3}{4x-4} = \frac{3(x-1)}{4(x-1)} = \frac{3\cancel{(x-1)}}{4\cancel{(x-1)}} = \frac{3}{4}$$

3. Reduce $\frac{x^2+2x+1}{3x+3}$

$$\frac{x^2+2x+1}{3x+3} = \frac{(x+1)(x+1)}{3(x+1)} = \frac{\cancel{(x+1)}(x+1)}{3\cancel{(x+1)}}$$
$$= \frac{x+1}{3}$$

4. Reduce $\frac{x^2-y^2}{x^3-y^3}$

$$\frac{x^2-y^2}{x^3-y^3} = \frac{(x-y)(x+y)}{(x-y)(x^2+xy+y^2)} =$$
$$\frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}(x^2+xy+y^2)} = \frac{x+y}{x^2+xy+y^2}$$

Warning: Do *not* cancel through an addition or subtraction sign. For example:

$$\frac{x+1}{x+2} \neq \frac{\cancel{x}+1}{\cancel{x}+2} \neq \frac{1}{2}$$

or

$$\frac{x+6}{6} \neq \frac{x+\cancel{6}}{\cancel{6}} \neq x$$

Multiplying Algebraic Fractions

- To *multiply algebraic functions,* first factor the numerator and denominators that are polynomials; then cancel where possible. Multiply the remaining numerators together and denominators together. (If you've canceled properly, your answer will be in reduced form.)

Examples:

1. $\frac{2x}{3} \cdot \frac{y}{5} = \frac{2x}{3} \cdot \frac{y}{5} = \frac{2xy}{15}$

2. $\frac{x^2}{3y} \cdot \frac{2y}{3x} = \frac{x^{\cancel{2}}}{3\cancel{y}} \cdot \frac{2\cancel{y}}{3\cancel{x}} = \frac{2x}{9}$

3. $\frac{x+1}{5y+10} \cdot \frac{y+2}{x^2+2x+1} = \frac{x+1}{5(y+2)} \cdot \frac{y+2}{(x+1)(x+1)} =$

$\frac{\overset{1}{\cancel{x+1}}}{5\cancel{(y+2)}} \cdot \frac{\overset{1}{\cancel{y+2}}}{\cancel{(x+1)}(x+1)} = \frac{1}{5(x+1)}$

Dividing Algebraic Fractions

- To *divide algebraic fractions,* invert the fraction following the division sign and multiply. Remember you can cancel only after you invert.

Examples:

1. $\frac{3x^2}{5} \div \frac{2x}{y} = \frac{3x^2}{5} \cdot \frac{y}{2x} = \frac{3x^{\cancel{2}\,1}}{5} \cdot \frac{y}{2\cancel{x}} = \frac{3xy}{10}$

2. $\frac{4x-8}{6} \div \frac{x-2}{3} = \frac{4x-8}{6} \cdot \frac{3}{x-2} = \frac{4(x-2)}{6} \cdot \frac{3}{x-2} =$

$\frac{4\cancel{(x-2)}^{1}}{\cancel{6}_{2}} \cdot \frac{\cancel{3}^{1}}{\cancel{x-2}_{1}} = \frac{4}{2} = 2$

Adding or Subtracting Algebraic Fractions

- *To add or subtract algebraic fractions having a common denominator,* simply keep the denominator and combine (add or subtract) the numerators. Reduce if necessary.

Examples:

1. $\frac{4}{x} + \frac{5}{x} = \frac{4+5}{x} = \frac{9}{x}$

2. $\frac{x-4}{x+1} + \frac{3}{x+1} = \frac{x-4+3}{x+1} = \frac{x-1}{x+1}$

3. $\frac{3x}{y} - \frac{2x-1}{y} = \frac{3x-(2x-1)}{y} = \frac{3x-2x+1}{y} = \frac{x+1}{y}$

- To *add or subtract algebraic fractions having different denominators,* first find a lowest common denominator (LCD), change each fraction to an equivalent fraction with the common denominator, then combine each numerator. Reduce if necessary.

Examples:

1. $\frac{2}{x} + \frac{3}{y} =$

LCD = xy

$\frac{2}{x} \cdot \frac{y}{y} + \frac{3}{y} \cdot \frac{x}{x} = \frac{2y}{xy} \cdot \frac{3x}{xy} = \frac{2y+3x}{xy}$

2. $\frac{x+2}{3x} + \frac{x-3}{6x} =$

LCD = $6x$

$\frac{x+2}{3x} \cdot \frac{2}{2} + \frac{x-3}{6x} = \frac{2x+4}{6x} + \frac{x-3}{6x} = \frac{2x+4+x-3}{6x} = \frac{3x+1}{6x}$

If there is a common variable factor with more than one exponent, use its greatest exponent.

3. $\frac{2}{y^2} - \frac{3}{y} =$

LCD = y^2

$\frac{2}{y^2} - \frac{3}{y} \cdot \frac{y}{y} = \frac{2}{y^2} - \frac{3y}{y^2} = \frac{2-3y}{y^2}$

4. $\frac{4}{x^3y} + \frac{3}{xy^2} =$

LCD = x^3y^2

$\frac{4}{x^3y} \cdot \frac{y}{y} + \frac{3}{xy^2} \cdot \frac{x^2}{x^2} = \frac{4y}{x^3y^2} + \frac{3x^2}{x^3y^2} = \frac{4y+3x^2}{x^3y^2}$

To find the lowest common denominator, it is often necessary to factor the denominators and proceed as follows.

Example:

$$\frac{x}{3x+3} + \frac{2x}{x+1} = \frac{x}{3(x+1)} + \frac{2x}{x+1}$$

$$\text{LCD} = 3(x+1)$$

$$\frac{x}{3(x+1)} + \frac{2x}{x+1} \cdot \frac{3}{3} = \frac{x}{3(x+1)} + \frac{6x}{3(x+1)} =$$

$$\frac{x+6x}{3(x+1)} = \frac{7x}{3(x+1)}$$

Inequalities

An *inequality* is a statement in which the relationships are not equal. Instead of using an equal sign (=), as in an equation, you use > (greater than) and < (less than), or ≥ (greater than or equal to) and ≤ (less than or equal to).

When working with inequalities, treat them exactly like equations, *except*, when you multiply or divide both sides by a negative number, *reverse* the direction of the sign. For example:

1. Solve for x: $2x + 4 > 6$.

$$\begin{array}{rcl} 2x+4 & > & 6 \\ \underline{\quad -4} & & \underline{-4} \\ 2x & > & 2 \\ \frac{2x}{2} & > & \frac{2}{2} \\ x & > & 1 \end{array}$$

2. Solve for x: $-7x > 14$.

Divide by –7 and reverse the sign.

$$\frac{-7x}{-7} < \frac{14}{-7}$$
$$x < -2$$

3. Solve for x: $3x + 2 \geq 5x - 10$.

$$\begin{array}{rcr} 3x+2 & \geq & 5x-10 \\ \underline{\quad -2} & & \underline{\quad\quad -2} \\ 3x & \geq & 5x-12 \\ \underline{-5x} & & \underline{-5x\quad\quad} \\ -2x & \geq & -12 \end{array}$$

Divide both sides by –2 and reverse the sign.

$$\frac{-2x}{-2} \leq \frac{-12}{-2}$$
$$x \leq 6$$

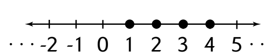
Graphing Inequalities

Examples:

When graphing inequalities involving only integers, dots are used.

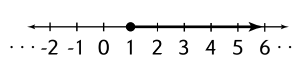
1. Graph the set of x such that $1 \leq x \leq 4$ and x is an integer.

$\{x: 1 \leq x \leq 4, x \text{ is an integer}\}$

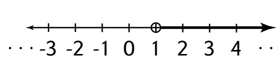


When graphing inequalities involving real numbers, lines, rays, and dots are used. A *dot* is used if the number is included. A *hollow dot* is used if the number is not included.

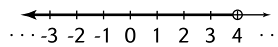
2. Graph the set of x such that $x \geq 1$. $\{x: x \geq 1\}$



3. Graph the set of x such that $x > 1$. $\{x: x > 1\}$



4. Graph the set of x such that $x < 4$. $\{x: x < 4\}$



This ray is often called an *open ray* or *half line.* The hollow dot distinguishes an open ray from a ray.

Solving Inequalities Containing Absolute Value

To solve an inequality containing absolute value, follow the same steps as solving equations with absolute value, except *reverse* the direction of the sign when setting the absolute value opposite a negative.

For example,

1. Solve for x: $|x - 1| > 2$.

Isolate the absolute value.

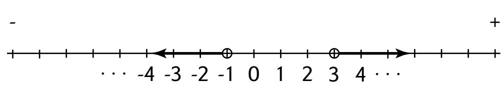
$$|x - 1| > 2$$

Set the contents of the absolute value portion to both 2 and –2. Be sure to change the direction of the sign when using –2.

Solve for x.

$$\begin{array}{rcl} x - 1 > 2 & & x - 1 < -2 \\ \underline{\quad +1 \;\; +1} & \text{or} & \underline{\quad +1 \quad +1} \\ x \qquad > 3 & & x \qquad < -1 \end{array}$$

Graph answer:



2. Solve for x: $3|x| - 2 \le 1$.

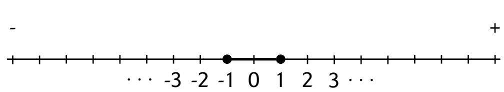
Isolate the absolute value.

$$\begin{array}{rcl} 3|x| - 2 & \le & 1 \\ \underline{\qquad +2} & & \underline{+2} \\ 3|x| & \le & 3 \\ \dfrac{3|x|}{3} & \le & \dfrac{3}{3} \\ |x| & \le & 1 \end{array}$$

Set the contents of the absolute value portion to both 1 and –1. Be sure to change the direction of the sign when using –1.

$$x \le 1 \text{ and } x \ge -1$$

Graph answer:



Solving for Two Unknowns—Systems of Equations

If you solve for two equations with the same two unknowns in each one, you can solve for both unknowns. There are three common methods for solving: addition/subtraction, substitution and graphing.

Addition/Subtraction Method

To use the addition/subtraction method:

1. Multiply one or both equations by some number to make the number in front of one of the variables (the unknowns) the same in each equation.
2. Add or subtract the two equations to eliminate one variable.
3. Solve for the other unknown.
4. Insert the value of the first unknown in one of the original equations to solve for the second unknown.

For example:

1. Solve for x and y:

$$\begin{aligned} 3x + 3y &= 24 \\ 2x + y &= 13 \end{aligned}$$

First multiply the bottom equation by 3. Now the y is preceded by a 3 in each equation.

$$\begin{aligned} 3x + 3y &= 24 & \qquad 3x + 3y &= 24 \\ 3(2x) + 3(y) &= 3(13) & \qquad 6x + 3y &= 39 \end{aligned}$$

Now you can subtract equations, eliminating the y terms.

$$\begin{aligned} 3x + 3y &= 24 \\ \underline{-6x + -3y} &\underline{= -39} \\ -3x \quad &= -15 \\ \frac{-3x}{-3} \quad &= \frac{-15}{-3} \\ x \quad &= 5 \end{aligned}$$

Now insert $x = 5$ in one of the original equations to solve for y.

$$\begin{aligned} 2x + y &= 13 \\ 2(5) + y &= 13 \\ 10 + y &= 13 \\ \underline{-10 \quad} &\underline{\quad -10} \\ y &= 3 \end{aligned}$$

Answer: $x = 5$, $y = 3$

Of course, if the number in front of a variable is already the same in each equation, you do not have to change either equation. Simply add or subtract.

2. Solve for x and y:

$x + y = 7$

$x - y = 3$

$$\begin{array}{rl} x + y = & 7 \\ \underline{x - y =} & \underline{3} \\ 2x \quad = & 10 \\ \frac{2x}{2} \quad = & \frac{10}{2} \\ x \quad = & 5 \end{array}$$

Now, inserting 5 for x in the first equation gives:

$$\begin{array}{rr} 5 + y = & 7 \\ \underline{-5 \qquad} & \underline{-5} \\ y = & 2 \end{array}$$

Answer: $x = 5$, $y = 2$

You should note that this method does not work when the two equations are, in fact, the same.

3. Solve for a and b:

$$3a + 4b = 2$$
$$6a + 8b = 4$$

The second equation is actually the first equation multiplied by 2. In this instance, the system is unsolvable.

4. Solve for p and q:

$$3p + 4q = 9$$
$$2p + 2q = 6$$

Multiply the second equation by 2.

$$(2)2p + (2)2q = (2)6$$
$$4p + 4q = 12$$

Now subtract the equations.

$$\begin{array}{rrrr} 3p\ + & 4q & = & 9 \\ \underline{(-)\ 4p\ +} & \underline{-4q} & \underline{=} & \underline{-12} \\ -p & & = & -3 \\ p & & = & 3 \end{array}$$

Now that you know $p = 3$, you can plug in 3 for p in either of the two original equations to find q.

$$\begin{aligned} 3p + 4q &= 9 \\ 3(3) + 4q &= 9 \\ 9 + 4q &= 9 \\ 4q &= 0 \\ q &= 0 \end{aligned}$$

Answer: $p = 3$, $q = 0$

Substitution Method

Sometimes a system is more easily solved by the substitution method. This method involves substituting one equation into another.

For example,

Solve for x and y when $x = y + 8$ and $x + 3y = 48$.

1. From the first equation, substitute $(y + 8)$ for x in the second equation.

$$(y + 8) + 3y = 48$$

2. Now solve for y. Simplify by combining y's.

$$\begin{aligned} 4y + 8 &= 48 \\ -8 &= -8 \\ \hline 4y \quad &= 40 \\ \frac{4y}{4} \quad &= \frac{40}{4} \\ y \quad &= 10 \end{aligned}$$

3. Now insert $y = 10$ in one of the original equations.

$$\begin{aligned} x &= y + 8 \\ x &= 10 + 8 \\ x &= 18 \end{aligned}$$

Answer: $y = 10$, $x = 18$

Graphing Method

Another method of solving equations is by graphing each equation on a coordinate graph. The coordinates of the intersection are the solution to the system. If you are unfamiliar with coordinate graphing, carefully review the "Basic Coordinate Geometry" section before attempting this method.

For example, solve the following system by graphing:

$$\begin{aligned} x &= 4 + y \\ x - 3y &= 4 \end{aligned}$$

1. First, find three values for x and y that satisfy each equation. (Although only two points are necessary to determine a straight line, finding a third point is a good way of checking.)

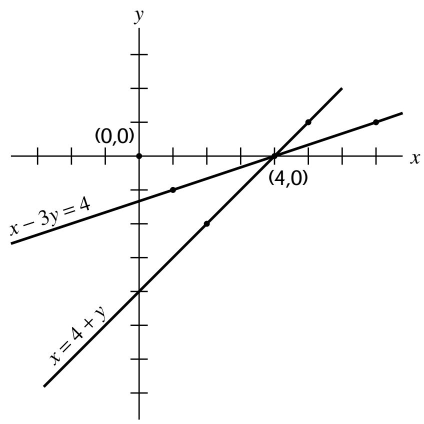
$x = 4 + y$

x	y
4	0
2	–2
5	1

$x - 3y = 4$

x	y
1	–1
4	0
7	1

2. Now graph the two lines on the coordinate plane, as shown in the following figure.



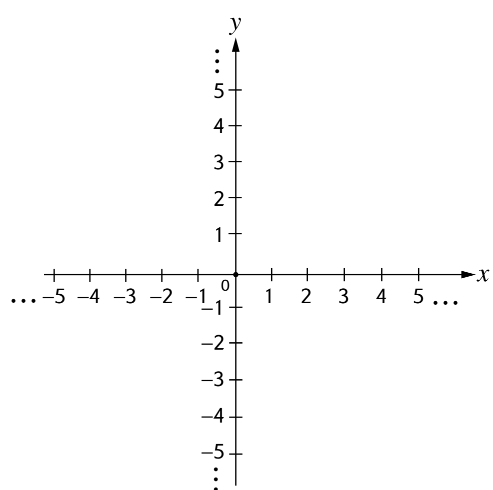
3. The point where the two lines cross (4, 0) is the solution of the system.
4. If the lines are parallel, they do not intersect, and there is no solution to the system.

Note: If lines are parallel, they have the same slope.

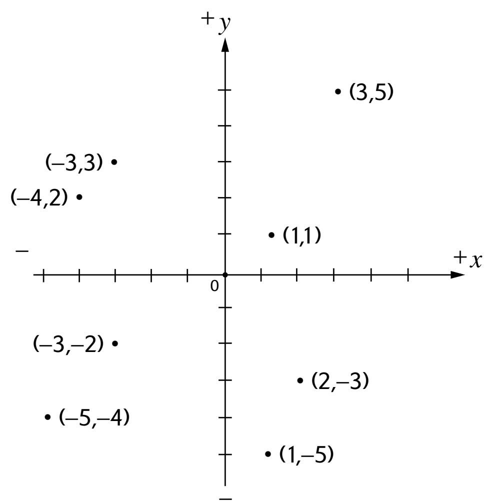
Basic Coordinate Geometry

Coordinate Graphs (*x*–*y* Graphs)

A *coordinate graph* is formed by two perpendicular number lines. These lines are called *coordinate axes*. The horizontal axis is called the *x-axis* or the *abscissa*. The vertical line is called the *y-axis* or the *ordinate*. The point at which the two lines intersect is called the *origin* and is represented by the coordinates (0, 0), often marked simply 0.

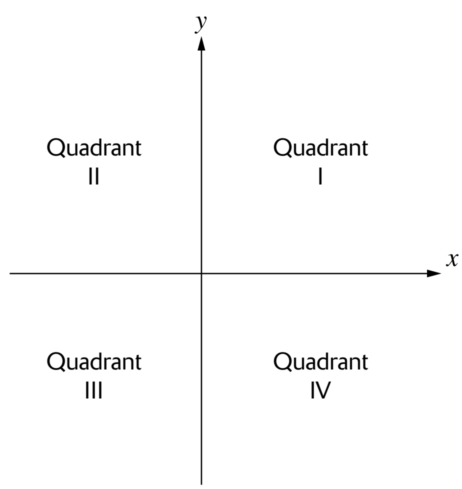


Each point on a coordinate graph is located by an ordered pair of numbers called *coordinates*. Notice the placement of points on the following graph and the coordinates, or ordered pairs, that show their location. Numbers are not usually written on the *x*- and *y*-axes.



On the *x*-axis, the numbers to the right of 0 are positive and to the left of 0 are negative. On the *y*-axis, numbers above 0 are positive and numbers below 0 are negative. The first number in the ordered pair is called the *x-coordinate* and shows how far to the right or left of 0 the point is. The second number is called the *y-coordinate* and shows how far up or down the point is from 0. The coordinates, or ordered pairs, are shown as (x, y). The order of these numbers is very important, as the point (3, 2) is different from the point (2, 3). Also, don't combine the ordered pair of numbers, because they refer to different directions.

The coordinate graph is divided into four quarters called *quadrants*. These quadrants are labeled as follows.



- In quadrant I, x is always positive and y is always positive.
- In quadrant II, x is always negative and y is always positive.
- In quadrant III, x is always negative and y is always negative.
- In quadrant IV, x is always positive and y is always negative.

Graphing Equations on the Coordinate Plane

To graph an equation on the coordinate plane, find the solutions by giving a value to one variable and solving the resulting equation for the other variable. Repeat this process to find other solutions. (When giving a value for one variable, start with 0; then try 1, and so forth.) Then, graph the solutions.

For example:

1. Graph the equation $x + y = 6$.
If x is 0, then y is 6.

$$\begin{aligned}(0) + y &= 6\\ y &= 6\end{aligned}$$

If x is 1, then y is 5.

$$\begin{array}{rcr}(1) + y &=& 6\\ \underline{-1} && \underline{-1}\\ y &=& 5\end{array}$$

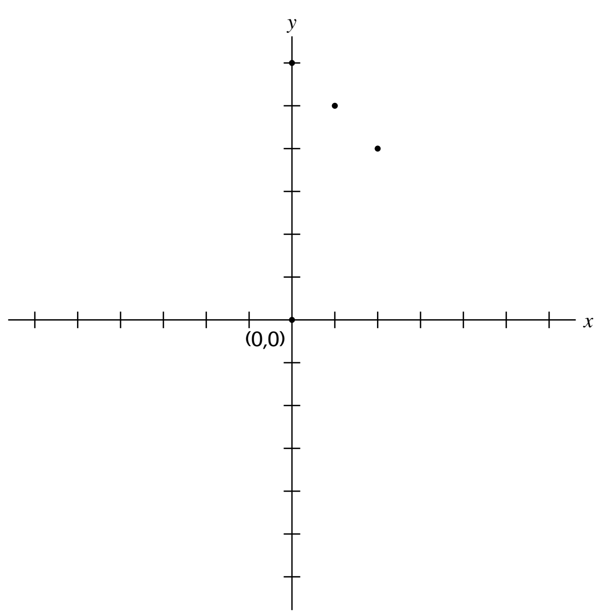
If x is 2, then y is 4.

$$\begin{array}{rcr}(2) + y &=& 6\\ \underline{-2} && \underline{-2}\\ y &=& 4\end{array}$$

Using a simple chart is helpful.

x	y
0	6
1	5
2	4

Now, plot these coordinates as shown in the following figure.



These solutions, when plotted, form a straight line. Equations whose graphs of their solution sets form a straight line are called linear equations. Equations that have a variable raised to a power, show division by a variable, involve variables with square roots, or have variables multiplied together do not form a straight line when their solutions are graphed. These are called nonlinear equations.

2. Graph the equation $y = x^2 + 4$.
If x is 0, then y is 4.

$$y = (0)^2 + 4$$
$$y = 0 + 4$$
$$y = 4$$

If x is 1, then y is 5.

$$y = (1)^2 + 4$$
$$y = 1 + 4$$
$$y = 5$$

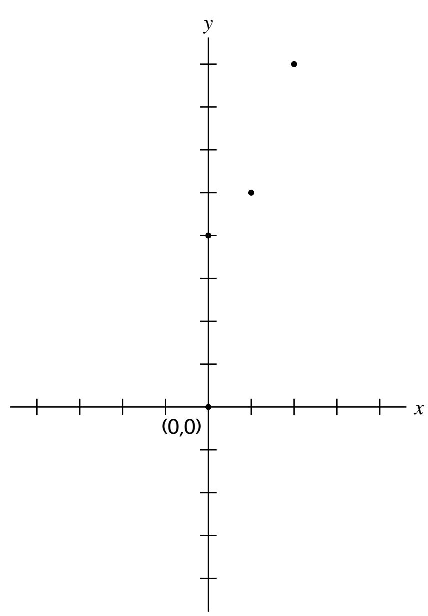
If x is 2, then y is 8.

$$y = (2)^2 + 4$$
$$y = 4 + 4$$
$$y = 8$$

Use a simple chart.

x	y
0	4
1	5
2	8

Now, plot these coordinates as shown in the following figure.



These solutions, when plotted, give a curved line (nonlinear). The more points plotted, the easier it is to see the curve and describe the solution set.

Slope and Intercept of Linear Equations

Two relationships between the graph of a linear equation and the equation itself must be pointed out. One involves the slope of the line, and the other involves the point where the line crosses the y-axis. To see either of these relationships, the terms of the equation must be in a certain order.

$$(+)(1)y = (\ )x + (\ )$$

When the terms are written in this order, the equation is said to be in *y*-intercept form or slope intercept. *y*-intercept form is written $y = mx + b$, and the two relationships involve m and b.

For example:

Write the equations in *y*-intercept form.

1. $x - y = 3$

$-y = -x + 3$

$y = x - 3$

2. $y = -2x + 1$ (already in *y*-intercept form)

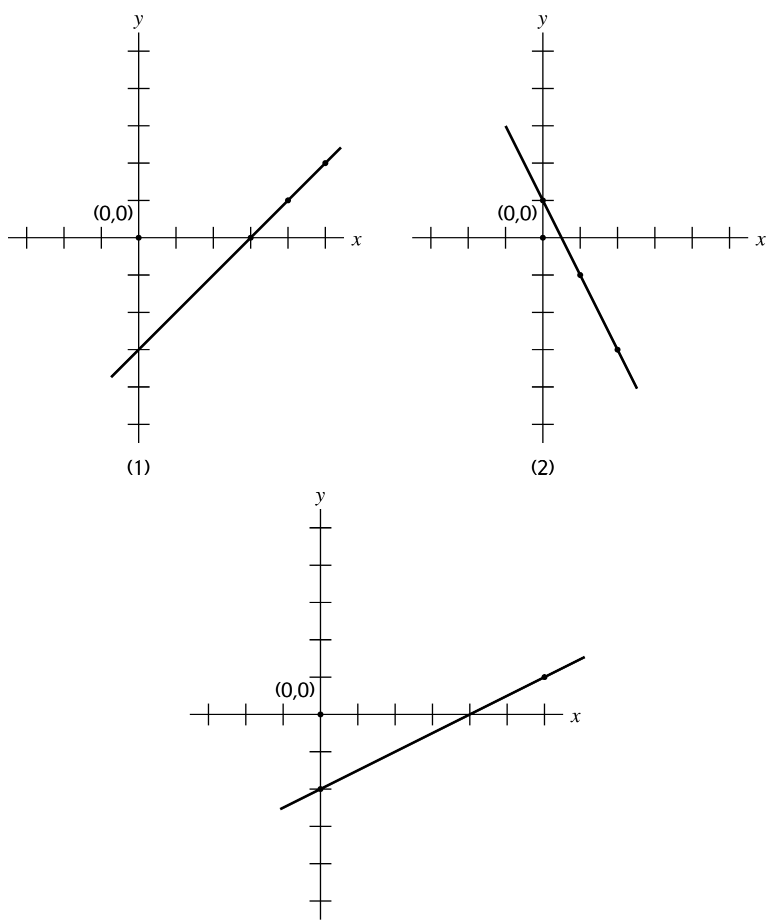
3. $x - 2y = 4$

$-2y = -x + 4$

$2y = x - 4$

$y = \frac{1}{2}x - 2$

As shown in the graphs of the three problems in the following figure, the lines cross the *y*-axis at –3, +1 and –2, the last term in each equation.



(3)

If a linear equation is written in the form $y = mx + b$, b is the y-intercept.

The slope of a line is defined as:

$$\frac{\text{the change in } y}{\text{the change in } x}$$

The word change refers to the difference in the value of y (or x) between two points on a line.

$$\text{slope in line } AB = \frac{y_A - y_B}{x_A - x_B} = \frac{y \text{ at point } A - y \text{ at point } B}{x \text{ at point } A - x \text{ at point } B}$$

Note: Points A and B can be any two points on a line; there is no difference in the slope.

1. Find the slope of $x - y = 3$ using coordinates.

   To find the slope of the line, pick any two points on the line, such as A(3, 0) and B (5, 2), and calculate the slope.

$$\text{slope} = \frac{y_A - y_B}{x_A - x_B} = \frac{(0)-(2)}{(3)-(5)} = \frac{-2}{-2} = 1$$

2. Find the slope of $y = -2x - 1$ using coordinates.
   Pick two points, such as A(1, –3) and B(–1, 1), and calculate the slope.

$$\text{slope} = \frac{y_A - y_B}{x_A - x_B} = \frac{(-3)-(1)}{(1)-(-1)} = \frac{-3-1}{1+1} = \frac{-4}{2} = -2$$

Looking back at the equations for examples 1, 2 and 3 written in y-form, it should be evident that the slope of the line is the same as the numerical coefficient of the x term.

1.

$$y = x - 3$$

slope = 1 y-intercept = –3

2.

$$y = -2x + 1$$

slope = –2 y-intercept = 1

3.

$$y = \tfrac{1}{2}x - 2$$

slope = $\tfrac{1}{2}$ y-intercept = –2

Graphing Linear Equations Using Slope and Intercept

Graphing an equation using its slope and y-intercept is usually quite easy.

1. State the equation in y-intercept form.
2. Locate the y-intercept on the graph (that is, one of the points on the line).
3. Write the slope as a ratio (fraction), and use it to locate other points on the line.
4. Draw a line through the points.

For example:

1. Graph the equation $x - y = 2$ using slope and y-intercept.

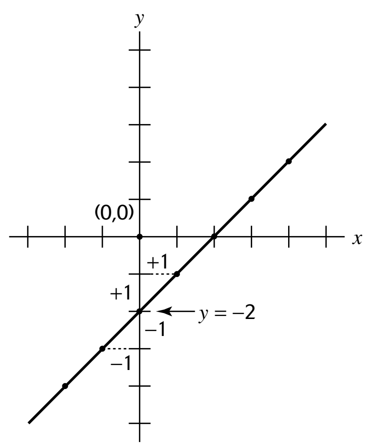
$$x - y = 2$$
$$-y = -x + 2$$
$$y = x - 2$$

Locate –2 on the x-axis and, from this point, count as shown in the following figure:

slope = 1

or $\frac{1}{1}$ $\frac{\text{(for every 1 up)}}{\text{(go 1 to the right)}}$

or $\frac{-1}{-1}$ $\frac{\text{(for every 1 down)}}{\text{(go 1 to the left)}}$



2. Graph the equation $2x - y = -4$ using slope and y-intercept.

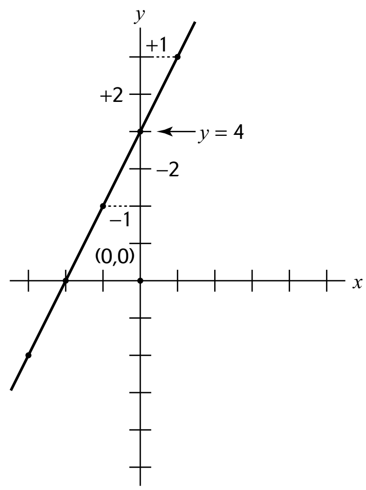
$$2x - y = -4$$
$$-y = -2x - 4$$
$$y = 2x + 4$$

Locate +4 on the y-axis and, from this point, count as shown in the following figure:

slope = 2

or $\frac{2}{1}$ $\frac{\text{(for every 2 up)}}{\text{(go 1 to the right)}}$

or $\frac{-2}{-1}$ $\frac{\text{(for every 2 down)}}{\text{(go 1 to the left)}}$



Finding the Equation of a Line

To find the equation of a line when working with ordered pairs, slopes and intercepts, use the following approach.

1. Find the slope, m.
2. Find the y-intercept, b.
3. Substitute the slope and intercept into the slope-intercept form, $y = mx + b$.
4. Change the slope-intercept form to standard form, $Ax + By = C$.

For example:

1. Find the equation of the line when $m = –4$ and $b = 3$.
   Find the slope, m.

$$m = –4 \text{ (given)}$$

   Find the y-intercept, b.

$$b = 3 \text{ (given)}$$

   Substitute the slope and intercept into the slope-intercept form $y = mx + b$.

$$y = –4x + 3$$

   Change the slope-intercept form to standard form $Ax + By = C$. Since

$$y = –4x + 3$$

   Adding $4x$ to each side gives:

$$4x + y = 3$$

2. Find the equation of the line passing through the point (6, 4) with a slope of –3.
   Find the slope, m.

$$m = –3 \text{ (given)}$$

   Find the y-intercept, b.
   Substitute $m = –3$ and the point (6, 4) into the slope-intercept form to find b.

$$\begin{aligned} y &= mx + b \text{ where } y = 4,\ m = -3, \text{ and } x = 6 \\ 4 &= (-3)(6) + b \\ 4 &= -18 + b \\ 18 + 4 &= b \\ 22 &= b \end{aligned}$$

Substitute the slope and intercept into the slope-intercept form: $y = mx + b$.

Since $m = -3$ and $b = 22$, $y = -3x + 22$.

Change the slope-intercept form to standard form: $Ax + By = C$.

Since $y = -3x + 22$, adding $3x$ to each side gives $3x + y = 22$.

3. Find the equation of the line passing through points (5, –4) and (3, –2).

Find the slope, m.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{(-4)-(-2)}{(5)-(3)} = \frac{-4+2}{2} = \frac{-2}{2} = -1$$

Find the y-intercept, b.

Substitute the slope and either point into slope-intercept form.

$$\begin{aligned} y &= mx + b \text{ where } m = -1,\ x = 5, \text{ and } y = -4 \\ -4 &= (-1)(5) + b \\ -4 &= -5 + b \\ 5 + -4 &= b \\ 1 &= b \end{aligned}$$

Substitute the slope and intercept into the slope-intercept form: $y = mx + b$.

Since $m = -1$ and $b = 1$, $y = -1x + 1$ or $y = -x + 1$.

Change the slope-intercept form to standard form: $Ax + By = C$.

Since $y = -x + 1$, adding x to each side gives $x + y = 1$.

Functions

Definition of a Function

A function is a formula or rule that shows the association between elements in one set to the elements in another set. The elements from the first set are the ones that are being plugged in (the domain), and the elements from the second set are the outcome or result (the range). In a function, all of the elements of the domain are unique or different.

Graphs of a Function

To graph a function, simply plug in values for the domain (if not already given) and find the range (results). Then, place these "ordered pairs" in the coordinate graph.

For example:

1. Graph the function $f(x) = x + 1$.

(You could think of this as $y = x + 1$.)

Now, plug in values for x (the domain) and work out the result ($f(x)$).

If x is 0, then $f(x)$ is 1.

$$\begin{aligned} f(x) &= (0) + 1 \\ f(x) &= 1 \end{aligned}$$

If x is 1, then $f(x)$ is 2.

$$f(x) = (1) + 1$$
$$f(x) = 2$$

If x is 2, then $f(x)$ is 3.

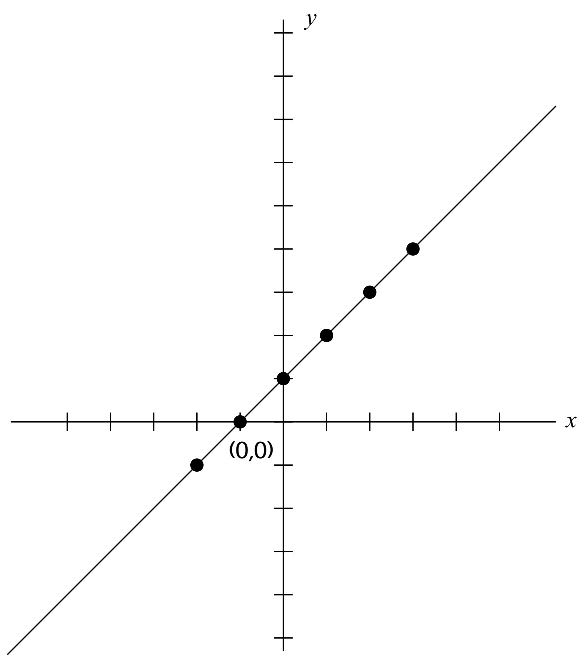
$$f(x) = (2) + 1$$
$$f(x) = 3$$

and so on.

Using a simple chart is helpful.

x	*f(x)*
0	1
1	2
2	3
3	4
–1	0
–2	–1

Next, plot the points as follows:



Since this is a linear equation or linear function, you could have plotted only three points.

2. Graph the function $f(x) = 3x - 2$.
   Now, plug in values for x (the domain) and work out the result ($f(x)$).

If x is 0, then $f(x)$ is –2.

$$f(x) = 3(0) - 2$$
$$f(x) = 0 - 2$$
$$f(x) = -2$$

If x is 1, then $f(x)$ is 1.

$$f(x) = 3(1) - 2$$
$$f(x) = 3 - 2$$
$$f(x) = 1$$

If x is 2, then $f(x)$ is 4.

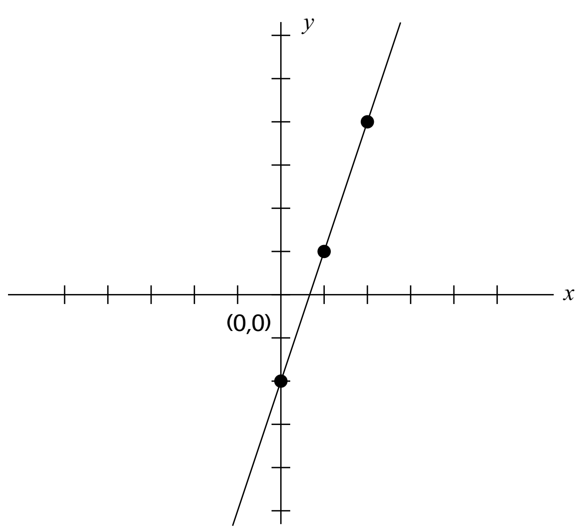
$$f(x) = 3(2) - 2$$
$$f(x) = 6 - 2$$
$$f(x) = 4$$

and so on.

Using a simple chart is helpful.

x	***f(x)***
0	–2
1	1
2	4

Next, plot the points as follows:



Since this is a linear equation or linear function, you need to plot only three points.

Notice that when x is 0, the graph crosses the y-axis at that point. If y were 0, the graph would cross the x-axis at that point.

Take a look at a few quadratic functions; for example:

1. Graph the quadratic function $y = x^2 + 2$.

Plug in simple values for x and you get...

If x is 0, then y is 2.

$$y = (0)^2 + 2$$
$$y = 2$$

If x is 1, then y is 3.

$$y = (1)^2 + 2$$
$$y = 1 + 2$$
$$y = 3$$

If x is 2, then y is 6.

$$y = (2)^2 + 2$$
$$y = 4 + 2$$
$$y = 6$$

If x is –1, then y is 3.

$$y = (-1)^2 + 2$$
$$y = 1 + 2$$
$$y = 3$$

If x is –2, then y is 6.

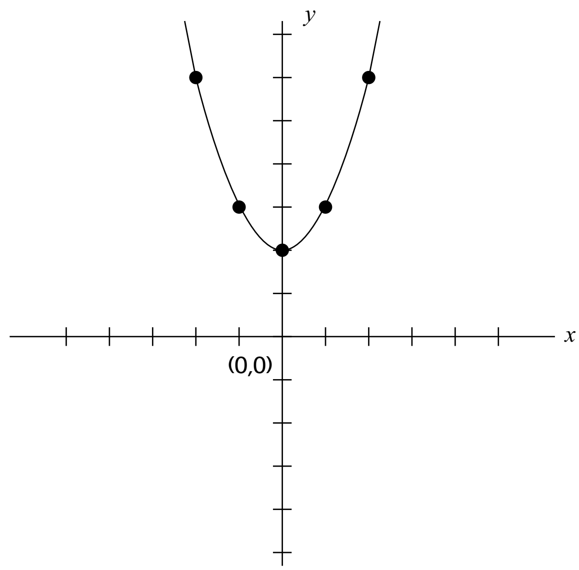
$$y = (-2)^2 + 2$$
$$y = 4 + 2$$
$$y = 6$$

and so on.

Using a simple chart is helpful.

x	***y***
0	2
1	3
2	6
–1	3
–2	6

Next, plot the points as follows:



These quadratic functions, when plotted, give a curved line (nonlinear). The more points plotted, the easier it is to see the curve and describe the solution set (or range). Fortunately, you will not actually be graphing or plotting points on the SAT since the questions are multiple choice or grid-in. They do, however, ask for a possible solution or solution set.

2. Graph the quadratic function $f(x) = x^2 - 2x + 1$

Now plug in values for x (the domain) and work out the result ($f(x)$).

If x is 0, then $f(x)$ is 1.

$$f(x) = (0)^2 - 2(0) + 1$$
$$f(x) = 0 + 1$$
$$f(x) = 1$$

If x is 1, then $f(x)$ is 0.

$$f(x) = (1)^2 - 2(1) + 1$$
$$f(x) = 1 - 2 + 1$$
$$f(x) = 0$$

If x is 2, then $f(x)$ is 1.

$$f(x) = (2)^2 - 2(2) + 1$$
$$f(x) = 4 - 4 + 1$$
$$f(x) = 1$$

If x is 3, then $f(x)$ is 4.

$$f(x) = (3)^2 - 2(3) + 1$$
$$f(x) = 9 - 6 + 1$$
$$f(x) = 4$$

If x is -1, then $f(x)$ is 4.

$$f(x) = (-1)^2 - 2(-1) + 1$$
$$f(x) = 1 + 2 + 1$$
$$f(x) = 4$$

If x is –2, then $f(x)$ is 9.

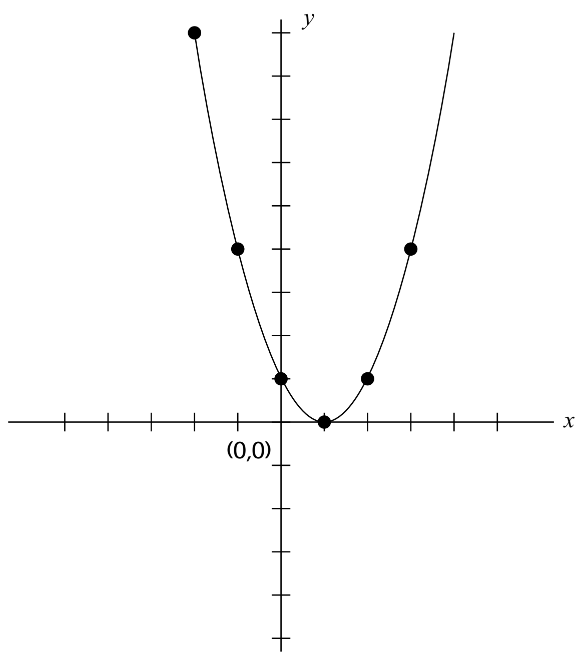
$$f(x) = (-2)^2 - 2(-2) + 1$$
$$f(x) = 4 + 4 + 1$$
$$f(x) = 9$$

and so on.

Using a simple chart is helpful.

x	***f(x)***
0	1
1	0
2	1
3	4
–1	4
–2	9

Next, plot the points as follows:



Finding the Value of Functions

The value of a function is really the value of the range.

For example:

What is the value of $f(x) = 5x - 7$ if $x = 4$?

Plugging in 4 for x gives

$$f(x) = 5(4) - 7$$
$$f(x) = 20 - 7$$
$$f(x) = 13$$

So $f(4) = 13$ in this function.

You could be asked to solve for x. For example:

What are the possible values of x in the function $f(x) = x^2 - 4$ if $f(x) = 0$?

Plugging in 0 for $f(x)$ gives $0 = x^2 - 4$

Next, solve the equation by factoring the difference of two squares:

$$0 = (x + 2)(x - 2)$$
$$0 = (x + 2) \text{ or } 0 = (x - 2)$$

So x could equal 2 or –2.

(You could have also solved the original equation [function] by adding 4 to each side, leaving $4 = x^2$, so $x = 2$ or –2.)

Function Involving New "Symbols" and Definitions

You could be given a special symbol in a function and asked to solve the function. Don't be alarmed, the special symbol (something like #, @, *) will be defined in terms of operations with which you are familiar. The key to solving special symbol problems is focusing on the definition (watch for the "=" if the words "defined as" are not used).

For example:

If $x \# y = 3x + y^2$, what is the value of $2 \# 3$?

Take a careful look at the given function:

$$x \# y = 3x + y^2$$

Notice the **positions** of x and y and simply plug in 2 for x and 3 for y in the definition:

$$= 3x + y^2$$
$$= 3(2) + (3)^2$$
$$= 6 + 9$$
$$= 15$$

So $2 \# 3 = 15$.

Sample SAT-Type Problems

Now you can try actual SAT-like problems in each topic area. Note: Remember that problems range from easy to average to difficult.

Algebra and Functions

Operations with Signed Numbers

1. What is the value of $\frac{(5-7)(-8+4)}{2-(-6)}$?

A. –8
B. –1
C. $\frac{2}{3}$
D. 1
E. 8

2. The value of the expression $\frac{(5-7)(-8+4)}{2-(-6)}$ is

A. –33
B. –28.5
C. 7.75
D. 20.25
E. 23.5

3. If $a < b < c$, which of the following must be less than 0?

A. $(c - b)^3$
B. $(a - c)^2$
C. $(c - a)b$
D. $\frac{a-c}{b-c}$
E. $(a - b)(c - b)$

4. If $x = -3$, then $-(-(-x + 2)) =$

Explanations

1. D. $\frac{(5-7)(-8+4)}{2-(-6)} = \frac{(-2)(-4)}{8} = \frac{8}{8} = 1$

2. A. $\frac{(5-7)(-8+4)}{2-(-6)} = (-8)(4) + \frac{9-5}{-4} = -32 + \frac{4}{-4} = -32 - 1 = -33$

3. E. Using the given information $a < b < c$, you try each answer choice:

A. $(c - b)^3 = (\textit{positive})^3 = \textit{positive}$
B. $(a - c)^2 = (\textit{negative})^2 = \textit{positive}$
C. $(c - a)b = (\textit{positive})(\textit{unknown}) =$?; cannot tell whether answer is positive or negative
D. $\frac{a-c}{b-c} = \frac{\textit{negative}}{\textit{negative}} = \textit{positive}$
E. $(a - b)(c - b) = (\textit{negative})(\textit{positive}) = \textit{negative}$

4. 5. Begin by substituting –3 into the expression. The result is $-(-(3 + 2)) = -(-5)$. The correct answer is **5.**

Substitution for Variables

5. If $N = \frac{3 - x^2}{x - 5}$, what is the value of N when $x = -2$?

A. $\frac{-12}{7}$
B. $\frac{1}{7}$
C. $\frac{1}{3}$
D. $\frac{6}{5}$
E. $\frac{12}{7}$

6. If $A = \frac{h}{2}(b_1 + b_2)$, what is the value of A when $h = 7$, $b_1 = 11$, and $b_2 = 9$?

A. 28
B. 32
C. 45
D. 58
E. 70

7. If $a \oplus b = \frac{\sqrt{a+b} - b}{a \div b}$, what is the value of 4 @ 5?

A. $\frac{-5}{2}$
B. $\frac{-4}{5}$
C. $\frac{2}{5}$
D. $\frac{5}{4}$
E. $\frac{5}{2}$

8. If $3a = 4b$, and $b = 1.25$, then $3(a + b) =$

Explanations

5. **B**. Substitute –2 for *x*.

$$N = \frac{3 - x^2}{x - 5} = \frac{3 - (-2)^2}{(-2) - 5} = \frac{3 - 4}{-7} = \frac{-1}{-7} = \frac{1}{7}$$

6. **E.** Substitute $h = 7$, $b_1 = 11$, and $b_2 = 9$

$$A = \frac{h}{2}(b_1 + b_2) = \frac{7}{2}(11 + 9) = \frac{7}{2}(20) = 70$$

7. **A.** Substitute 4 for *a* and 5 for *b*.

$$a \oplus b = \frac{\sqrt{a+b} - b}{a \div b} = \frac{\sqrt{4+5} - 5}{4 \div 5} = \frac{3 - 5}{\frac{4}{5}} = (-2) \cdot \frac{5}{4} = \frac{-5}{2}$$

8. **8.75 or $\frac{35}{4}$.** Applying the distributive property to the expression $3(a + b)$ gives $3a + 3b$. Substituting the quantity $4b$ for the quantity $3a$ gives the expression of $4b + 3b$. $4b + 3b = 7b$. By substitution of 1.25 for b, you have $7(1.25) = 8.75$ or $\frac{35}{4}$.

Absolute Value

9. What is the value of $|2 - 5| - |3 + -2|$?

A. –2
B. –1
C. 0
D. 2
E. 4

10. If $|x| = -x$, then x must be

A. positive
B. nonnegative
C. negative
D. nonpositive
E. zero

11. If $|3-2x|=7$, then $x =$

A. –5 only
B. –2 only
C. 2 only
D. 2 and –5 only
E. 5 and –2 only

12. If $|-3x-1|=8$, what is one possible value of x?

Explanations

9. D. $|2-5|-|3+-2|=|-3|-|1|=3-1=2$

10. D. $|x|=-x$. Since the absolute of a number cannot be negative, the absolute value must be nonnegative. If $-x$ is nonnegative, then x must be nonpositive.

11. E. $|3-2x|=7$. This breaks down into two separate equations:

$$3-2x=-7 \quad \text{or} \quad 3-2x=7$$
$$3-2x-3=-7-3 \qquad 3-2x-3=7-3$$
$$-2x=-10 \qquad -2x=4$$
$$\frac{-2x}{-2}=\frac{-10}{-2} \qquad \frac{-2x}{-2}=\frac{4}{-2}$$
$$x=5 \quad \text{or} \quad x=-2$$

12. $\mathbf{\frac{7}{3}}$ **or 2.33** For an equation of this type, let the expression inside the absolute value symbol equal either 8 or – 8. Solving the two equations below,

$-3x-1=8$ implies that $-3x=9$, and $x=-3$

or $-3x-1=-8$ implies that $-3x=-7$, and $x=\frac{7}{3}$

The correct answer would be either –3 or $\frac{7}{3}$ or 2.33. So you would grid either $\frac{7}{3}$ or 2.33.

Working with Algebraic Expressions

13. How much greater than $5-7x$ is $3x+2$?

A. $3-10x$
B. $7-10x$
C. $-3-4x$
D. $10x+3$
E. $10x-3$

14. $\frac{(2xy^2)(5x^2y)^2}{15xy^3}=$

A. $\frac{2x^2}{3y}$

B. $\frac{2x^4}{3y}$

C. $\frac{2x^5}{3y}$

D. $\frac{10x^4}{3y}$

E. $\frac{35x^4}{y}$

15. $(x+3)(x-4)-(x+2)(x+5)=$

A. $-8x-22$
B. $-8x-8$
C. $-6x-22$
D. $-22x-8$
E. $6x-22$

16. If $\frac{5}{3}-\frac{2}{x}=\frac{x}{3}-\frac{2}{5}$, then what is one of the values of x?

Explanations

13. **E.** $3x + 2 - (5 - 7x) = 3x + 2 - 5 + 7x = 10x - 3.$

14. **D.** $\dfrac{(2xy^2)(5x^2y)^2}{15xy^3} = \dfrac{2xy^2 \bullet 25x^4y^2}{15xy^5} = \dfrac{50x^5y^4}{15xy^5} = \dfrac{10x^4}{3y}$

15. **A.** $(x + 3)(x - 4) - (x + 2)(x + 5) = x^2 - x - 12 - (x^2 + 7x + 10)$

$$= x^2 - x - 12 - x^2 - 7x - 10$$

$$= -8x - 22$$

16. **1.2 or $\frac{6}{5}$.** Since the two sides of this equation are of the same format, and the left side has a 5 in the numerator of one of the fractions, try 5 for x in the right member of the equation. Also, since the bottom of one of the fractions in the right member has a 5 in the denominator, try 5 for x in the left member of the equation. The statement will look like:

$$\frac{5}{3} - \frac{2}{5} = \frac{5}{3} - \frac{2}{5}$$

This is always true because of the reflexive property of equality. **5 is one of the solutions.**

Solving this equation is a more difficult approach. Arriving at the quadratic equation of

$3(5x - 6)(x - 5) = 0$, it follows that $(5x - 6) = 0$ and $x = \frac{6}{5}$. Therefore, $\frac{6}{5}$ (or 1.2) is a correct answer also.

Manipulating Integer and Rational Expressions

17. $\dfrac{a}{b} + \dfrac{x}{y} =$

A. $\dfrac{a + x}{b + y}$

B. $\dfrac{ay + bx}{2by}$

C. $\dfrac{ab + xy}{by}$

D. $\dfrac{ay + bx}{by}$

E. $ay + bx$

18. If $x > 0$, $y > 0$, and $z > 0$, then $x + \dfrac{1}{z + \frac{1}{y}} =$

A. $\dfrac{xz + yx + 1}{z}$

B. $xy + zy + 1$

C. $\dfrac{xzy + x + y}{zy + 1}$

D. $\dfrac{xzy + 1}{y}$

E. $\dfrac{xzy + x + y}{y}$

19. If $\dfrac{8x + 15y}{x + y} = 10$, what is the value of $\dfrac{x}{y}$?

A. $\dfrac{2}{5}$

B. $\dfrac{10}{23}$

C. $\dfrac{23}{10}$

D. $\dfrac{5}{2}$

E. It cannot be determined from the given information.

20. If $2^{3/2} \bullet 2^{1/3} = 2^x$, what is the value of x?

Explanations

17. **D.** $\frac{a}{b}+\frac{x}{y}=\frac{a}{b}\bullet\frac{y}{y}+\frac{x}{y}\bullet\frac{b}{b}=\frac{ay}{by}+\frac{bx}{by}=\frac{ay+bx}{by}$

18. **C.**

$$x+\frac{1}{z+\frac{1}{y}}=x+\frac{1}{\frac{zy+1}{y}}=x+\frac{y}{zy+1}=x\bullet\frac{zy+1}{zy+1}+\frac{y}{zy+1}$$

$$=\frac{xzy+x}{zy+1}+\frac{y}{zy+1}=\frac{xzy+x+y}{zy+1}$$

19. **D.** $\frac{8x+15y}{x+y}=10$ Rewrite this as: $\frac{8x+15y}{x+y}=\frac{10}{1}$

Then, cross multiplying yields: $1(8x+15y)=10(x+y)$

$$8x+15y=10x+10y$$

$$8x+15y-8x=10x+10y-8x$$

$$15y=2x+10y$$

$$15y-10y=2x+10y-10y$$

$$5y=2x$$

$$\frac{5y}{2y}=\frac{2x}{2y}$$

$$\frac{5}{2}=\frac{x}{y}$$

20. $\mathbf{\frac{11}{6}}$**.** Since this is a product of two numbers with the same base, the rule is to add the exponents.

$\frac{3}{2}+\frac{1}{3}=\frac{9}{6}+\frac{2}{6}=\frac{11}{6}$ The correct answer is $\mathbf{\frac{11}{6}}$**.**

Solving Rational Equations and Inequalities

21. If $\frac{x}{3}+\frac{x}{4}=\frac{7}{12}$, what is the value of x?

A. $\frac{1}{12}$

B. $\frac{1}{7}$

C. 1

D. $\frac{49}{24}$

E. $\frac{7}{2}$

22. Which of the following is a possible solution to $\frac{x+3}{4}\geq\frac{3-x}{3}$?

A. –5

B. $\frac{-5}{7}$

C. 0

D. $\frac{2}{9}$

E. $\frac{7}{8}$

23. If $\frac{x^2+7x+12}{5x+15}=3-x$, what is the value of x?

A. –6

B. $\frac{6}{11}$

C. $\frac{11}{10}$

D. $\frac{11}{6}$

E. 2

24. If $a=3$ and $\frac{3}{x}+\frac{5}{a}=7$, what would be the value of x?

Explanations

21. **C.** $\frac{x}{3} + \frac{x}{4} = \frac{7}{12}$ Get rid of the denominators by multiplying all terms by 12 to get:

$$12 \cdot \frac{x}{3} + 12 \cdot \frac{x}{4} = 12 \cdot \frac{7}{12}$$
$$4x + 3x = 7$$
$$7x = 7$$
$$x = 1$$

22. **E.** $\frac{x+3}{4} \geq \frac{3-x}{3}$ Get rid of the denominators by multiplying both sides by12.

$$12 \cdot \frac{x+3}{4} \geq 12 \cdot \frac{3-x}{3}$$
$$3(x + 3) \geq 4(3 - x)$$
$$3x + 9 \geq 12 - 4x$$
$$3x + 9 + 4x \geq 12 - 4x + 4x$$
$$7x + 9 \geq 12$$
$$7x + 9 - 9 \geq 12 - 9$$
$$7x \geq 3$$
$$\frac{7x}{7} \geq \frac{3}{7}$$

$x \geq \frac{3}{7}$ The only answer choice $\geq \frac{3}{7}$ is **E**, $\frac{7}{8}$.

23. **D.** $\frac{x^2 + 7x + 12}{5x + 15} = 3 - x$ Factoring numerator and denominator of fraction on left:

$$\frac{(x+3)(x+4)}{5(x+3)} = 3 - x$$
$$\frac{x+4}{5} = \frac{3-x}{1}$$

cross multiplying yields $1(x + 4) = 5\,(3 - x)$

$$x + 4 = 15 - 5x$$
$$x + 4 + 5x = 15 - 5x + 5x$$
$$6x + 4 = 15$$
$$6x + 4 - 4 = 15 - 4$$
$$6x = 11$$
$$\frac{6x}{6} = \frac{11}{6}$$
$$x = \frac{11}{6}$$

24. $\mathbf{\frac{9}{16}}$**.** Begin by substituting 3 for a. Next, subtract $\frac{5}{3}$ from each member of the equation. You should have the following equation: $\frac{3}{x} = \frac{16}{3}$. Next, multiply both members by $3x$. $3x$ is the least common denominator for the equation. You should now have $9 = 16x$. Solving for x, you get $x = \frac{9}{16}$.

Working with Linear Functions—Graphs and Equations

25. The graph of $y = \frac{-3}{4}x + 7$ is perpendicular to the graph of which of the following equations?

A. $y = \frac{-3}{4}x - 7$

B. $y = \frac{-4}{3}x + 5$

C. $y = \frac{3x}{4} + 8$

D. $y = \frac{-4}{3}x - 7$

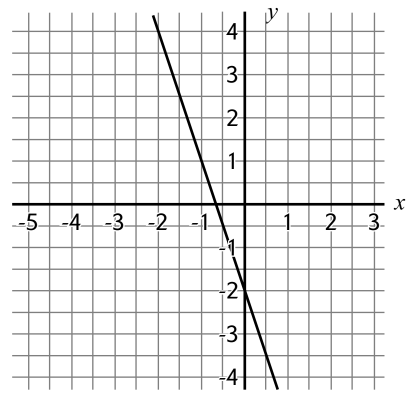
E. $y = \frac{4}{3}x + 12$

26. If the point $(6,y)$ is on the graph of the equation $y = \frac{2}{3}x - 5$, what is the value of y?

A. –9
B. –4
C. –1
D. 1
E. $\frac{33}{2}$

27. What is the y-coordinate of the point at which the graph of $2x - 3y = 12$ crosses the y-axis?

A. –12
B. –4
C. –3
D. 3
E. 12



28. If the line shown in the graph above is reflected about the y-axis and then translated upward 4 units, the resulting equation is $y = ax + k$. What would be the value of k?

Explanations

25. **E.** The graph of an equation of the form $y = mx + b$ has a slope of m. So the slope of the graph of the equation $y = \frac{-3}{4}x + 7$ has a slope of $\frac{-3}{4}$.

Two lines are perpendicular if their slopes are opposite reciprocals. Therefore the slope of the line perpendicular to the graph of the given equation is $\frac{4}{3}$. The only equation among the answer choices having the correct slope is Choice **E.**

26. **C.** If the point $(6,y)$ is on the graph of the equation $y = \frac{2}{3}x - 5$, substituting 6 for x gives $y = \frac{2}{3} \cdot 6 - 5 = 4 - 5 = -1$.

27. **B.** The point at which the graph crosses the y-axis will have an x-coordinate of 0.

Substitute 0 for x. $2x - 3y = 12$

$2(0) - 3y = 12$

$-3y = 12$

$\frac{-3y}{-3} = \frac{12}{-3}$

$y = -4$

28. **2.** The y-intercept of the line in the graph is at –2 or (0, –2). The reflection of (0, –2) about the y-axis remains (0, –2). When this point is translated upward by 4 units, the y-intercept will then be at 2. Since k in the equation represents the y-intercept, $k = 2$.

Solving Radical Equations

29. If $\sqrt{x} - 3 = 6$, what is the value of x?

A. 9
B. 16
C. 25
D. 49
E. 81

30. For which of the ordered pairs (x, y) is $\sqrt{x} - \sqrt{y} = 3$?

A. (1, 16)
B. (3, 0)
C. (25, 64)
D. (64, 25)
E. (100, 64)

31. If $\sqrt{x+4} = x - 2$, what is the value of x?

A. –3 only
B. 0 and –3
C. 0 only
D. 0 and 5
E. 5 only

32. If $\sqrt{x+2} - x = -4$, what is the value of x?

Explanations

29. **E.**

$$\sqrt{x} - 3 = 6$$
$$\sqrt{x} - 3 + 3 = 6 + 3$$
$$\sqrt{x} = 9$$
$$\left(\sqrt{x}\right)^2 = 9^2$$
$$x = 81$$

30. **D.** Using the given equation $\sqrt{x} - \sqrt{y} = 3$, you try each answer choice.

A. (1, 16) $\sqrt{1} - \sqrt{16} = 1 - 4 \neq 3$

B. (3, 0) $\sqrt{3} - \sqrt{1} = \sqrt{3} - 1 \neq 3$

C. (25, 64) $\sqrt{25} - \sqrt{64} = 5 - 8 = -3 \neq 3$

D. (64, 25) $\sqrt{64} - \sqrt{25} = 8 - 5 = 3$, so Choice **D** is correct.

31. **E.** $\sqrt{x+4} = x - 2$. Squaring both sides, you get:

$$\left(\sqrt{x+4}\right)^2 = (x-2)^2$$
$$x + 4 = (x - 2)(x - 2)$$
$$x + 4 = x^2 - 4x + 4$$
$$x + 4 - x - 4 = x^2 - 4x + 4 - x - 4$$
$$0 = x^2 - 5x$$

$0 = x(x - 5)$, so it appears that $x = 0$ or $x = 5$

If you were to substitute each of these values of x in the original equation, only $x = 5$ actually works. The answer of $x = 0$ is known as an extraneous root.

32. **7.** First isolate the radical term by adding x to each side of the equation. Next square both sides of the equation. The new equation is $x + 2 = x^2 - 18x + 16$. Next, set the equation equal to zero. The equation will be $0 = x^2 - 9x + 14$. Solving this equation by factoring the trinomial you get $(x - 2)(x - 7) = 0$. Setting each binomial equal to zero, you get $x = 2$ or $x = 7$. Substituting 2 for x in the original equation, the statement is false, so 2 isn't a solution for the original equation. Substituting 7 for x, the original equation is true. The correct answer is **7**.

Basic Factoring

33. $\frac{3x-6}{5x-10} + \frac{2}{5} =$

A. $25x - 50$
B. $\frac{3}{5}$
C. 1
D. $\frac{3x-4}{5x-5}$
E. $\frac{8}{5}$

34. If $x^2 - y^2 = 12$ and $x - y = 3$, what is the value of $x + y$?

A. –3
B. 0
C. 4
D. 9
E. It cannot be determined from the given information.

35. If $x + 2$ is a factor of $x^2 + kx + 8$, where k is a constant, what is the value of k?

A. –2
B. 2
C. 4
D. 6
E. 8

36. If $(x + c)(x - 2) = x^2 + 3x - 10$, what is the value of c?

Explanations

33. **C.** $\frac{3x-6}{5x-10} + \frac{2}{5} = \frac{3(x-2)}{5(x-2)} + \frac{2}{5} = \frac{3}{5} + \frac{2}{5} = 1$

34. **C.** $x^2 - y^2 = 12$. Factoring the left side, you have:

$(x + y)(x - y) = 12$. Substitute 3 for $x - y$.

$$(x+y) \cdot 3 = 12$$
$$\frac{(x+y) \cdot 3}{3} = \frac{12}{3}$$
$$x + y = 4$$

35. **D.** Since $x + 2$ is a factor of $x^2 + kx + 8$,

$(x + 2)\ (x + ?) = x^2 + kx + 8$

You know the product of 2 and ? has to equal 8. so ? = 4.

You also know that the sum of 2 and ? has to equal k.

So $2 + 4 = 6$. Therefore, $k = 6$.

36. **5.** The sum of cx and $-2x$ is equal to $3x$. Therefore, $cx = 2x + 3x = 5x$. Therefore, c must equal 5. The correct answer is 5. You can also use $-2x = -10$, which implies $x = 5$.

Direct and Inverse Variation

37. If w is inversely proportional to x, and $w = \frac{1}{9}$ when $x = 6$, what is the value of w when $x = \frac{3}{4}$?

A. $\frac{3}{8}$

B. $\frac{8}{9}$

C. $\frac{9}{8}$

D. $\frac{12}{7}$

E. $\frac{18}{5}$

38. If m is directly proportional to r^3, and $m = \frac{1}{12}$ when $r = \frac{1}{2}$, what is the value of m when $r = \frac{3}{2}$?

A. $\frac{2}{3}$

B. $\frac{4}{9}$

C. $\frac{9}{4}$

D. $\frac{15}{2}$

E. $\frac{35}{3}$

39. If h is directly proportional to x^2, but inversely proportional to y, and $h = \frac{9}{8}$ when $x = \frac{1}{2}$ and $y = \frac{1}{3}$, what is the value of h when $x = \frac{1}{3}$ and $y = \frac{1}{2}$?

A. $\frac{2}{15}$

B. $\frac{3}{11}$

C. $\frac{4}{15}$

D. $\frac{1}{3}$

E. $\frac{3}{2}$

40. Speed varies inversely with elapsed time. Using the speed check signs along the road that are one mile apart, and holding the car at a constant speed of 60 miles per hour according to the speedometer, the passenger finds it takes 60 seconds to travel one mile. During the next mile, the driver holds the speed at a constant 72 miles per hour. How much time, in minutes, should the passenger calculate for the car to travel the second mile?

Explanations

37. **B**. Since w is inversely proportional to x, $w = \frac{c}{x}$, where c is some constant.

Substituting $w = \frac{1}{9}$ and $x = 6$:

$$\frac{1}{9} = \frac{c}{6}$$

$$6 \cdot \frac{1}{9} = 6 \cdot \frac{c}{6}$$

$$\frac{2}{3} = c$$

So our equation is now: $w = \dfrac{\left(\frac{2}{3}\right)}{x}$

Substituting $x = \frac{3}{4}$: $w = \dfrac{\left(\frac{2}{3}\right)}{\left(\frac{3}{4}\right)} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$

38. **C.** Since m is directly proportional to r^3, $m = cr^3$, where c is some constant.

Substituting $m = \frac{1}{12}$ and $r = \frac{1}{2}$:

$$\frac{1}{12} = c \cdot \left(\frac{1}{2}\right)^3$$

$$\frac{1}{12} = c \cdot \frac{1}{8}$$

$$8 \cdot \frac{1}{12} = c \cdot \frac{1}{8} \cdot 8$$

$$\frac{2}{3} = c$$

So our equation is now: $m = \frac{2}{3} r^3$

Substitute $r = \frac{3}{2}$: $m = \frac{2}{3}\left(\frac{3}{2}\right)^3 = \frac{2}{3} \cdot \frac{27}{8} = \frac{9}{4}$

39. **D.** Since h is directly proportional to x^2, but inversely proportional to y,

$h = \frac{cx^2}{y}$ for some constant c.

Then, $h = \frac{9}{8}$ when $x = \frac{1}{2}$ and $y = \frac{1}{3}$:

$$\frac{9}{8} = \frac{c\left(\frac{1}{2}\right)^2}{\frac{1}{3}}$$

$$\frac{9}{8} = c \cdot \frac{1}{4} \cdot \frac{3}{1}$$

$$\frac{9}{8} = c \cdot \frac{3}{4}$$

$$\frac{4}{3} \cdot \frac{9}{8} = c \cdot \frac{3}{4} \cdot \frac{4}{3}$$

$$\frac{3}{2} = c$$

So our equation is now: $h = \frac{\frac{3}{2}x^2}{y}$

Substitute $x = \frac{1}{3}$ and $y = \frac{1}{2}$: $h = \frac{\frac{3}{2}\left(\frac{1}{3}\right)^2}{\frac{1}{2}}$

$$h = \frac{3}{2} \cdot \frac{1}{9} \cdot \frac{2}{1} = \frac{1}{3}$$

40. **50.** Since the speed varies inversely with time, the model for calculation is

speed $\times$ time is constant ($st = k$) The constant k is (60) (60) or 3600. If the speed is 72 miles per hour, then 72 t = 3600. $t = \frac{3600}{72}$. The correct answer is **50.**

Function Notation and Evaluation

41. If $f(x) = x^2 + 5x$, what is the value of $f(-2)$?

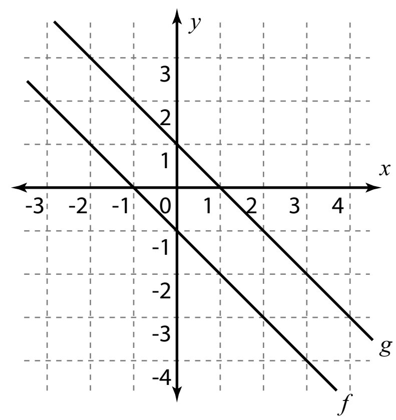
A. -14
B. -6
C. 6
D. 14
E. 16

42. For which of the following functions will $f(-2) = f(2)$?

A. $f(x) = 4x^3$
B. $f(x) = \frac{x}{5}$
C. $f(x) = 3x - 1$
D. $f(x) = 5 + |x|$
E. $f(x) = x^3 - 4$

43. If $f(x) = 3x - 9$ and $g(x) = \frac{x}{3} + 2$, then $f(g(x)) =$

A. $x - 7$
B. $x - 3$
C. $x - 1$
D. $\frac{x}{9} - 7$
E. $9x + 1$



44. In the graph above, the functions f and g are lines. What is the value of $g(2) - f(2)$?

Explanations

41. **B.** $f(x) = x^2 + 5x$. Substitute -2 for x: $f(-2) = (-2)^2 + 5(-2) = 4 + -10 = -6$

42. **D.** $f(-2)$ and $f(2)$ will be equal only if all powers of x in the function are even or in the case of any absolute values of the variable; this applies to the function $f(x) = 5 + |x|$ in Choice **D**.

43. **B.** $f\left(g(x)\right) = f\left(\frac{x}{3} + 2\right) = 3\left(\frac{x}{3} + 2\right) - 9 = x + 6 - 9 = x - 3$

44. **2.** Mapping the values for $g(2)$ and $f(2)$ you get -1 and -3 respectively. Subtract:

$-1 - (-3)$. The correct answer is **2.**

Concepts of Range and Domain

45. If $f(x) = 5\sqrt{x+2}$, what are values of x for which $f(x)$ is real number?

A. $x < -2$
B. $x \geq -2$
C. $x \geq 0$
D. $x \geq 2$
E. all real numbers

46. If $f(x) = x^2 + 3|x| + \frac{1}{2}$, which of the following is in the range of function f?

A. $-2\frac{1}{5}$
B. $\frac{-3}{4}$
C. 0
D. $\frac{1}{8}$
E. $1\frac{5}{7}$

47. Which of the following functions have the same domain as well as identical ranges?

I. $f(x) = |x + 5|$
II. $g(x) = \sqrt{x + 3}$
III. $h(x) = (x - 2)^2$

A. I and II
B. II and III
C. I and III
D. I, II, and III
E. None have the same domains and identical ranges.

48. The range of the function $y = f(x)$ is defined by $-2 \le y \le 6$. If this function is reflected about the y-axis and then translated downward two units, what will be the largest value in the range?

Explanations

45. **B.** The problem is really just asking for the domain of the function f.

$f(x) = 5\sqrt{x+2}$ only has meaning when the term $x + 2 \ge 0$, since you cannot find the square root of a negative number in the real number system.

Since $x + 2 \ge 0$, you have $x \ge -2$.

46. **E.** When substituting negative numbers for x in $f(x) = x^2 + 3|x| + \frac{1}{2}$ you always get positive values for $f(x)$. The smallest value of $f(x)$ will occur when $x = 0$; in this case, the value of $f(0) = \frac{1}{2}$. Therefore all values of $f(x)$ will be greater than or equal to 0. Only Choice **E** fits this condition.

47. **C.** Determining the domain and range for each function yields:

I. $f(x) = |x + 5|$ domain: all real numbers range: $f(x) \ge 0$
II. $g(x) = \sqrt{x + 3}$ domain: $x \ge -3$ range: $g(x) \ge 0$
III. $h(x) = (x - 2)^2$ domain: all real numbers range: $h(x) \ge 0$

Therefore the functions in I and III have the same domains as well as the same ranges.

48. **4.** The range will not change as a result of reflection about the y-axis. When this function is moved downward two units, the range values will all decrease by two units and be defined by $-4 \le y \le 4$. Therefore, the greatest value of the range is 4. The correct answer is **4.**

Working with Positive Roots.

49. $\frac{6}{\sqrt{3}} =$

A. $\frac{\sqrt{2}}{2}$
B. $\sqrt{2}$
C. $2\sqrt{3}$
D. $3\sqrt{2}$
E. $6\sqrt{3}$

50. $\sqrt{12} + \sqrt{75} =$

A. $\sqrt{87}$
B. $7\sqrt{3}$
C. $7\sqrt{6}$
D. 21
E. $29\sqrt{3}$

51. $\left(\sqrt{6}-\sqrt{3}\right)^2 =$

A. 3
B. 9
C. $9-3\sqrt{2}$
D. $9-5\sqrt{2}$
E. $9-6\sqrt{2}$

52. The volume of a cube is 64. If the edge length of this cube is increased by 2 units, what will be the volume of the resulting cube?

Explanations

49. C. $\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

50. B. $\sqrt{12}+\sqrt{75} = \sqrt{4 \cdot 3}+\sqrt{25 \cdot 3} = 2\sqrt{3}+5\sqrt{3} = 7\sqrt{3}$

51. E. $\left(\sqrt{6}-\sqrt{3}\right)^2 = \left(\sqrt{6}-\sqrt{3}\right)\left(\sqrt{6}-\sqrt{3}\right) = 6-2\sqrt{18}+3$

$$\begin{aligned} &= 9-2\sqrt{18} \\ &= 9-2\sqrt{9 \cdot 2} \\ &= 9-2 \cdot 3\sqrt{2} \\ &= 9-6\sqrt{2} \end{aligned}$$

52. 216. If the volume of a cube is 64, the edge length will be the cube root of 64 which is 4 units. Increase this edge length by 2 units to obtain the edge length of the new cube. This value is 6 units. The volume of a cube with an edge of 6 units is 6 cubed, or 216. The correct answer is **216.**

Solving Quadratic Equations

53. If $x^2 + 3 = 19$, what is the value of x?

A. –4 only
B. 4 only
C. –4 and 4
D. $\sqrt{21}$ only
E. 16 only

54. If 3 is one of the roots of the equation $x^2 - kx - 15 = 0$, where k is a constant, what is the other root of the equation?

A. –15
B. –5
C. –3
D. 3
E. 5

55. If $x^2 + 3x = 10$, what is the value of x?

A. –2 only
B. –2 and 5
C. 2 only
D. $\frac{10}{3}$
E. 2 and –5

56. A function is defined by $y = x^2 - 5x + 6$. What is the smallest value of x where this function crosses the x-axis?

Explanations

53. **C.** $x^2 + 3 = 19$

$x^2 + 3 - 3 = 19 - 3$

$x^2 = 16$

$x = \pm 4$

54. **B.** Since 3 is a root (or solution) of the equation $x^2 - kx - 15 = 0$,

$x - 3$ must be a factor of $x^2 - kx - 15$:

$(x - 3)(x + ?) = x^2 - kx - 15$, so ? must equal 5

so you know $(x - 3)(x + 5) = x^2 - kx - 15 = 0$

Therefore, the other solution comes from $x + 5 = 0$, or $x = -5$

55. **E.** $x^2 + 3x = 10$

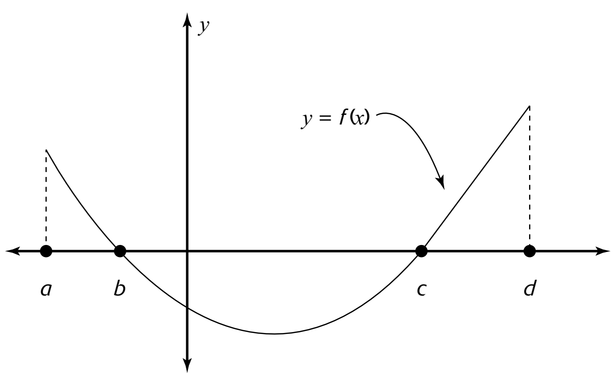
$x^2 + 3x - 10 = 10 - 10$

$x^2 + 3x - 10 = 0$

$(x - 2)(x + 5) = 0$, so $x = 2$ and $x = -5$

56. **2.** A graph crosses the x-axis where y is equal to zero. Setting y equal to 0, obtains the equation $x^2 - 5x + 6 = 0$. The left member if the equation is factorable, so the equation can be written $(x - 3)(x - 2) = 0$. Since one or both factors must equal 0, x is equal to 3 or 2. The correct answer, since the question asked for the smallest, is **2.**

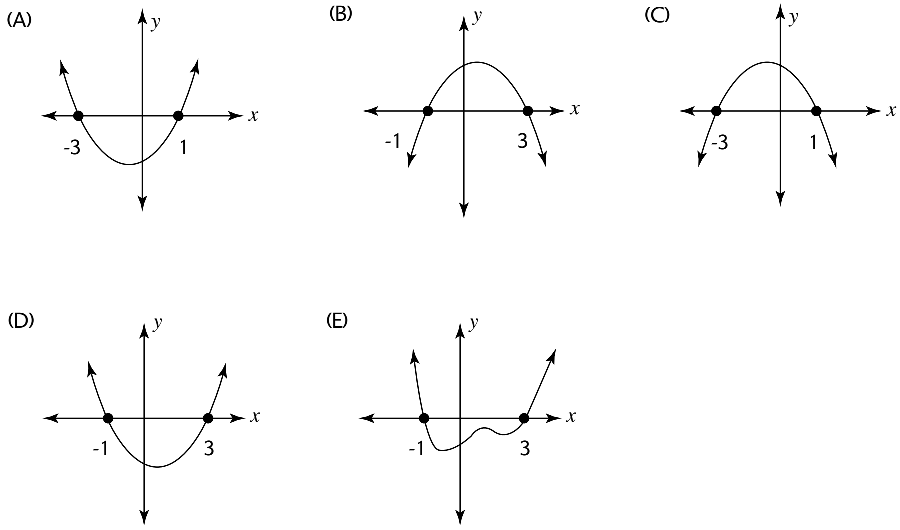
Working with Quadratic Functions and Graphs

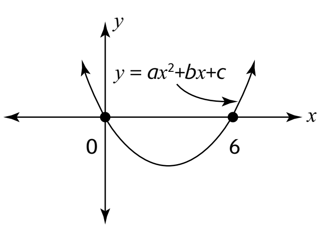


57. A portion of a quadratic function $f(x)$ is shown in the figure above. For what value of x will $f(x) \geq 0$?

A. $b < x < c$
B. $x < b, x > c$
C. $a \leq x \leq b, c \leq x \leq d$
D. $b \leq x \leq c$
E. $a < x < b, c < x < d$

58. If $f(x) = x^2 - 2x - 3$, which of the following could be the graph of f?





59. In the figure above, a portion of the graph of $y = ax^2 + bx + c$ is shown.

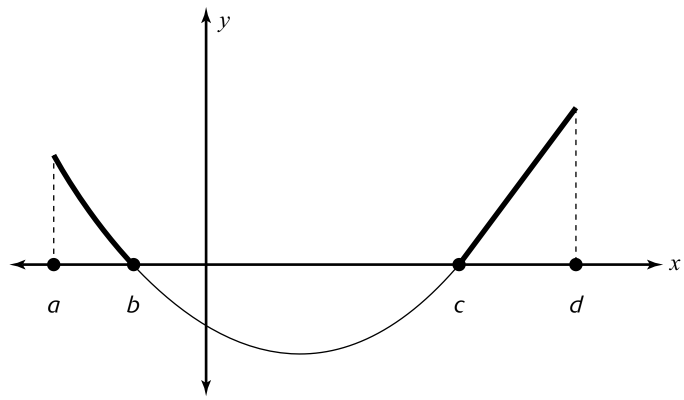
For which of the following values of x will their y-coordinates be equal?

A. –1 and 1
B. 1 and 4
C. 2 and 5
D. –2 and 7
E. –3 and 9

60. If $f(x) = -x^2 - 4x - k$ and contains the point (–4, 3), what is the y coordinate of the function at the point on the axis of symmetry of the graph?

Explanations

57. **C.**



In the figure above, $f(x) \geq 0$ in the dark shaded regions—above or on the x-axis. These regions can be described by: $a \leq x \leq b$ and $c \leq x \leq d$.

58. **D.** For the graph of $f(x) = x^2 - 2x - 3$, its x-intercepts can be found by setting $f(x) = 0$ and then solving for x.

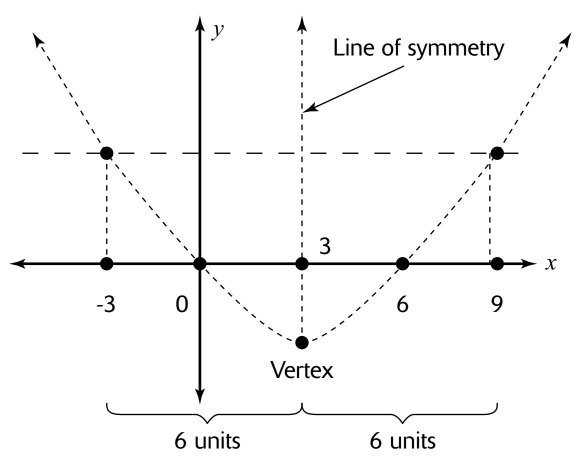
$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$**,** so the graph of f crosses the x-axis at -1 and 3.

Since the coefficient of the x^2 term is positive, the graph of f must open upwards, as in Choice **D**.

59. **E.** Since the graph of the parabola $y = ax^2 + bx + c$ crosses the x-axis at 0 and 6,

the x-coordinate of its vertex must be 3. Because of the symmetry of the parabola around the vertical line $x = 3$, points with x-coordinates equal distances left and right of this line of symmetry must have the same y-coordinate. The only answer choice that has two values of x meeting this condition is **E;** note that -3 is 6 units left of the line of symmetry at $x = 3$, while 9 is 6 units right of the line of symmetry.



60. **1.** The axis of symmetry is the line through the vertex of the graph that divides the graph into two pieces that, when folded along this axis line, exactly match each other. For this reason, the two points on the x-axis that are on the graph of the function are equidistant from the axis.

Begin by substituting the point into the function for x and $f(x)$. The equation will be $3 = -16 + 16 - k$.

From this equation the value of k is –3. Now substitute –3 into the function for k and factor the resulting polynomial. The function is $f(x) = -x^2 - 4x - 3$, and the factored form is $f(x) = -1(x + 1)(x + 3)$. The values for x where the graph intersects the x-axis is the values where the factors $(x + 1)$ and $(x + 3)$ are equal to 0. Therefore this graph passes through –1 and 3. Since the axis line must pass through the midpoint of these numbers, the axis of symmetry is all points where $x = -2$.

To find the value of the function where $x = -2$, evaluate $f(-2)$. This value is $-(-2)^2 - 4(-2) - 3$ or $-4 + 8 - 3$. The correct answer is **1.** The y coordinate of the vertex point is another method to solve the problem.

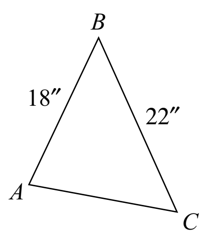
Geometry and Measurement

Geometry Diagnostic Test (Geometry and Measurement)

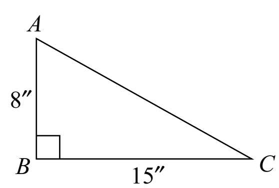
Questions

1. Lines that stay the same distance apart and never meet are called _______ lines.

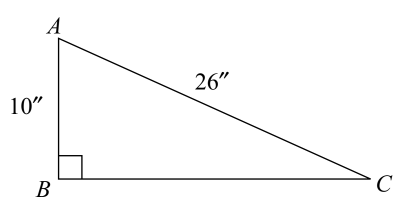
2. Lines that meet to form 90° angles are called _______ lines.



3. In the preceding triangle, *AC* must be smaller than _______ inches.



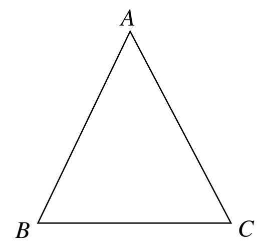
4. What is the length of *AC* in the preceding figure?



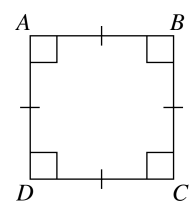
5. What is the length of *BC* in the preceding figure?

6. Name each of the following polygons:

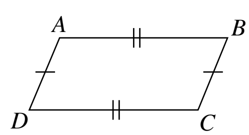
A.



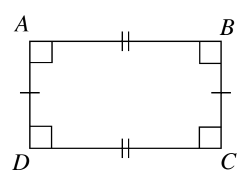
B.



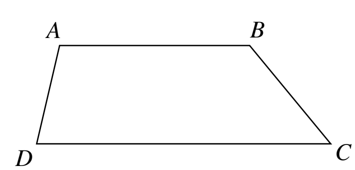
C.

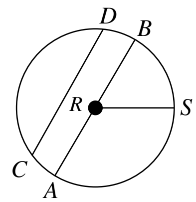


D.



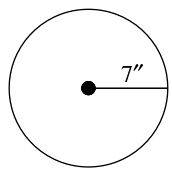
E.





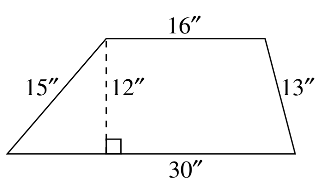
7. Fill in the blanks for circle *R* in the preceding figure:

A. $\overline{RS}$ is called the _______.
B. $\overline{AB}$ is called the _______.
C. $\overline{CD}$ is called a _______.



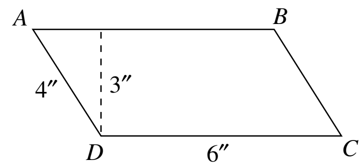
8. Find the area and circumference for the circle in the preceding figure ($\pi \approx \frac{22}{7}$):

A. area =
B. circumference =



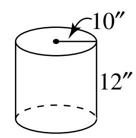
9. Find the area and perimeter of the preceding figure:

A. area =
B. perimeter =

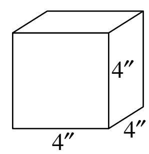


10. Find the area and perimeter of the preceding figure (*ABCD* is a parallelogram):

A. area =
B. perimeter =

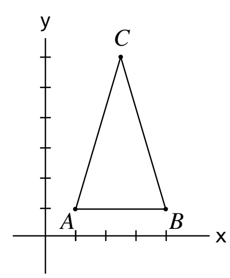


11. Find the volume of the preceding figure if $V = (\pi r^2)h$. (Use 3.14 for π):



12. What is the surface area and volume of the preceding cube?

A. surface area =
B. volume =



13. What is the area of $\triangle ABC$ in the preceding figure?

Answers

1. parallel

2. perpendicular

3. 40 inches

(Since $AB + BC = 40$ inches, $AC < AB + BC$ and $AC < 40$ inches.)

4. $AC = 17$ inches

5. Since $\triangle ABC$ is a right triangle, use the Pythagorean theorem:

$$a^2 + b^2 = c^2$$
$$10^2 + b^2 = 26^2$$
$$100 + b^2 = 676$$
$$b^2 = 576$$
$$b = 24"$$

6.

A. triangle
B. square
C. parallelogram
D. rectangle
E. trapezoid

7.

A. radius
B. diameter
C. chord

8.

A. area $= \pi r^2$
$= \pi(7^2)$
$= \frac{22}{7}(7)(7)$
$= 154$ square inches

B. circumference $= \pi d$
$= \pi(14)$ ($d = 14"$, because $r = 7"$)
$= \frac{22}{7}(14)$
$= 22(2)$
$= 44$ inches

9.

A. area $= \frac{1}{2}(a+b)h$
$= \frac{1}{2}(16+30)12$
$= \frac{1}{2}(46)12$
$= 23(12)$
$= 276$ square inches

B. perimeter $= 16 + 13 + 30 + 15 = 74$ inches

10.

A. area $= bh$
$= 6(3)$
$= 18$ square inches

B. perimeter $= 6 + 4 + 6 + 4 = 20$ inches

11. volume $= (\pi r^2)h$
$= (\pi \cdot 10^2)(12)$
$= 3.14(100)(12)$
$= 314(12)$
$= 3{,}768$ cubic inches

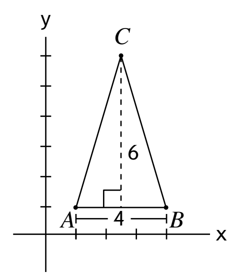
12.

A. All six surfaces have an area of 4×4, or 16 square inches because each surface is a square. Therefore, $16(6) = 96$ square inches is the surface area.

B. Volume = side × side × side, or $4^3 = 64$ cubic inches.

13. 12

The area of a triangle is $\frac{1}{2}$ × base × height.



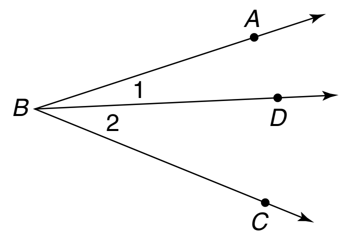
Base AB of the triangle is 4 units. The height of the triangle is 6 units, so the

$$\text{area of triangle} = \frac{1}{2} \times 4 \times 6$$
$$= \frac{1}{2} \times 24$$
$$= 12$$

Geometry Review

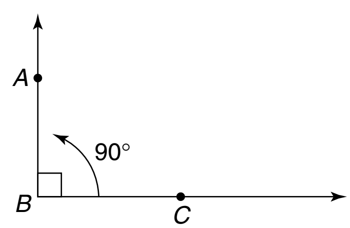
Types of Angles

Adjacent angles are any angles that share a common side and a common vertex (point)



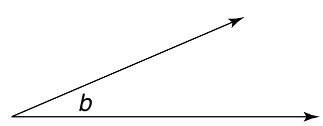
In the diagram, ∠1 and ∠2 are adjacent angles.

A *right angle* has a measure of 90°. The symbol ⌞ in the interior of an angle designates the fact that a right angle is formed.



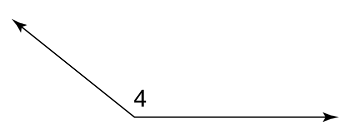
In the diagram, ∠*ABC* is a right angle.

Any angle the measure of which is less than 90° is called an acute angle.



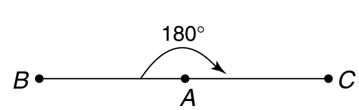
In the diagram, ∠*b* is acute.

Any angle whose measure is larger than 90° but smaller than 180° is called an *obtuse angle*.



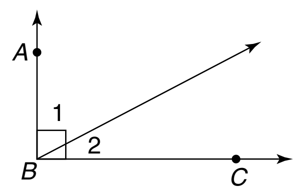
In the diagram, ∠4 is an obtuse angle.

A *straight angle* has a measure of 180°.



In the diagram, ∠*BAC* is a straight angle (also called a line).

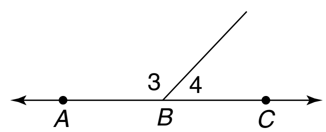
Two angles whose sum is 90° are called *complementary angles*.



In the diagram, because $\angle ABC$ is a right angle, $\angle 1 + \angle 2 = 90°$.

Therefore, $\angle 1$ and $\angle 2$ are complementary angles. If $\angle 1 = 55°$, its complement, $\angle 2$, would be $90° - 55° = 35°$.

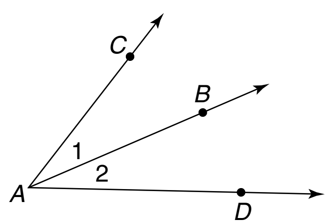
Two angles whose sum is 180° are called *supplementary angles*. Two adjacent angles that form a straight line are supplementary.



In the diagram, since $\angle ABC$ is a straight angle, $\angle 3 + \angle 4 = 180°$.

Therefore, $\angle 3$ and $\angle 4$ are supplementary angles. If $\angle 3 = 122°$, its supplement, $\angle 4$, would be: $180° - 122° = 58°$.

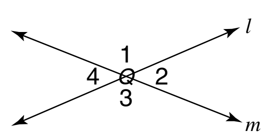
A ray from the vertex of an angle that divides the angle into two equal pieces is called an *angle bisector*.



In the diagram, $\overrightarrow{AB}$ is the angle bisector of $\angle CAD$.

Therefore, $\angle 1 = \angle 2$.

If two straight lines intersect, they do so at a point. Four angles are formed. Those angles opposite each other are called *vertical angles*. Those angles sharing a common side and a common vertex are, again, *adjacent angles*. Vertical angles are always equal.



In the diagram, line l and line m intersect at point Q, $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are formed.

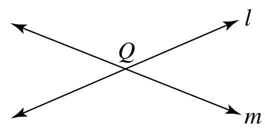
$$\left.\begin{array}{l}\angle 1 \text{ and } \angle 3\\ \angle 2 \text{ and } \angle 4\end{array}\right\}\text{ are vertical angles}$$

$$\left.\begin{array}{l}\angle 1 \text{ and } \angle 2\\ \angle 2 \text{ and } \angle 3\\ \angle 3 \text{ and } \angle 4\\ \angle 1 \text{ and } \angle 4\end{array}\right\}\text{ are adjacent angles}$$

$$\text{Therefore, } \begin{array}{l}\angle 1 = \angle 3\\ \angle 2 = \angle 4\end{array}$$

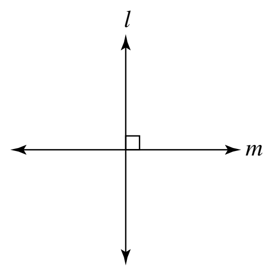
Types of Lines

- Two or more lines that cross each other at a point are called *intersecting lines*. That point is on each of those lines.



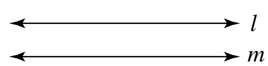
In the diagram, lines l and m intersect at Q.

- Two lines that meet to form right angles (90° angles) are called *perpendicular lines*. The symbol $\perp$ is used to denote perpendicular lines.



In the diagram, $l \perp m$.

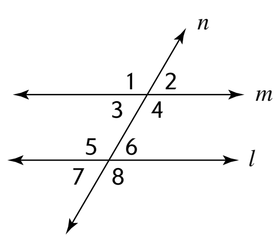
- Two or more lines that remain the same distance apart at all times are called *parallel lines*. Parallel lines never meet. The symbol $\|$ is used to denote parallel lines.



In the diagram, $l \| m$.

Parallel Lines Cut by Transversal

- When two parallel lines are both intersected by the third line, it is termed *parallel lines, cut by a transversal*. In the diagram below, line n is the transversal, and lines m and l are parallel. Eight angles are formed. There are many facts and relationships about these angles.



1. *Adjacent angles*. Angles 1 and 2 are adjacent and they form a straight line; therefore, they are supplementary. $\angle 1 + \angle 2 = 180°$.

   Likewise: $\angle 2 + \angle 4 = 180°$ $\quad \angle 7 + \angle 8 = 180°$
   $\angle 3 + \angle 4 = 180°$ $\quad \angle 5 + \angle 7 = 180°$
   $\angle 1 + \angle 3 = 180°$ $\quad \angle 6 + \angle 8 = 180°$
   $\angle 5 + \angle 6 = 180°$

2. *Vertical angles*. Angles 1 and 4 are vertical angles; therefore, they are equal. $\angle 1 = \angle 4$.

   Likewise: $\angle 2 = \angle 3$
   $\angle 5 = \angle 8$
   $\angle 7 = \angle 6$

3. *Corresponding angles*. If we could physically pick up line *l* and place it on line *m*, the angles that would coincide with each other would be equal in measure. They are called corresponding angles.

   Therefore: $\angle 1 = \angle 5$ $\quad \angle 3 = \angle 7$
   $\angle 2 = \angle 6$ $\quad \angle 4 = \angle 8$

4. *Alternate interior and exterior angles*. Alternate angles are on the opposite side of the transversal. Interior angles are those contained within the parallel lines. Exterior angles are those on the outsides of the parallel lines.

   Therefore: $\angle 3$ and $\angle 6$ are alternate interior angles.
   $\angle 3 = \angle 6$
   $\angle 4$ and $\angle 5$ are alternate interior angles.
   $\angle 4 = \angle 5$
   $\angle 2$ and $\angle 7$ are alternate exterior angles.
   $\angle 2 = \angle 7$
   $\angle 1$ and $\angle 8$ are alternate exterior angles.
   $\angle 1 = \angle 8$

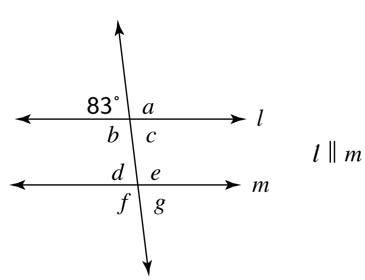
5. *Consecutive interior angles*. Consecutive interior angles are on the same side of the transversal.

   Therefore: $\angle 3$ and $\angle 5$ are consecutive interior angles.
   $\angle 3 = \angle 5 = 180°$
   $\angle 4$ and $\angle 6$ are consecutive interior angles.
   $\angle 4 = \angle 6 = 180°$

   The sum of the measures of each pair of consecutive angles = 180°.

   Using all of these facts, if we are given the measure of one of the eight angles, the other angle measures can all be determined.

   For example:



Note that since the lines are parallel, you can *see* which angles are equal, even if you cannot remember the rules.

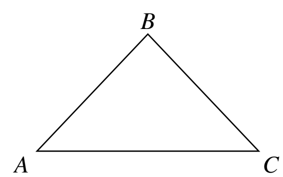
Polygons

Closed shapes or figures with three or more sides are called *polygons*. (*Poly* means many; *gon* means sides; thus, *polygon* means many sides.)

Triangles

This section deals with those polygons having the fewest number of sides. A *triangle* is a three-sided polygon. It has three angles in its interior. The sum of these angles is *always* 180°. The symbol for triangle is $\triangle$. A triangle is named by all three letters of its vertices.

The following figure shows $\triangle ABC$:

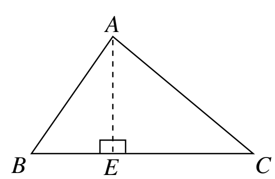


There are various types of triangles:

- A triangle having all three sides equal (meaning all three sides having the same length) is called an *equilateral triangle*.
- A triangle having two sides equal is called an *isosceles triangle*.
- A triangle having none of its sides equal is called a *scalene triangle*.
- A triangle having a right (90°) angle in its interior is called a *right triangle*.

Facts about triangles:

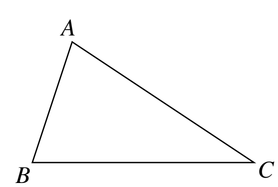
- Every triangle has a base (bottom side) and a height (or altitude). Every height is the *perpendicular* (forming a right angle) distance from a vertex to its opposite side (the base).

  In this diagram of $\triangle ABC$, $\overline{BC}$ is the base, and $\overline{AE}$ is the height. $\overline{AE} \perp \overline{BC}$.
- The sum of the lengths of any two sides of a triangle must be larger than the length of the third side.

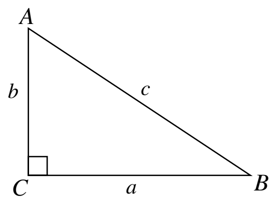
  In the diagram of $\triangle ABC$:

  $$AB + BC > AC$$
  $$AB + AC > BC$$
  $$AC + BC > AB$$

Pythagorean theorem:

- In any right triangle, the relationship between the lengths of the sides is stated by the Pythagorean theorem.



The parts of a right triangle are:

$\angle C$ is the right angle.

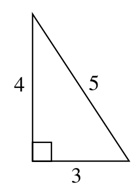
- The side opposite the right angle is called the *hypotenuse* (side c). (The hypotenuse is always the longest side.) The other two sides are called the *legs* (sides a and b).

The three lengths a, b, and c are always numbered such that:

$$a^2 + b^2 = c^2$$

For example: If $a = 3$, $b = 4$, and $c = 5$:

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25$$

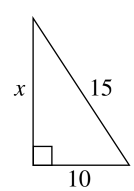


- Therefore, 3–4–5 is called a Pythagorean triple. There are other values for a, b, and c that always work. Some are 1–1–$\sqrt{2}$, 5–12–13, and 8–15–17. Any multiple of one of these triples also works. For example, multiplying the 3–4–5 solution set shows that 6–8–10, 9–12–15, and 15–20–25 are also Pythagorean triples.
- If perfect squares are known, the lengths of these sides can be determined easily. A knowledge of the use of algebraic equations can also be used to determine the lengths of the sides.

For example:

Find the length of x in the triangle.

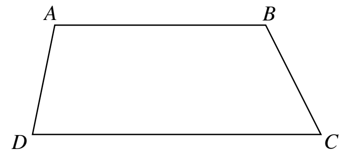
$$a^2 + b^2 = c^2$$
$$x^2 + 10^2 = 15^2$$
$$x^2 + 100 = 225$$
$$x^2 = 125$$
$$x = \sqrt{125}$$
$$\sqrt{125} = \sqrt{25} \times \sqrt{5} = 5\sqrt{5}$$
$$\text{So, } x = 5\sqrt{5}$$



Quadrilaterals

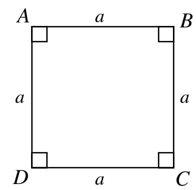
A polygon having four sides is called a *quadrilateral*. There are four angles in its interior. The sum of these interior angles is always 360°. A quadrilateral is named using the four letters of its vertices.

The following figure shows quadrilateral *ABCD*.

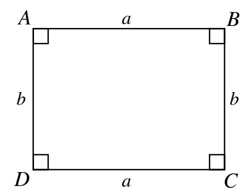


Types of quadrilaterals

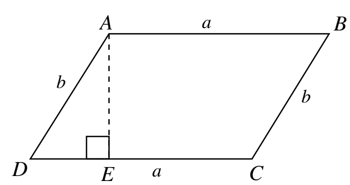
- A *square* has four equal sides and four right angles.



- A *rectangle* has opposite sides that are equal and four right angles.

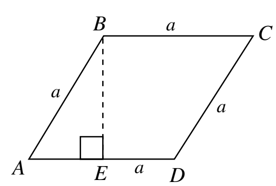


- A *parallelogram* has opposite sides equal and parallel, opposite angles equal, and consecutive angles supplementary. Every parallelogram has a height.

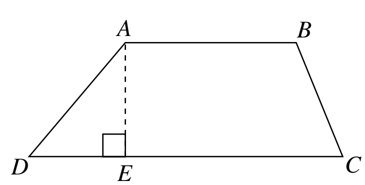


$\overline{AE}$ is the height of the parallelogram, $\overline{AB} \| \overline{CD}$, and $\overline{AD} \| \overline{BC}$.

- A *rhombus* is a parallelogram with four equal sides. A rhombus has a height. $\overline{BE}$ is the height.



- A *trapezoid* has only one pair of parallel sides. A trapezoid has a height. $\overline{AE}$ is the height. $\overline{AB} \| \overline{DC}$.



Other Polygons

- A *pentagon* is a 5-sided polygon.
- A *hexagon* is a 6-sided polygon.
- An *octagon* is an 8-sided polygon.
- A *nonagon* is a 9-sided polygon.
- A *decagon* is a 10-sided polygon.

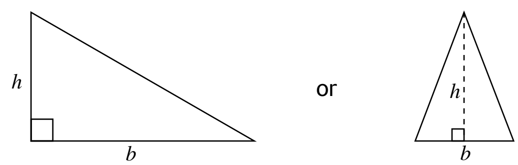
Perimeter

Perimeter means the total distance all the way around the outside of any shape. The perimeter of any polygon can be determined by adding the lengths of all the sides. The total distance around is the sum of all sides of the polygon. No special formulas are really necessary.

Area

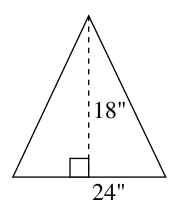
Area (A) means the amount of space inside the polygon. The formulas for each area are as follows:

Triangle: $A = \frac{1}{2}bh$



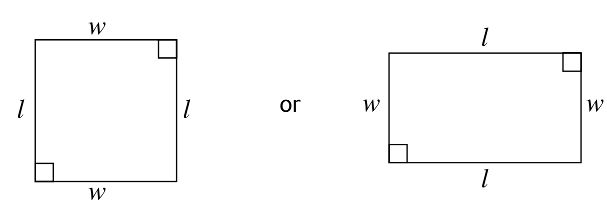
For example:

$$A = \frac{1}{2}bh$$

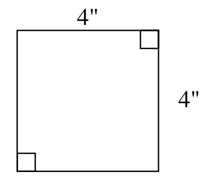


$$\text{A} = \frac{1}{2}(24)(18) = 216 \text{ sq. in.}$$

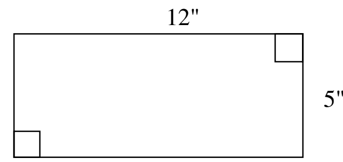
Square or rectangle: $A = lw$



For example:

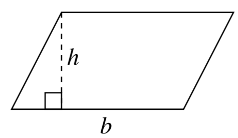


$$A = l(w) = 4(4) = 16 \text{ sq. in.}$$



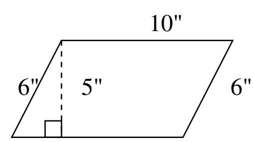
$$A = l(w) = 12(5) = 60 \text{ sq. in.}$$

Parallelogram: $A = bh$



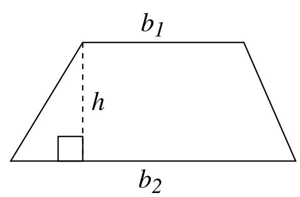
For example:

$$A = b(h)$$



$$A = 10(5) = 50 \text{ sq. in.}$$

Trapezoid: $A = \frac{1}{2}\left(b_1 + b_2\right)h$

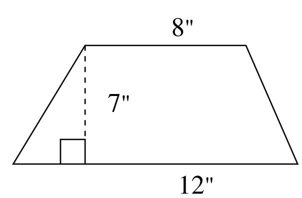


For example:

$$A = \frac{1}{2}(b_1 + b_2)h$$

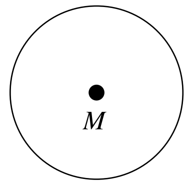
$$A = \frac{1}{2}(8 + 12)(7)$$

$$= \frac{1}{2}(20)(7) = 70 \text{ sq. in.}$$



Circles

A closed shape whose side is formed by one curved line, all points on which are equidistant from the center point, is called a *circle*. Circles are named by the letter of their center point.

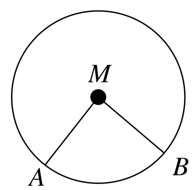


This is circle M. M is the center point because it is the same distance away from all points on the circle.

Parts of a Circle

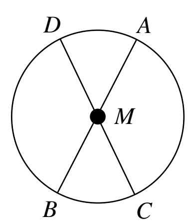
- *Radius* is the distance from a center to any point on a circle. In any circle, all radii (plural) are the same length.

$\overline{MA}$ is a radius.
$\overline{MB}$ is a radius.



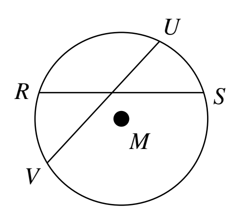
- *Diameter* is the distance across a circle, through the center. In any circle, all diameters are the same length. Each diameter is two radii.

$\overline{AB}$ is a diameter.
$\overline{CD}$ is a diameter.



- The *chord* is a line segment whose end points lie on the circle itself.

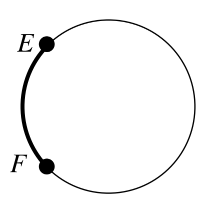
$\overline{RS}$ is a chord.
$\overline{UV}$ is a chord.



The diameter is the longest chord in any circle.

- An *arc* is the distance between any two points on the circle itself. An arc is a piece of the circle. The symbol $\frown$ is used to denote an arc. It is written on top of the two endpoints that form the arc. Arcs are measured in degrees. There are 360° around a circle.

This is $\overset{\frown}{EF}$
Minor $\overset{\frown}{EF}$ is the shorter distance between E and F.
Major $\overset{\frown}{EF}$ is the longer distance between E and F.
When $\overset{\frown}{EF}$ is written, the minor arc is assumed.



Circumference and Area

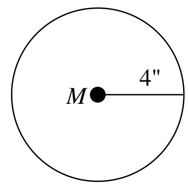
- *Circumference* is the distance around a circle. Since there are no sides to add up, a formula is needed. π (pi) is a Greek letter that represents a specific number. In fractional or decimal form, the commonly used approximations are: $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$.

  The formula for circumference is: $C = \pi d$ or $C = 2\pi r$.

  For example:

  In circle M, $d = 8$ because $r = 4$.

$$\begin{aligned} C &= \pi d \\ &= \pi(8) \\ &= 3.14(8) \\ &= 25.12 \text{ inches} \end{aligned}$$

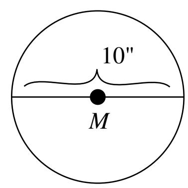


- The *area* of a circle can be determined by: $A = \pi r^2$.

  For example:

  In circle M, $r = 5$ because $d = 10$.

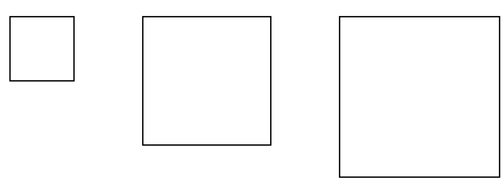
$$\begin{aligned} A &= \pi(r^2) \\ &= \pi(5^2) \\ &= 3.14(25) \\ &= 78.5 \text{ sq. in.} \end{aligned}$$



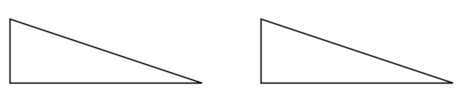
Congruence and Similarity

Two plane (flat) geometric figures are said to be congruent if they are identical in size and shape. They are said to be similar if they have the same shape, but are not identical in size. For example:

All squares are similar.



The following triangles are congruent.



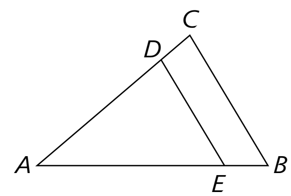
A more precise working definition of similar figures follows:

- Similar figures have corresponding angles equal and corresponding sides that are in proportion. Corresponding sides are those sides that are across from the equal angles.

For example:

Triangles *ADE* and *ACB* are similar

Side $DE = 4$ and corresponding side $CB = 6$. If side $AD = 6$, then side $AC =$



Since the triangles are similar, the corresponding sides are in proportion. The corresponding sides in the case are *AD* and *AC, DE* and *CB,* and *AE* and *AB.* The corresponding angles are $\angle DAE$ and $\angle CAB$, $\angle ADE$ and $\angle ACB$, and $\angle AED$ and $\angle ABC$. Because the proportion of side *DE* to side *CB* is 4 to 6 or $\frac{4}{6}$, which reduces to $\frac{2}{3}$, the same ratio holds for all corresponding sides. Therefore,

$$\frac{AD}{AC} = \frac{DE}{CB} \text{ or } \frac{AD}{AC} = \frac{2}{3}$$

and since $AD = 6$, $\frac{6}{AC} = \frac{2}{3}$.

Cross multiplying gives:

$$6(3) = 2(AC)$$
$$18 = 2(AC)$$

Divide each side by 2.

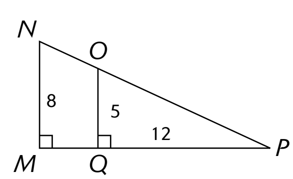
$$\frac{18}{2} = \frac{2(AC)}{2}$$
$$9 = AC \text{ or } AC = 9$$

Please note that this question could have been introduced as follows:

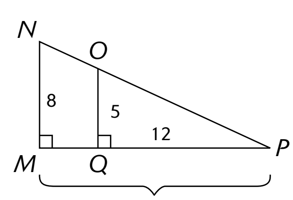
In the triangle shown, $DE \| CB$. If $AD = 4$, $CB = 6$, and $AD = 6$, what is the length of *AC*?

A line parallel to one side within a triangle produces similar triangles. Therefore, triangles *ADE* and *ACB* are similar and the problem can be solved as above.

A few more examples:



1. In the figure above, what is the length of *PM*?



In the figure above, since *MN* and *OQ* are both perpendicular to *PM,* they are parallel to each other. Therefore, triangles *OPQ* and *NPM* are similar. Since *OQ* to *NM* is in the ratio 5 to 8, you can set up the following proportion to find *PM.*

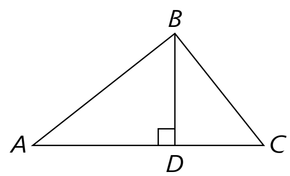
$$\frac{OQ}{NM} = \frac{PQ}{PM} \text{ or } \frac{5}{8} = \frac{12}{PM}$$

Cross multiplying gives

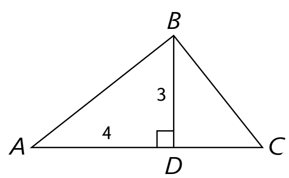
$$5(PM) = 8(12)$$
$$5(PM) = 96$$

Divide each side by 5.

$$\frac{5(PM)}{5} = \frac{96}{5}$$
$$PM = 19\frac{1}{5} \text{ or } 19.2$$



2. In the figure above, triangles *ADB* and *BDC* are similar. $\angle DAB = \angle DBC$, and $\angle ABD = \angle BCD$. What is the length of *DC*?



In the figure above, since triangles *ADB* and *BDC* are similar, the only real difficulty is matching the corresponding parts. Since $\angle BAD = \angle CBD$, they are corresponding and sides across from them, *BD* and *DC*, are corresponding. Since $\angle DCB = \angle DBA$, *AD* and *BD* are corresponding. (Note that *BD* is used twice, as it is part of both triangles). Because *BD* to *AD* is in the ratio 3 to 4, you can set up the following proportion and solve accordingly.

$$\frac{BD}{AD} = \frac{DC}{BD} \text{ or } \frac{3}{4} = \frac{DC}{3}$$
$$3(3) = 4(DC)$$
$$9 = 4(DC)$$
$$\frac{9}{4} = \frac{4(DC)}{4}$$
$$2\frac{1}{4} = DC \text{ or } DC = 2\frac{1}{4} \text{ or } 2.25$$

Three-Dimensional Shapes

Volume

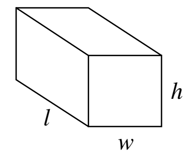
In three-dimensional shapes, *volume* refers to the capacity to hold. The formula for volume of each shape is different.

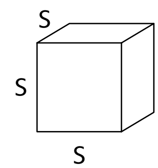
The volume of any prism (a three-dimensional shape having many sides, but two bases) can be determined by:

Volume (V) = (area of base)(height of prism)

Specifically, for a rectangular solid:

$$V = (lw)(h)$$
$$= lwh$$

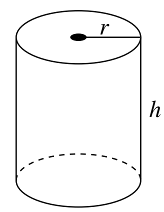




The formula for the volume of the cube is often written $V = s \times s \times s = s^3$

Specifically, for a cylinder (circular bases):

$$V = (\pi r^2)h$$
$$= \pi r^2 h$$

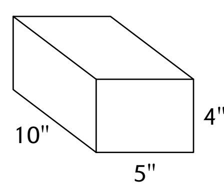


Volume is labeled in cubic units.

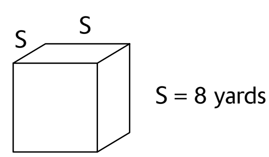
Some examples:

Find the volumes of the solid figures below whose dimensions are indicated.

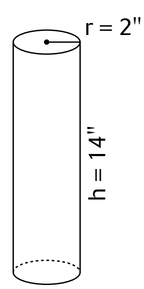
1. Rectangular Solid
$V = lwh = (10)(5)(4) = 200$ cu. in.



2. Cube
$V = s^3 = 8 \times 8 \times 8 = 512$ cu. yd.



3. Cylinder
$V = \pi r^2 h = \frac{22}{\cancel{7}} \times \frac{2}{1} \times \frac{2}{1} \times \frac{\overset{2}{\cancel{14}}}{1} = 22(8) = 176$ cu. in.

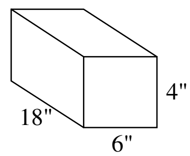


Surface Area

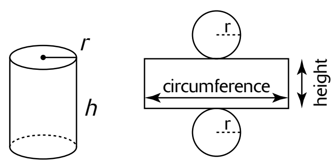
The surface area of a three-dimensional solid is the area of all the surfaces that form the solid. Find the area of each surface, and then add those areas. The surface area of a rectangular solid can be found by adding the areas of all six surfaces. For example:

The surface area of this prism is:

top:	$18 \times 6 = 108$
bottom:	$18 \times 6 = 108$
left side:	$6 \times 4 = 24$
right side:	$6 \times 4 = 24$
front:	$18 \times 4 = 72$
back:	$18 \times 4 = \underline{72}$
	408 sq. in.

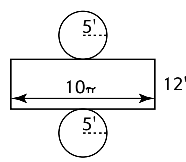


To determine the *surface area of a right circular cylinder*, it is best envisioned "rolled out" onto a fat surface as below.



Now find the area of each individual piece. The area of each circle equals πr^2. Note that the length of the rectangle equals the circumference of the circle. The rectangle's area equals circumference times height. Adding the three parts gives the surface area of the cylinder: For example:

Find the surface area of a cylinder with radius 5' and height 12'.



The area of the circle = $\pi(r^2) = \pi(5^2) = 25\pi$.

The area of the bottom circle is the same, 25π.

The length of the rectangle is the circumference of the circle, or $2\pi r = 2\pi(5) = 10\pi$.

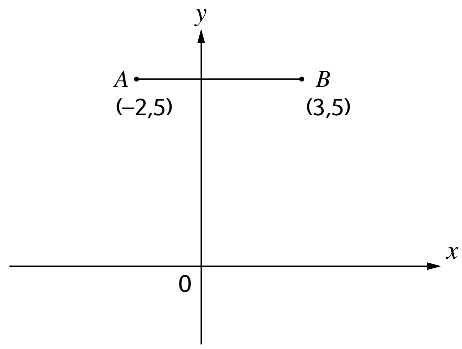
Therefore the area of the rectangle equals its height times $10\pi = 12 \times 10\pi = 120\pi$.

Totaling all the piece gives $25\pi + 25\pi + 120\pi = 170\pi$.

Coordinate Geometry and Measurement

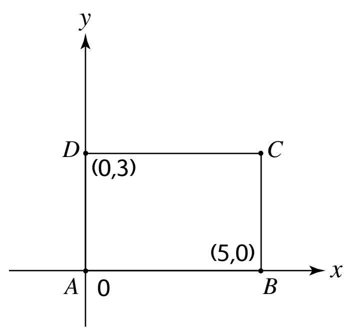
Refer to "Basic Coordinate Geometry" in the "Algebra Review" section if you need to review coordinate graphs.

Coordinate graphs can be used in measurement problems. For example:



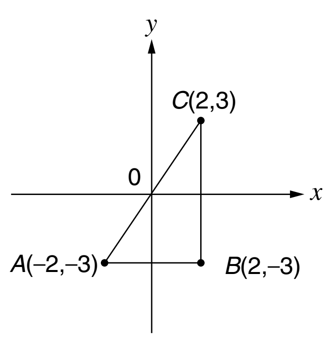
1. What is the length of AB in the preceding graph?

Since the coordinates of the points are (–2, 5) and (3, 5), the first, or x-coordinate is the clue to the distance of each point from the y-axis. The distance to point B from the y-axis is 3, and the distance to point A from the y-axis is 2. (–2 is 2 in the negative direction.) So 3 + 2 gives a length of 5.



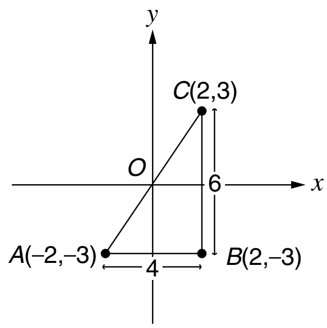
2. What is the area of rectangle $ABCD$ in the preceding graph?

The formula for the area of a rectangle is base × height. Since point A is at (0, 0) and point B is at (5, 0), the base is 5. Since point D is at (0, 3), the height is 3, so the area is 5 × 3 = 15.



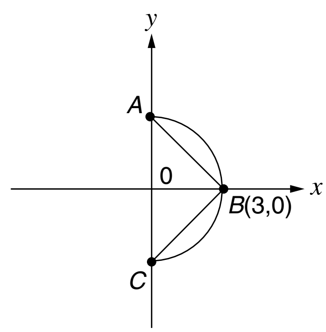
3. What is the area of $\triangle ABC$ in the preceding figure?

The area of a triangle is $\frac{1}{2}$ × base × height.



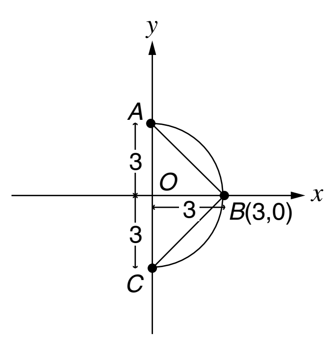
Base *AB* of the triangle is 4 units (because from *A* to the *y*-axis is 2 units and from the *y*-axis to *B* is another 2 units). Height *BC* of the triangle is 6 units (3 units from *B* to the *x*-axis and another 3 units to *C*). Note that $\angle B$ is a right angle. So

$$\begin{aligned}\text{area of triangle} &= \frac{1}{2}\times 4\times 6\\ &= \frac{1}{2}\times 24\\ &= 12\end{aligned}$$



4. In the preceding figure, what is the perimeter of $\triangle ABC$ inscribed within the semicircle with center *O*?

To find the perimeter of the triangle, you need the lengths of the three sides. You know that radius *OB* is 3 units long. The *OA* and *OC* are each 3 units because they are also radii. Therefore, side *AC* of the triangle is 6 units.



In triangle AOB, you know that OA is 3 and OB is 3. From the Pythagorean theorem,

$$\begin{aligned}
a^2 + b^2 &= c^2 \\
(OA)^2 + (OB)^2 &= (AB)^2 \\
3^2 + 3^2 &= (AB)^2 \\
9 + 9 &= (AB)^2 \\
18 &= AB \\
\sqrt{18} &= AB \\
\sqrt{9 \times 2} &= AB \\
3\sqrt{2} &= AB
\end{aligned}$$

(If you spotted that triangle AOB is an isosceles right triangle with sides in the ratio $1:1:\sqrt{2}$, you wouldn't have needed to use the Pythagorean theorem.)

By symmetry, you know that $AB = CB$. So $CB = 3\sqrt{2}$ and

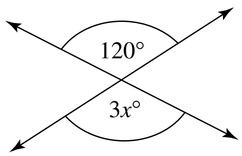
$$\begin{aligned}
\text{perimeter} &= CA + AB + CB \\
&= 6 + 3\sqrt{2} + 3\sqrt{2} \\
&= 6 + 6\sqrt{2}
\end{aligned}$$

Sample SAT-Type Problems

Now you can try actual SAT-like problems in each topic area. Note: Remember that problems range from easy to average to difficult.

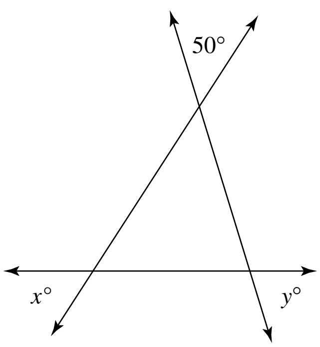
Geometry and Measurement

Vertical Angles



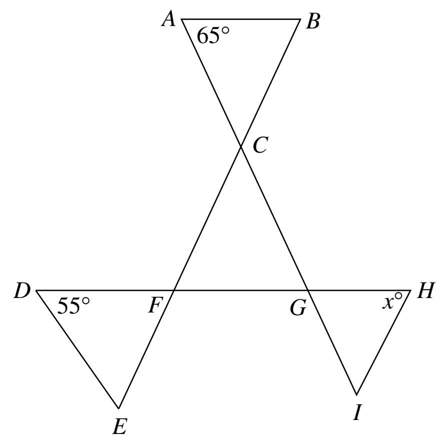
1. In the figure above, what is the value of x?

A. 20
B. 30
C. 40
D. 90
E. 120



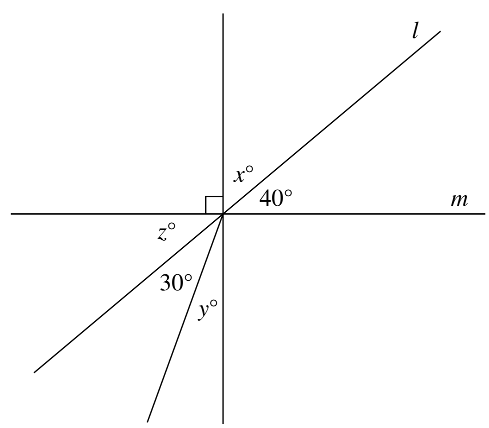
2. In the figure above, what is the value of $x + y$?

A. 50
B. 75
C. 100
D. 110
E. 130



3. In the figure above, $\overline{CA} = \overline{CB}$, $\overline{FD} = \overline{FE}$, and $\overline{GH} = \overline{GI}$. What is the value of x?

A. 45
B. 55
C. 60
D. 65
E. 70

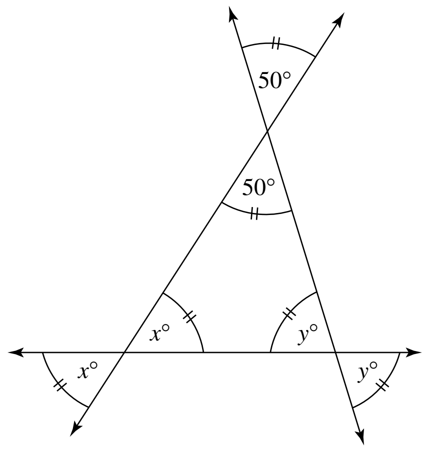


4. What is the value of $x° + y°$ in the figure above?

Explanations

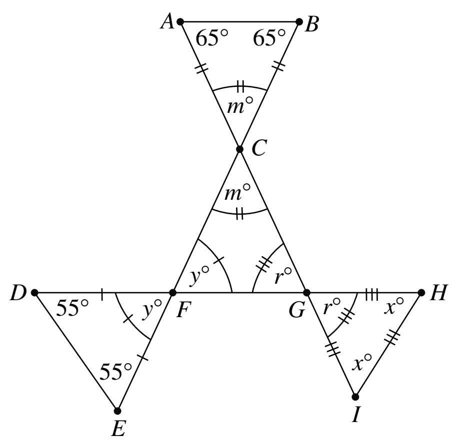
1. **C.** Since vertical angles have equal measure: $3x = 120$, so $x = 40$.

2. **E.**



Since vertical angles have equal measure, the interior angles of the triangle have measures 50, x, and y. Then $x + y + 50 = 180$, so $x + y = 130$.

3. **C.**



Since $\overline{CA} = \overline{CB}$, $\overline{FD} = \overline{FE}$, and $\overline{GH} = \overline{GI}$, $\angle A = \angle B = 65$, $\angle D = \angle E = 55$,

and $\angle H = \angle I = x$.

In $\triangle ABC$: $65 + 65 + m = 180$, so $m = 50$

In $\triangle DEF$: $55 + 55 + y = 180$, so $y = 70$

In $\triangle FCG$: $m + y + r = 180$, then substitute 50 for m and 70 for y to get

$50 + 70 + r = 180$, so $r = 60$.

Finally, in $\triangle GHI$: $r + 2x = 180$ substitute 60 for r to get

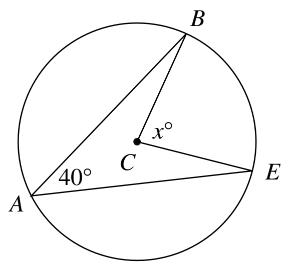
$$60 + 2x = 180$$

$$2x = 120$$

$$x = 60$$

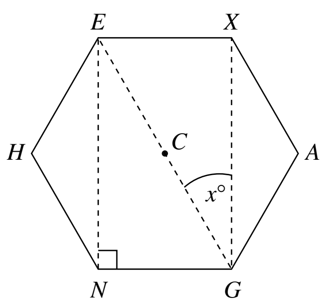
4. **70.** Since lines *l* and *m* form an angle of 40 degrees, the angle across (vertical angle) is also 40 degrees. Therefore, the measure of angle *z* is also 40 degrees. Since there is an indicated right angle, $z° + 30° + y° = 90°$. It follows that $40° + 30° + y° = 90°$ and therefore, $y = 20°$. Also, $x°$ is the complement of $40°$. $x = 50°$. It follows that $50° + 20° = 70°$. The correct answer is 70°.

Angles in Figures



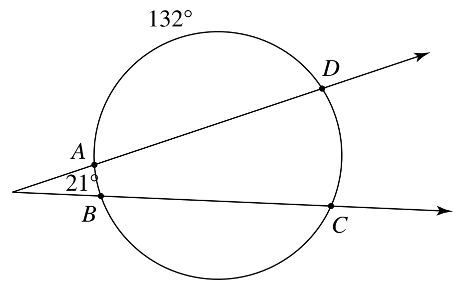
5. In circle *C* above, what is the value of *x*?

A. 40
B. 50
C. 60
D. 80
E. 90



6. In the regular hexagon above with center at point *C*, what is the value of *x*?

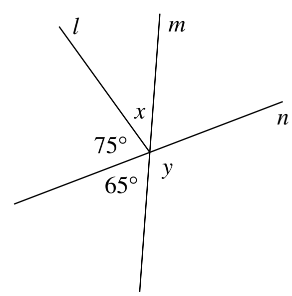
A. 15
B. 30
C. 40
D. 45
E. 60



7. In the figure above, the ratio of the degree measures of $\overset{\frown}{DC}$ to $\overset{\frown}{AB}$ is 5 to 2.

What is the degree measure of $\overset{\frown}{BC}$?

A. 14
B. 28
C. 70
D. 150
E. 230



8. What is the difference between *x* and *y* in the figure above?

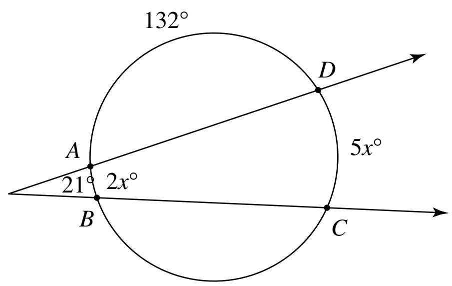
Explanations

5. **D.** $\angle A$ is an inscribed angle, so $40 = \frac{1}{2}\overset{\frown}{BE}$; therefore, $\overset{\frown}{BE} = 80$. Since $\angle C$ is a central angle, $\overset{\frown}{BE} = \angle C = 80$.

6. **B.** Since ΔNCG is equilateral, all angles of this triangle are 60°. In particular $\angle CGN$ is 60°. $\angle ENG$ is 90°, so $\angle XGN$ is 90° also. Then $\angle CGN + x = 90°$.

Therefore $x = 30$.

7. **D.**



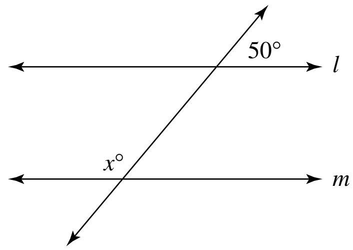
$21 = \frac{1}{2}(5x - 2x)$ An angle formed by 2 secant lines intersecting outside of the circle is one-half of the difference of the 2 arcs intercepted by the secants.

$$21 = \frac{1}{2}(3x)$$
$$2 \cdot 21 = 2 \cdot \frac{1}{2}(3x)$$
$$42 = 3x$$
$$\frac{42}{3} = \frac{3x}{3}$$
$$14 = x$$

Then,
$$\begin{aligned} 132 + \overset{\frown}{AB} + \overset{\frown}{DC} + \overset{\frown}{BC} &= 360 \\ 132 + 2x + 5x + \overset{\frown}{BC} &= 360 \\ 132 + 7x + \overset{\frown}{BC} &= 360 \\ 132 + 7(14) + \overset{\frown}{BC} &= 360 \quad \text{Substitute 14 for } x. \\ 132 + 98 + \overset{\frown}{BC} &= 360 \\ 230 + \overset{\frown}{BC} &= 360 \\ \overset{\frown}{BC} &= 130 \end{aligned}$$

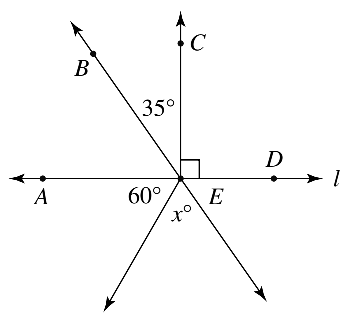
8. **75.** Using vertical angles, the angle opposite the 65° angle is 65°. Therefore, 65° + x + 75° = 180°. Solving for x, the value of x is 40°. The 65° angle and y form a straight angle (or linear pair), so these angles are supplementary and their sum is 180°. Solving for y, the value for y is 115°. Now subtract these two values and the correct answer is 75°.

Perpendicular and Parallel Lines



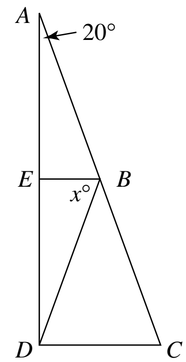
9. In the figure above with $l \parallel m$, what is the value of x?

A. 40
B. 50
C. 60
D. 130
E. 150



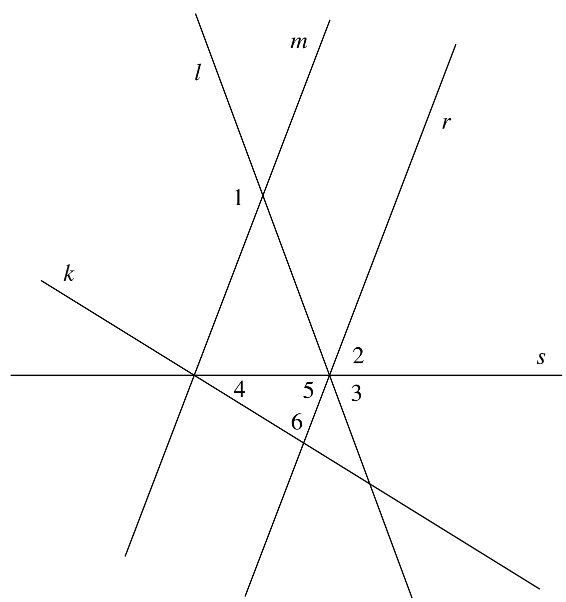
10. What is the value of x in the figure above?

A. 35
B. 55
C. 65
D. 115
E. 125



11. In the figure above, $\overline{AE} \perp \overline{EB}$, $\overline{AD} \perp \overline{DC}$, and $BD = BC$. What is the value of x?

A. 20
B. 50
C. 60
D. 70
E. 125

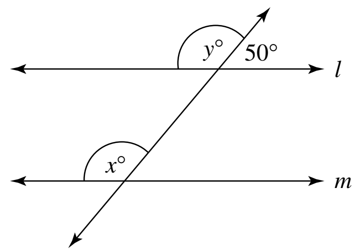


Note: Figure not drawn to scale.

12. In the figure above, $k \perp m$, $m \parallel r$, $\angle 1 = 120°$, and $\angle 3 = 60°$. What is the measure of angle 4?

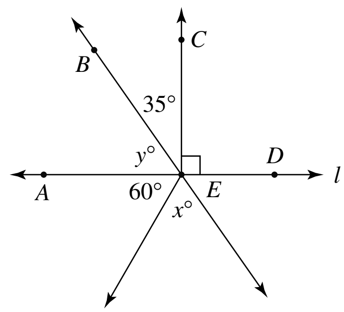
Explanations

9. **D.**



$y + 50 = 180$, so $y = 130$. Since $l \parallel m$, the corresponding angles have to be equal. Therefore, $y = x = 130$.

10. **C.**

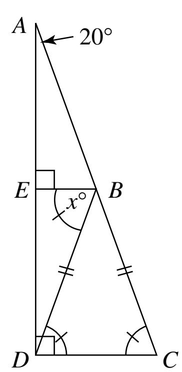


Since $\overline{CE} \perp \overline{AD}$, $\angle AEC$ is also a right angle. Therefore, $y + 35 = 90$, so $y = 65$.

Then, $y + 60 + x = 180$. Substitute 65 for y: $65 + 60 + x = 180$, $115 + x = 180$,

so $x = 65$.

11. **D.**



Since $\overline{AD} \perp \overline{DC}$, $\angle ADC = 90$. In $\triangle ADC$: $\angle A + \angle ADC + \angle C = 180$

Substitute 20 for $\angle A$ and 90 for: $\angle ADC$ $20 + 90 + \angle C = 180$, so $\angle C = 70$

Since $BD = BC$, $\angle BDC = \angle C = 70$.

$\overline{BE}$ and $\overline{DC}$ are both perpendicular to $\overline{AD}$, so $\overline{BE} \parallel \overline{DC}$; thus $x = \angle BDC = 70$.

12. **30°.** Angle 2 and angle 3 form an angle that is an alternate exterior angle with angle 1, so $\angle 1 = \angle 2 + \angle 3$. By substitution, $\angle 2 + \angle 3 = 120°$. Since $\angle 3 = 60°$ and $\angle 2 = 60°$, $\angle 5 = 60°$ because of vertical angles. Since $m \parallel r$ and $k \perp m$, it follows that angle 6 must be a right angle. Because 6 is a consecutive angle with the intersection of k and m. Therefore, $\angle 6 = 90°$. The triangle consisting of angles 4, 5, and 6 is a 30 – 60 – 90 right triangle, and the measure of angle 4 is 30°. The correct answer is 30°.

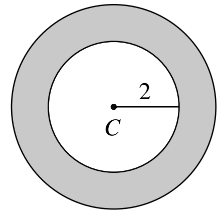
Circumference, Area, Radius, Diameter

13. What is the radius of a circle whose circumference is 16π?

A. $2\sqrt{2}$
B. 4
C. $4\sqrt{2}$
D. 6
E. 8

14. What is the circumference of a circle whose area is 8π?

A. $2\sqrt{2}$
B. 4
C. $4\sqrt{2}$
D. $2\pi\sqrt{2}$
E. 8π



15. For the concentric circles above, with center at *C*, the shaded region has an area of 11π. What is the radius of the larger circle?

A. $\sqrt{2}$
B. $\sqrt{7}$
C. 3
D. $\sqrt{15}$
E. 4

16. The ratio of the areas of two circles is 9π to 4π. What is the ratio of the circumferences of these two circles?

Explanations

13. E. Circumference of a circle is found using the formula $c = 2\pi r$.

Substitute 16π for c: $16\pi = 2\pi r$,

so $r = 8$.

14. E. Area of circle is found by using the formula $A = r^2$.

Substitute 8π for A: $8\pi = \pi r^2$

$$\frac{8\pi}{\pi} = \frac{\pi r^2}{\pi}$$

$$8 = r^2$$

$$\sqrt{8} = r$$

$$2\sqrt{2} = r$$

The circumference $= 2\pi r = 2\pi \cdot 2\sqrt{2} = 4\pi\sqrt{2}$.

15. D. If r is the radius of the larger circle, then:

$$Area_{shaded} = Area_{big\ circle} - Area_{small\ circle}$$

$$11\pi = \pi r^2 - \pi(2)^2$$

$$11\pi = \pi^2 - 4\pi$$

$$11\pi + 4\pi = \pi r^2 - 4\pi + 4\pi$$

$$15\pi = \pi r^2$$

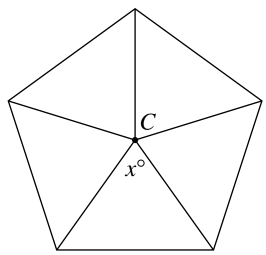
$$\frac{15\pi}{\pi} = \frac{\pi r^2}{\pi}$$

$$15 = r^2$$

$$\sqrt{15} = r$$

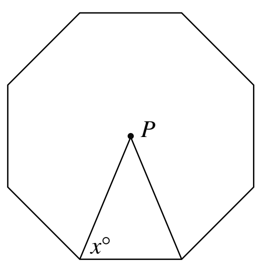
16. $\mathbf{\frac{3}{2}}$**.** Since the ratio of the areas is $\frac{9\pi}{4\pi}$, and area is πr^2 for a circle, the radius of the larger circle must be 3 and the radius of the smaller circle must be 2. Circumference of a circle is equal to $2\pi r$. Therefore, the ratio of the circumference must be $\frac{6\pi}{4\pi}$. In reduced form, the ratio is $\frac{3}{2}$.

Perimeter, Area, Angle Measure of Polygon



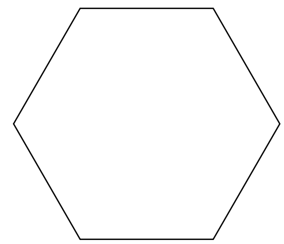
17. In the regular pentagon above, with center at C, what is the value of x?

A. 36
B. 45
C. 60
D. 72
E. It cannot be determined from given information.



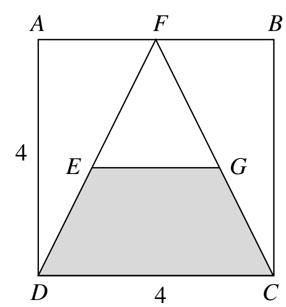
18. In the regular octagon above, with center at P, what is the value of x?

A. 22.5
B. 45
C. 60
D. 67.5
E. 135



19. In the figure above, the regular hexagon has a perimeter of 72. What is the area of the hexagon?

A. 108
B. $108\sqrt{3}$ (approximately 187.06)
C. 216
D. $216\sqrt{2}$ (approximately 305.47)
E. $216\sqrt{3}$ (approximately 374.12)



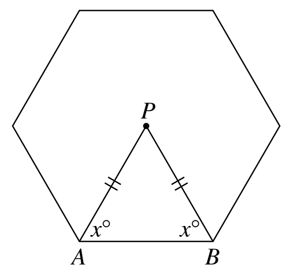
20. Quadrilateral $ABCD$ is a square with sides of 4 units as shown above. Triangle FDC is isosceles such that $DF = CF$. E is the midpoint of $\overline{FD}$ and G is the midpoint of $\overline{FC}$. What is the area of the shaded region?

Explanations

17. **D.** $x = \frac{360}{5} = 72$

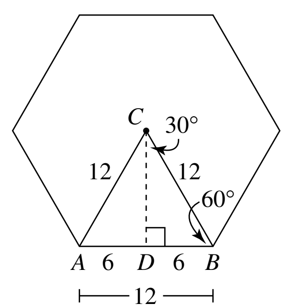
18. **D.** $\angle P = \frac{360}{8} = 45$. Since $PA = PB$, $\angle PAB = \angle PBA = x$.

In $\triangle APB$: $45 + \angle PAB + \angle PBA = 180$.



$45 + x + x = 180$, so $2x = 135$, and therefore $x = 67.5$

19. **E.**



Side of regular hexagon $= \frac{72}{6} = 12$. With hexagon center at C, $\triangle ACB$ is equilateral.

Method I: The area of an equilateral triangle with side length s is found by using the formula $\frac{s^2\sqrt{3}}{4}$.

So $\triangle ACB$ has area: $\frac{12^2\sqrt{3}}{4} = \frac{144\sqrt{3}}{4} = 36\sqrt{3}$.

The hexagon will then have area 6 times this: $6 \cdot 36\sqrt{3} = 216\sqrt{3}$.

Method II: In $\triangle ACB$, the altitude to base $\overline{AB}$ bisects the base at point D.

Then $\triangle CDB$ is a 30°–60°–90° triangle. With $DB = 6$, CD would be $6\sqrt{3}$.

So $\triangle CAB$ has area $\frac{1}{2} \cdot 12 \cdot 6\sqrt{3} = 36\sqrt{3}$. Then the hexagon will have an area

6 times this: $6 \cdot 36\sqrt{3} = 216\sqrt{3}$.

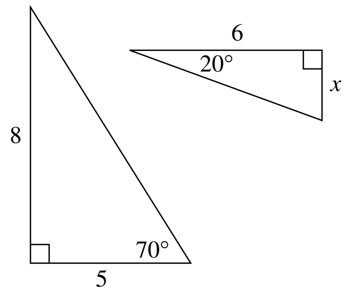
20. **6.** By constructing an altitude from F to the midpoint of $\overline{DC}$, you have the altitude of triangle DFC. The altitude is perpendicular to $\overline{DC}$. Since the polygon is a square, this altitude is the same length as the sides of the square. The altitude is 4 units. The altitude of the large triangle also serves as the altitude of the smaller triangle. The altitude of the smaller triangle is half of the altitude of the larger triangle. $\overline{EG}$ is half the length of $\overline{DC}$ and is 2 units also.

The area of $\triangle FDC = \frac{1}{2}(4)(4) = 8$ and the area of $\triangle FEG = \frac{1}{2}(2)(2) = 2$. $8 - 2 = 6$

Using the area formula for both triangles and subtracting, the answer is 6.

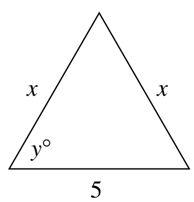
An alternate solution would be to find the area of isosceles trapezoid $DEGC$ using the area formula $A = \frac{1}{2}(b_1 + b_2)h$ where $b_1 = 4$, $b_2 = 2$, and $h = 2$.

Triangles—Right, Isosceles, Equilateral, Angle Measure, Similarity



21. In the figure above, what is the value of x?

A. $\frac{4}{15}$
B. $\frac{15}{4}$
C. $\sqrt{17}$
D. $\sqrt{53}$
E. $\frac{48}{5}$

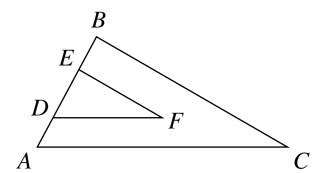


22. In the diagram above, $2x + 5 = 15$. What is the value of y?

A. 5
B. 30
C. 45
D. 60
E. It cannot be determined from the given information.

23. The measures of the interior angles of a triangle are in the ratio 2:3:4 respectively. What is the measure of the largest angle of this triangle?

A. 20°
B. 40°
C. 60°
D. 80°
E. 100°



24. In the figure shown above, $\overline{BC} \parallel \overline{EF}$ and $\overline{DF} \parallel \overline{AC}$. If $BC = 7$, $EF = 3$, and $DF = 2$, what is the measure of $\overline{AC}$?

Explanations

21. **C.** The missing angle in the first triangle is 20°; the missing angle in the second triangle is 70°. The two right triangles are therefore similar, so the lengths of their sides are proportional.

$$\frac{8}{5} = \frac{6}{x}$$

$$8x = 30$$

Cross multiply: $$\frac{8x}{8} = \frac{30}{8}$$

$$x = \frac{15}{4}$$

22. **D.** Solving for *x*: $2x + 5 = 15$

$$2x + 5 - 5 = 15 - 5$$

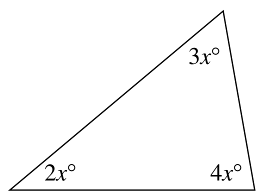
$$2x = 10$$

$$x = 5$$

Since the lengths of all 3 sides of the triangle are 5, the triangle is equilateral.

Therefore, $y = 60$.

23. **D.**



Labeling the angles in the triangle above as $2x$, $3x$, and $4x$, we have:

$$2x + 3x + 4x = 180$$

$$9x = 180$$

$$x = 20$$

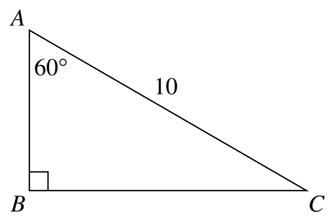
Then the largest angle has measure $4x = 4(20) = 80$.

24. $\mathbf{\frac{14}{3}}$ **or 4.66** Since there are two pair of parallel segments, there are corresponding angles that are congruent. $\angle ABC = \angle AEF$ and $\angle BAC = \angle BDF$. Therefore, by the AA postulate for similarity, $\triangle ABC \approx \triangle DEF$. Since these triangles are similar, their corresponding sides are proportional so that $\frac{BC}{EF} = \frac{AC}{DF}$. By substitution and solving this proportion by the product of the means is equal to the product of the extremes (cross multiplying), the statement should be:

$\frac{7}{3} = \frac{AC}{2}$. Therefore $3AC = 14$ and $AC = \frac{14}{3}$.

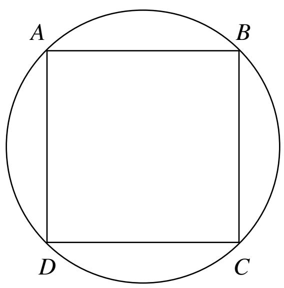
The correct answer is $\frac{14}{3}$ or 4.66.

Special Triangles 30°–60°–90°, 45°–45°–90°



25. In the figure above, what is the length of $\overline{BC}$?

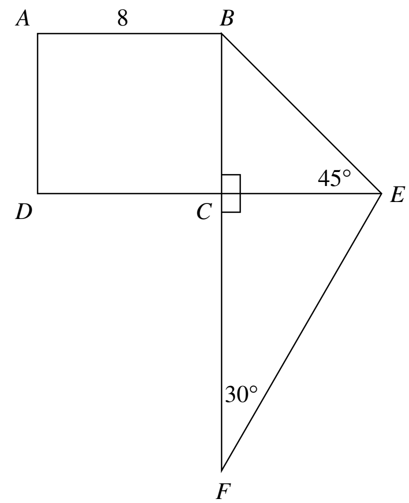
A. 5
B. $5\sqrt{2}$
C. $5\sqrt{3}$
D. $10\sqrt{2}$
E. $10\sqrt{3}$



26. In the figure above, square *ABCD* is inscribed within a circle of radius $3\sqrt{2}$.

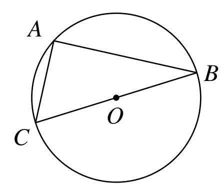
What is the perimeter of *ABCD?*

A. 12
B. $12\sqrt{2}$
C. 24
D. $24\sqrt{2}$
E. 36



27. In the figure above, *ABCD* is a rectangle having a perimeter of 30. What is the perimeter of the figure above?

A. $51\sqrt{5}$
B. $37 + 7\sqrt{5}$
C. $37 + 7\sqrt{2} + 7\sqrt{3}$
D. $37 + 14\sqrt{2}$
E. $51 + 7\sqrt{2} + 7\sqrt{3}$



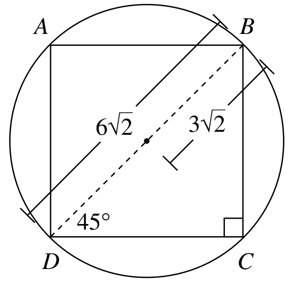
28. Circle *O* contains diameter $\overline{CB}$ as is shown in the diagram above. If $OB = 6$ and $AB = 6\sqrt{3}$, what is the length of $\overline{AC}$?

Explanations

25. **C.** Using the 30°–60°–90° pattern, $AB = \frac{1}{2}AC = \frac{1}{2}(10) = 5$.

Then $BC = \sqrt{3}(BC) = 5\sqrt{3}$.

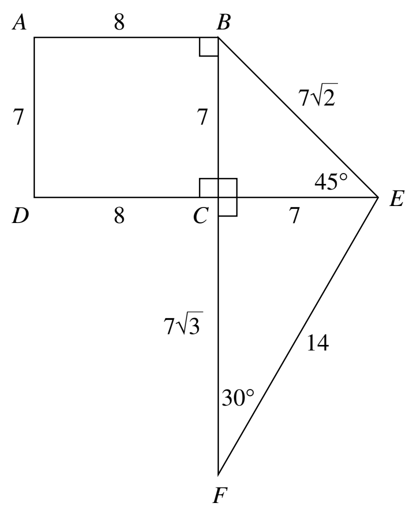
26. **C.**



Since the radius of the circle is $3\sqrt{2}$, its diameter is $6\sqrt{2}$, which is also the diagonal of square *ABCD*. Using the 45°–45°–90° pattern, each side of the square is 6.

Therefore the perimeter of the square is 4(6) = 24.

27. **C.**



To find the length of $\overline{AD}$: $30 = 2(AD) + 2(AB)$

$30 = 2(AD) + 2(8)$

$30 = 2(AD) + 16$

$14 = 2(AD)$

$7 = AD$

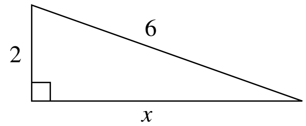
In the 45°–45°–90° triangle, $CE = BC = 7$ and $BE = \sqrt{2}(BC) = 7\sqrt{2}$.

In the 30°–60°–90° triangle, $BF = 2(CE) = 2(7) = 14$, and $CF = \sqrt{3}(CE) = 7\sqrt{3}$.

Finally, the perimeter of the figure is $8 + 7 + 8 + 7\sqrt{3} + 14 + 7\sqrt{2} = 37 + 7\sqrt{3} + 7\sqrt{2}$.

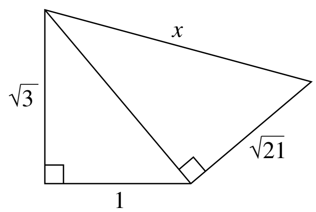
28. **6.** In a circle, an inscribed angle is equal to one half the measure of the intercepted arc. Since $\angle CAB$ is inscribed in a semi-circle, and the measure of a semi-circle is 180 degrees, $\angle CAB = 90°$ and the triangle is a right triangle. If the $OB = 6$, the diameter CB is 12. Now, if $AB = 6\sqrt{3}$, that is the length of the shortest side multiplied by $\sqrt{3}$ for a 30°–60°–90°. This makes $\triangle CAB$ a 30°–60°–90°. Therefore, $AC = 6$. If the Pythagorean theorem is used, $(AC)^2 = (BC)^2 - (AB)^2$. $(AC)^2 = (12)^2 - (6\sqrt{3})^2$. It follows that $(AC)^2 = 144 - 108$, and that $(AC)^2 = 36$. If $(AC)^2 = 36$, then $AC = 6$.

Pythagorean Theorem



29. In the figure above, what is the value of x?

A. $2\sqrt{2}$ (approximately 2.83)
B. $2\sqrt{3}$ (approximately 3.46)
C. $4\sqrt{2}$ (approximately 5.66)
D. $2\sqrt{10}$ (approximately 6.32)
E. 32

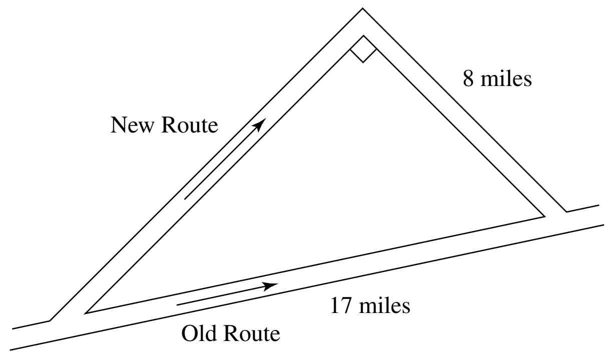


30. In the figure above, what is the value of x?

A. 2
B. $\sqrt{17}$ (approximately 4.12)
C. $\sqrt{23}$ (approximately 4.80)
D. 5
E. $\sqrt{35}$ (approximately 5.92)

31. Miguel starts at point *A,* riding his bicycle, traveling west for 3 miles, then north for 7 miles, followed by a 5-mile easterly ride, finishing up with an 11-mile southerly ride ending up at point *B*. What is the distance from point *A* to point *B* directly?

A. $2\sqrt{3}$ (approximately 3.46)
B. $2\sqrt{5}$ (approximately 4.47)
C. 6
D. $3\sqrt{7}$ (approximately 7.94)
E. 26



32. A jogging trail is being constructed to re-route runners around the exterior of an endangered plant reserve. The plan showing the old trail and the new trail is shown above.

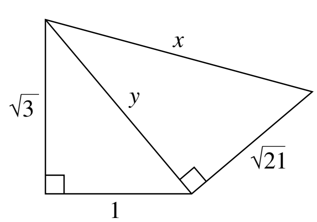
How many extra miles will the runner have to run once the new route is put into service?

Explanations

29. C. Using the Pythagorean theorem:

$$2^2 + x^2 = 6^2$$
$$4 + x^2 = 36$$
$$x^2 = 32$$
$$x = \sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$$

30. D.



Using the Pythagorean theorem for the triangle on the left:

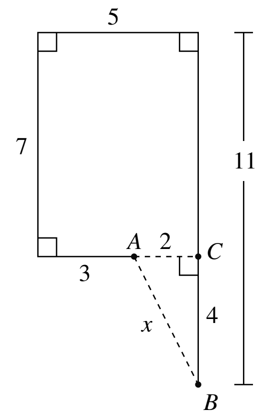
$$\left(\sqrt{3}\right)^2 + 1^2 = y^2$$
$$3 + 1 = y^2$$
$$4 = y^2$$
$$2 = y$$

Then for the triangle on the right: $y^2 + \left(\sqrt{21}\right)^2 = x^2$

Substitute 2 for *y*. $2^2 + \left(\sqrt{21}\right)^2 = x^2$

$$4 + 21 = x^2$$
$$25 = x^2$$
$$5 = x$$

31. B.

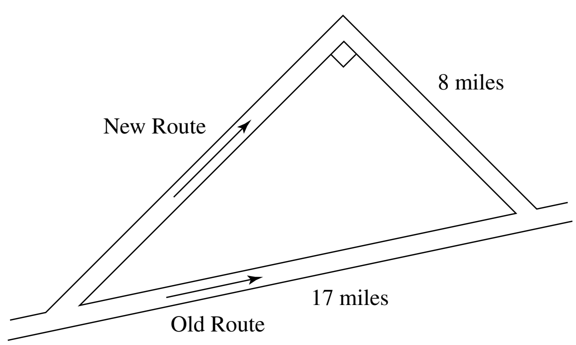


Creating right $\triangle ACB$, $AC = 5 - 3 = 2$, and $CB = 11 - 7 = 4$.

Using the Pythagorean theorem: $2^2 + 4^2 = (AB)^2$

$$4 + 16 = (AB)^2$$
$$20 = (AB)^2$$
$$\sqrt{20} = AB$$
$$\sqrt{4 \cdot 5} = AB$$
$$2\sqrt{5} = AB$$

32. 6 miles.



The new route and the old route form a right triangle as shown. The Pythagorean theorem states that the sum of the squares of the shorter sides is equal to the square of the longest side in a right triangle. Therefore, $8^2 + x^2 = 17^2$. Solving for *x*, $x^2 = 289 - 64$, $x^2 = 225$, and $x = 15$. This is also a special right triangle (Pythagorean triple) of 8, 15, and 17 miles. The length of the new route is 23 miles and the length of the old route is 17 miles. The difference is 6 miles.

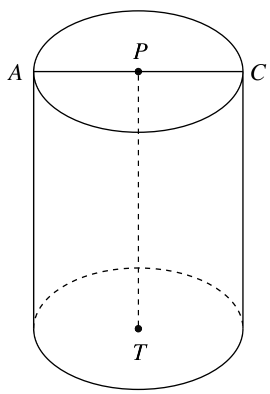
Volume and Surface Area of Solids

33. What is the total surface area of a right circular cylinder having radius 2 and height 3?

A. 8π
B. 10π
C. 14π
D. 16π
E. 20π

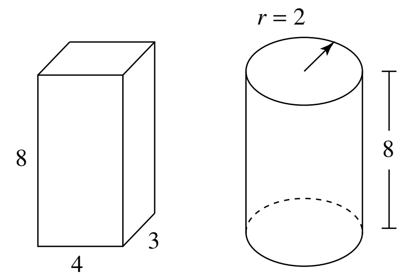
34. If the volume of a cube is 27, what is the total surface area of the cube?

A. 3
B. 9
C. 36
D. 54
E. 63



35. In the right circular cylinder in the figure above, with circle center at point P, $AC = PT$. If the volume of the cylinder is 54π, what is the radius of the cylinder?

A. 3
B. $3\sqrt{2}$ (approximately 4.24)
C. $\sqrt{26}$ (approximately 5.10)
D. $3\sqrt{3}$ (approximately 5.20)
E. 6



36. What is the difference between the surface area of a right rectangular prism with dimensions of $3 \times 4 \times 8$ and a right circular cylinder having dimensions of $r = 2$ and $h = 8$ as is shown in the figure above? Find the difference to the nearest whole number.

Explanations

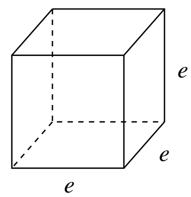
33. **E.** Total area of cylinder $= 2\pi r^2 + 2\pi rh$

$$= 2\pi \cdot 2^2 + 2\pi \cdot 2 \cdot 3$$

$$= 8\pi + 12\pi$$

$$= 20\pi$$

34. **D.**



With edge length e, $Vol_{cube} = e^3$

Substitute 27 for volume. $27 = e^3$

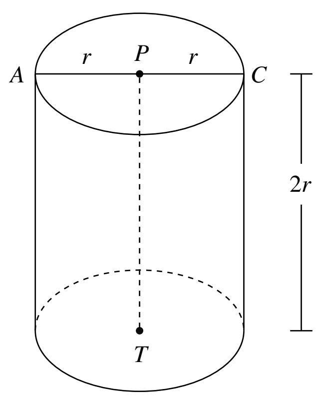
$3 = e$

Then total surface area $= 6e^2$

$= 6 \cdot 3^2$

$= 54$

35. **A.**



Since $AC = PT$, the height of the cylinder is twice the radius of the cylinder.

Therefore $h = 2r$. $Vol_{cylinder} = \pi r^2 h$

Substitute 54π for volume and $2r$ for h. $54\pi = \pi r^2 \cdot 2r$

$$54\pi = 2\pi r^3$$

$$\frac{54\pi}{2\pi} = \frac{2\pi r^3}{2\pi}$$

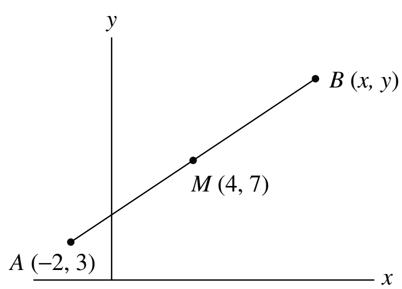
$$27 = r^3$$

$$3 = r$$

36. **10.** First find the surface area of the prism. This area is made up of three parts (two ends and the area around the prism). The area of an end is 12, so both ends have an area of 24. The area of the sides is calculated by the perimeter of the base multiplied by the height. This is 14×8 which is 112 square units. Together the area is $112 + 24 = 136$ square units. Next find the surface area of the cylinder.

In a similar manner, there are two ends and the tube-like side. The area of the base is πr^2, so this value will be 4π. Doubled, the area for both ends is 8π. Now find the area of the tube-like side. This is the circumference multiplied by the height. The circumference is $2\pi r$, so the tube-like area is 4π units. 8 units $\times$ 4π units $= 32\pi$ square units. $32\pi + 8\pi = 40\pi$. 40π is approximately 125.60 square units. The difference is $136 - 125.6$ or 10.4 square units. To the nearest whole number, the answer is 10.

Coordinate Geometry—Coordinates, Slope, Distance Formula, Midpoint Formula



37. In the figure above, point M is the midpoint of $\overline{AB}$. What are the coordinates of point B?

A. (–1, 1.5)
B. (1, 5)
C. (2, 3.5)
D. (6, 17)
E. (10, 11)

38. $\triangle ABC$ has its vertices at $A(-1, -3)$, $B(2, 5)$, and $C(5, 7)$. What is the slope of $\overline{AC}$?

A. $\frac{2}{3}$
B. $\frac{3}{5}$
C. 1
D. $\frac{5}{3}$
E. $\frac{8}{3}$

39. $\triangle MNP$ has its vertices at $M(-3, 5)$, $N(0, 10)$, and $P(1, 8)$. What is the length of the longest side of $\triangle MNP$?

A. $\sqrt{5}$ (approximately 2.24)
B. $2\sqrt{2}$ (approximately 2.82)
C. 5
D. $\sqrt{34}$ (approximately 5.83)
E. $\sqrt{173}$ (approximately 13.15)

40. What is the length (to the nearest tenth of a unit) of the shorter distance—the distance between (–2, –1) and (7, 6) or the distance between (2, –1) and (–8, –6)?

Explanations

37. **E.** Since the coordinates of the midpoint are found by averaging the coordinates of the midpoints:

$$\frac{x+-2}{2}=4 \qquad \frac{y+3}{2}=7$$

$$2\left(\frac{x+-2}{2}\right)=2\cdot 4 \quad \text{and} \quad 2\left(\frac{y+3}{2}\right)=2\cdot 7$$

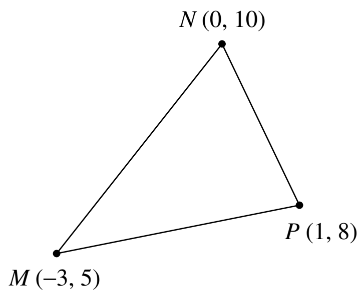
$$x+-2=8 \qquad y+3=14$$

$$x=10 \qquad y=11$$

So point B has coordinates (10, 11).

38. **D.** For points $A(-1,-3)$ and $C(5, 7)$ slope of $\overline{AC}=\frac{y_2-y_1}{x_2-x_1}=\frac{7-(-3)}{5-(-1)}=\frac{10}{6}=\frac{5}{3}$.

39. **D.**



Compute the lengths of the 3 sides of $\triangle MNP$.

$MP=\sqrt{(1-(-3))^2+(8-5)^2}=\sqrt{4^2+3^2}=\sqrt{16+9}=\sqrt{25}=5$

$MN=\sqrt{(0-(-3))^2+(10-5)^2}=\sqrt{3^2+5^2}=\sqrt{9+25}=\sqrt{34}$

$NP=\sqrt{(0-1)^2+(10-8)^2}=\sqrt{1^2+2^2}=\sqrt{1+4}=\sqrt{5}$

Therefore, $MN=\sqrt{34}$ is the longest side.

40. **11.1** Using the distance formula, or by setting the distances up as hypotenuses of triangles, the lengths can be found as follows:

One triangle will have dimensions of 9 in the horizontal direction and 7 in the vertical direction. The distance is found by finding the length of the hypotenuse; so this distance is $\sqrt{81+49}=\sqrt{130}$. This is approximately 11.4 units.

For the other triangle, the horizontal distance is 10 and the vertical distance is 5. Here the distance between the two points is $\sqrt{100+25}$. This distance is 11.1 or $\sqrt{125}$.

The shortest distance is between the points (2,–1) and (–8, –6). The correct answer is about 11.1 units.

Geometric Notation for Length, Segments, Lines, Rays and Congruence

41. Points A, B, and C are noncollinear. What is $\overline{AB}\cup\overline{BC}\cup\overline{AC}$?

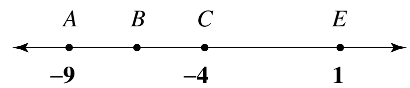
A. { }
B. {A.B,C}
C. $\angle BAC$
D. $\triangle ABC$
E. $\overline{AC}$

42. If $AB = BC$, which of the following statements must be true?

A. B is between A and C
B. $AB + BC = AC$
C. $\overline{AB}=\overline{BC}$
D. $\overline{AB}\cup\overline{BC}=\overline{AC}$
E. B is the midpoint of $\overline{AC}$

43. The union of points *A* and *B* and all point *C,* such that *C* is between *A* and *B,* results in which of the following geometric figures?

A. $\overrightarrow{AB}$
B. $\overline{AB}$
C. $\overrightarrow{CA}$
D. $\overline{AC}$
E. $\overleftrightarrow{BC}$

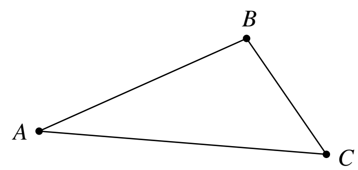


Note: Figure not drawn to scale.

44. Points *A, B, C, D,* and *E* lie on a line as shown above. *C* is the midpoint of $\overline{AE}$ and *B* is the midpoint of $\overline{AC}$. *D* is a point that lies three times further from *B* than from *E.* What is the distance *BD?*

Explanations

41. **D.**



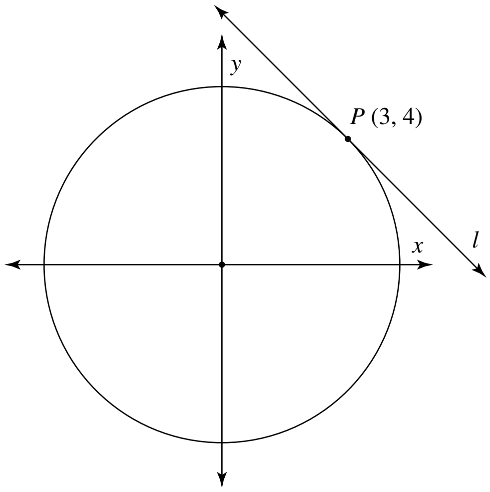
The union of all 3 segments in the figure above is just $\triangle ABC$.

42. **C.** If $AB = BC$, there is no guarantee that points *A* and *B* are even collinear. This eliminates all answer choices but **C.** If the distances are equal, then the segments must be congruent.

43. **B.** For *C* to be between points *A* and *B,* all 3 points must be collinear. The union of points *A* and *B* and all the points between *A* and *B* is just a segment with its endpoints at *A* and *B.* Therefore the correct answer is **B.**

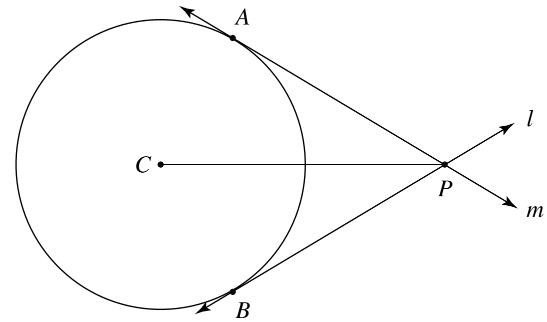
44. $\mathbf{\frac{45}{8}}$**.** The coordinate of *B* is found by using the midpoint formula. The midpoint of $\overline{AE}$ is –4. The midpoint of $\overline{AC}$ is $\frac{-9+-4}{2} = -\frac{13}{2}$. Now, the distance between *B* and *E* is $-\frac{13}{2} - 1$. $-\frac{13}{2} - 1 = -\frac{15}{2}$ Since distances are not negative, this distance is $\frac{15}{2}$. This distance must be divided into 4 parts, three of which are between *C* and *D* and the fourth between *D* and *E.* $\frac{1}{4}$of $\frac{15}{2}$ is $\frac{15}{8}$so the distance between *D* and *E* is $\frac{15}{8}$ and the distance between *B* and *D* is 3 times $\frac{15}{8}$ or $\frac{45}{8}$. It follows that $BD = \frac{45}{8}$. The correct answer is $\frac{45}{8}$.

Properties of Tangent Lines



45. In the figure above, l is tangent to the graph of $x^2 + y^2 = 25$ at point P. What is the slope of line l?

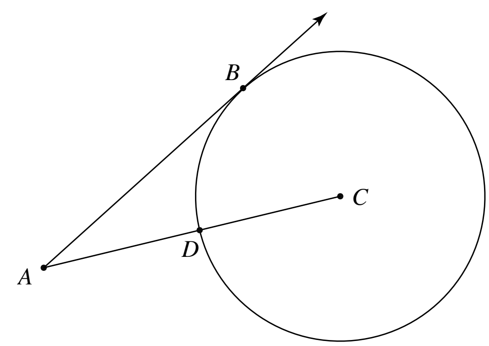
A. $\frac{-4}{3}$
B. $\frac{-3}{4}$
C. $\frac{3}{4}$
D. $\frac{4}{3}$
E. It cannot be determined from the given information.



46. In the figure above, lines l and m intersect at point P and are tangent to circle C at points B and A respectively. If the diameter of circle C is 6 and $PC = 5$, what is the length of $\overline{AP}$?

A. 4
B. $3\sqrt{2}$ (approximately 4.24)
C. $3\sqrt{3}$ (approximately 5.20)
D. $2\sqrt{13}$ (approximately 7.21)
E. 8

47.



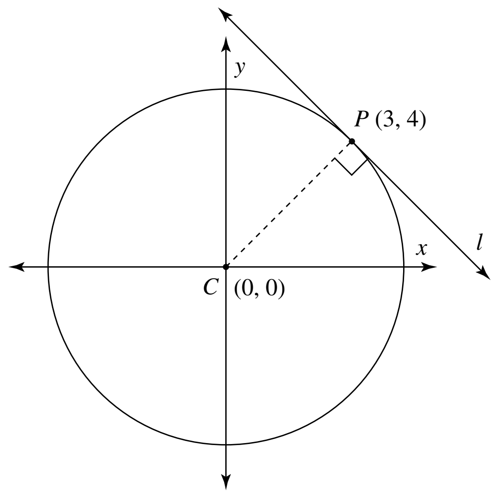
In the figure above, $\overrightarrow{AB}$ is tangent of circle C at point B. If $AB = 6$ and $AD = 2$, what is the radius of circle C?

A. 4
B. 6
C. 8
D. 10
E. 12

48. A secant line and a tangent line of a circle intersect in the exterior of a circle to form an angle of 64°. If the ratio of the intercepted arcs between the tangent and the secant is 11:3, what is the measure of the third arc formed?

Explanations

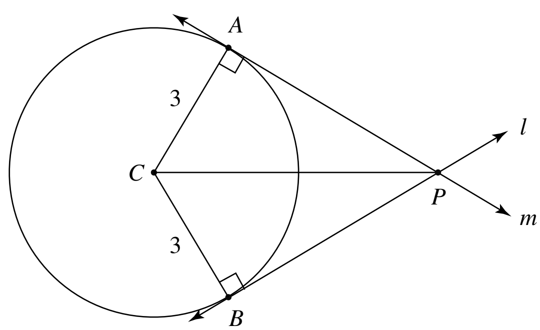
45. **B.**



The center of the given circle is (0, 0); label this point *C*. Then $\overline{CP} \perp l$. The slope of line *l* is the opposite reciprocal of the slope of $\overline{CP}$.

Slope of $\overline{CP} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{3-0} = \frac{4}{3}$. Thus the slope of line *l* is $\frac{-3}{4}$.

46. **A.**



Since the circle's diameter is 6, its radius is 3. $\angle CAP$ is a right angle – a radius drawn to a point of tangency creates a right angle. Then using the Pythagorean theorem:

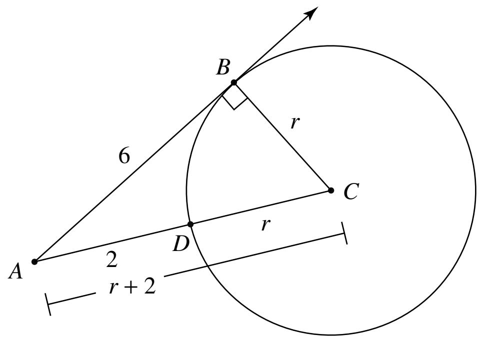
$3^2 + (AP)^2 = 5^2$

$9 + (AP)^2 = 25$

$(AP)^2 = 16$

$AP = 4$ You could also have just used a 3–4–5 right triangle.

47. C.



Labeling r as the circle's radius and noting that $\angle ABC$ is right angle, the Pythagorean theorem gives:

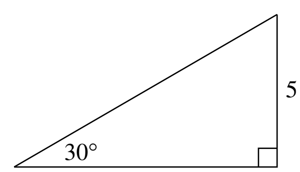
$$r^2 + 6^2 = (r + 2)^2$$
$$r^2 + 36 = r^2 + 4r + 4$$
$$r^2 + 36 - r^2 = r^2 + 4r + 4 - r^2$$
$$36 = 4r + 4$$
$$36 - 4 = 4r + 4 - 4$$
$$32 = 4r$$
$$8 = r$$

48. 136. The measure of an angle formed by a secant and a tangent is ½ the difference of the intercepted arcs. Therefore, $\frac{11x - 3x}{2} = 64$ and $x = 16$. It follows that $11x = 176$ and $3x = 48$. Since the measure of two arcs is 224, and all three arcs measure 360, the remainder of the circle must measure 136. The correct answer is **136.**

Problems in Which Trigonometry Could Be Used as an Alternate Solution Method

49. A square has a perimeter of 32. What is the length of the square's diameter?

- **A.** $4\sqrt{2}$ (approximately 5.66)
- **B.** 8
- **C.** $8\sqrt{2}$ (approximately 11.31)
- **D.** $8\sqrt{13}$ (approximately 13.86)
- **E.** 16

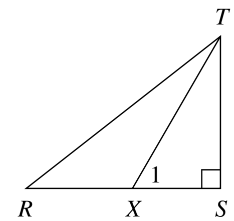


50. What is the area of the figure above?

- **A.** $5\sqrt{3}$ (approximately 8.66)
- **B.** $\frac{25\sqrt{2}}{2}$ (approximately 17.68)
- **C.** $\frac{25\sqrt{3}}{2}$ (approximately 21.65)
- **D.** $25\sqrt{3}$ (approximately 43.30)
- **E.** approximately 51.73

51. What is the area of a regular hexagon having a perimeter of 36?

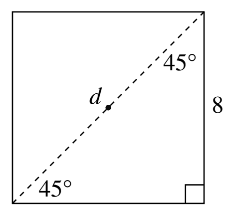
A. 54
B. $54\sqrt{2}$ (approximately 76.37)
C. $54\sqrt{3}$ (approximately 93.53)
D. 108
E. $108\sqrt{3}$ (approximately 187.06)



52. In the figure above, $\angle S = 90°$ and $\angle 1 = 60°$. $RT = \sqrt{3}\,TX$. What is the measure of $\angle R$?

Explanations

49. C.



With a perimeter of 32, each side of the square is 8. The diagonal of the square forms two 45°–45°–90° triangles so we could find the length of the diagonal as just $8\sqrt{2}$.

Using right triangle trigonometry (soh-cah-toa):

$\sin\theta = \dfrac{opposite}{hypotenuse}$ Therefore

$$\sin 45° = \frac{8}{d}$$
$$d(\sin 45°) = d\left(\frac{8}{d}\right)$$
$$d(\sin 45°) = 8$$
$$\frac{d(\sin 45°)}{\sin 45°} = \frac{8}{\sin 45°}$$
$$d = \frac{8}{\sin 45°} = 11.31 \text{ approximately}$$

Note that $8\sqrt{2} = 11.31$ approximately.

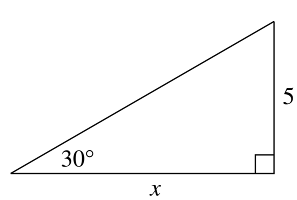
We could have used the Law of Sines instead of soh-cah-toa:

$$\frac{\sin\ of\ angle}{opp.\ side} = \frac{\sin\ of\ angle}{opp.\ side} \qquad \frac{\sin 45°}{8} = \frac{\sin 90°}{d}$$

Cross multiplying yields: $d(\sin 45°) = 8(\sin 90°)$

$$\frac{d(\sin 45°)}{\sin 45°} = \frac{8(\sin 90°)}{\sin 45°} \qquad d = \frac{8\sin 90°}{\sin 45°} = 11.31(\text{approx.})$$

50. B.



Using a 30°–60°–90° triangle, the side labeled x is just $5\sqrt{3}$.

Then the area of the triangle = $\frac{1}{2}bh = \frac{1}{2} \bullet 5\sqrt{3} \bullet 5 = \frac{25\sqrt{3}}{2} = 21.65$ approximately.

Using right triangle trigonometry (soh-cah-toa):

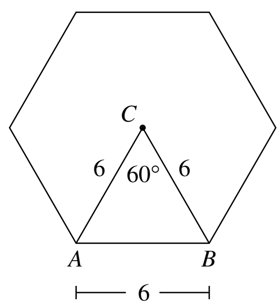
$\tan\theta = \frac{opposite}{adjacent}$ Therefore,

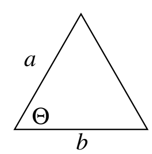
$$\tan 30^\circ = \frac{5}{x}$$
$$x(\tan 30^\circ) = x\left(\frac{5}{x}\right)$$
$$x(\tan 30^\circ) = 5$$
$$\frac{x(\tan 30^\circ)}{\tan 30^\circ} = \frac{5}{\tan 30^\circ}$$
$$x = \frac{5}{\tan 30^\circ}$$

This is the base of the triangle.

The area of the triangle = $\frac{1}{2}bh = \frac{1}{2}\left(\frac{5}{\tan 30^\circ}\right)5 = \frac{25}{2(\tan 30^\circ)} = 21.65$, approximately.

51. E.





Area $\Delta = \frac{1}{2}$ absin Θ

In the regular hexagon above, $\triangle ACB$ is equilateral. So $\angle C = 60°$ and the length of each side is $\frac{36}{6} = 6$.

The figure on the right above shows how to find the area of a triangle given the lengths of 2 sides and their included angle. Using this formula for $\triangle ACB$:

Area of $\triangle ACB = \frac{1}{2} \bullet 6 \bullet 6 \bullet \sin 60^\circ = 18(\sin 60^\circ)$.

Then the area of the hexagon is just 6 times the area of the triangle $\triangle ACB$.

Area of hexagon = 6 ($Area_{triangle}$) = 6 · 18 (sin 60°) = 93.53 approximately.

We could instead have drawn an altitude from C to $\overline{AB}$; then using our 30°–60°–90° pattern, height of $\triangle ACB$ would be $3\sqrt{3}$.

Area of $\triangle ACB = 9\sqrt{3}$.

Area of hexagon = $6\left(Area_{triangle}\right) = 6 \bullet 9\sqrt{3} = 54\sqrt{3}$.

52. 30°. Since $\triangle TSX$ is a right triangle with $\angle 1 = 60°$, it is a 30°–60°–90° triangle. Therefore, $TS = \frac{\sqrt{3}}{2}TX$. If $RT = \sqrt{3}\,TX$, then $\sin R = \frac{TS}{RT}$. It follows that $\sin R = \frac{\frac{\sqrt{3}}{2}TX}{\sqrt{3}\,TX}$.

TX will cancel in both numerator and denominator, and the remaining fraction will reduce to $\frac{1}{2}$. If the ratio of the opposite to the hypotenuse is 1:2, it follows that angle R must be a 30° angle. The correct answer is **30°.**

Qualitative Behavior of Graphs and Functions

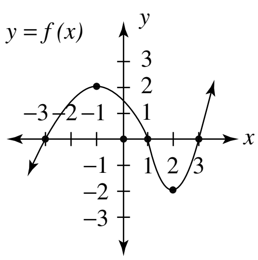
53. For which of the following functions will $f(-x) = -f(x)$?

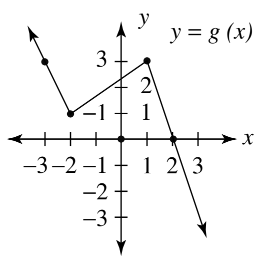
I. $f(x) = 4x^3$

II. $f(x) = \dfrac{-12}{x^2}$

III. $f(x) = 5x - 7x^3$

A. I only
B. II only
C. III only
D. I and III only
E. I, II, and III



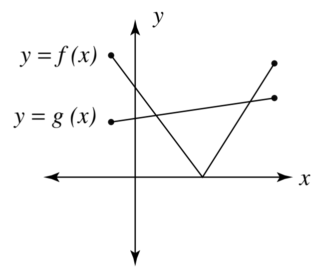


54. What is the value of $f(g(-2))$?

A. –1
B. 0
C. 1
D. 2
E. 3

55. For $f(x) = x^2 - 5x + 7$, $f(m) = f(m + 2)$ for some constant m. What is value of m?

A. –6
B. $\frac{2}{3}$
C. $\frac{3}{2}$
D. 2
E. 3



56. In the graph above, the functions $f(x)$ and $g(x)$ are shown. In how many cases are $f(x)$ and $g(x)$ equal?

Explanations

53. D. Answer Choice **II,** $f(x) = \dfrac{-12}{x^2}$, has an even power of the variable x. In this case, $f(-x) = f(x)$. For answer choices **I** and **III,** the functions have only odd powers of the variable, in which case substituting $-x$ for x will just change the sign of the answer, so $f(-x) = -f(x)$.

54. B. Using graphs of the function f and g: $f(g(-2)) = f(1) = 0$.

55. C. The given function is: $f(x) = x^2 - 5x + 7$

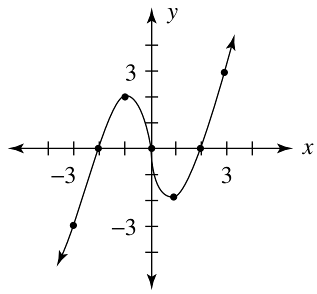
We are told that: $f(m) = f(m + 2)$

$$m^2 - 5m + 7 = (m + 2)^2 - 5(m + 2) + 7$$
$$m^2 - 5m + 7 = m^2 + 4m + 4 - 5m - 10 + 7$$
$$m^2 - 5m + 7 = m^2 - m + 1$$
$$m^2 - 5m + 7 - m^2 = m^2 - m + 1 - m^2$$
$$-5m + 7 + 5m = -m + 1 + 5m$$
$$7 = 4m + 1$$
$$6 = 4m$$
$$\frac{6}{4} = m \quad \text{so} \quad m = \frac{3}{2}$$

56. **2.** Functions are equal where they intersect. Since *f(x)* and *g(x)* intersect in two points, the correct answer is **2.**

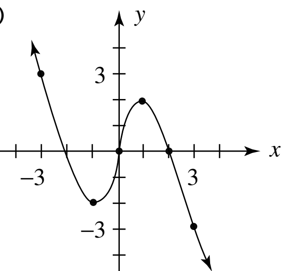
Transformations and Their Effect on Graphs and Functions

57.

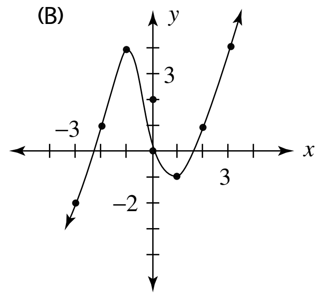


The graph of $y = f(x)$ is shown above. Which of the following could be the graph of $y = f(x - 1)$?

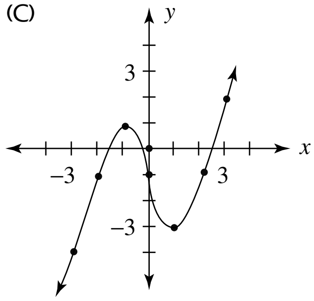
(A)



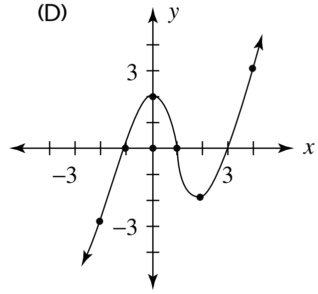
(B)



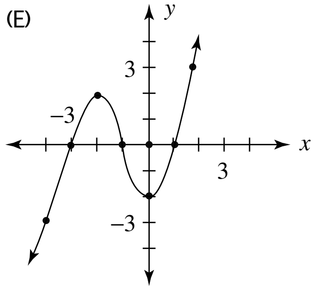
(C)



(D)



(E)

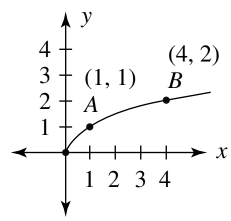


58. The point $P(2, -3)$ is on the graph of the function $y = f(x)$. The transformation $y = f(x - 2) + 5$ moves point P to which of the following new locations?

- **A.** (0, –8)
- **B.** (0, 2)
- **C.** (4, 2)
- **D.** (4, –8)
- **E.** (5, –5)

59. For the function f, $f(2) = 7$. If $g(x) = f(x + 3) + 4$, what is the value of $g(-1)$?

- **A.** 3
- **B.** 6
- **C.** 10
- **D.** 11
- **E.** 12



60. In the figure above, the graph of $y = f(x)$ is shown. The graph is first reflected about the *y-axis* and then translated upward 4 units. What will be the *y*-coordinate of point *B* after this procedure?

Explanations

57. **D.** The graph of $y = f(x - 1)$ will be the same shape as that of $y = f(x)$ but will move 1 unit to the **right**, as in Choice **D.**

58. **C.** The transformation $y = f(x - 2) + 5$ moves every point on the graph of $y = f(x)$, 2 units to the right and 5 units up. So the point (2, –3) gets moved to the point

$(2 + 2, -3 + 5) = (4, 2)$.

59. **D.** Since $f(2) = 7$, one of the points on the graph of f is (2, 7).

The function $g(x) = f(x + 3) + 4$ is just a transformation of the graph of $f(x)$ for which each point on the graph of f is moved 3 units left and 4 units up.

Therefore, the point (2, 7) is moved to the point $(2 - 3, 7 + 4) = (-1, 11)$. Since the point (–1, 11) is on the graph of g, $g(-1) = 11$.

60. **6.** The reflection of point *B* about the *y-axis* will change the coordinates to (–4, 2). When the point is translated upward 4 units, the *y* value will be increased by 4 and will then be 2 + 4 or 6. The correct answer is **6.**

PART III

MATH PRACTICE TESTS

Three Simulated Full-Length Practice Mathematics Tests

This section contains three simulated full-length practice new SAT I math tests. The practice tests are followed by complete answers, explanations, and analysis techniques. The format, levels of difficulty, question structure, and number of questions are similar to those on the new SAT I. Since the test is new, the number and order of question types may vary.

The SAT I is copyrighted and may not be duplicated, and these questions are not taken directly from the actual tests or released sample problems. The sections in these practice exams are labeled by subject for your convenience in reviewing. They are not labeled on the actual SAT I.

When you take these exams, try to simulate the test conditions by following the time allotments carefully.

CUT HERE

Test IA

1	A B C D E
2	A B C D E
3	A B C D E
4	A B C D E
5	A B C D E
6	A B C D E
7	A B C D E
8	A B C D E
9	A B C D E
10	A B C D E
11	A B C D E
12	A B C D E
13	A B C D E
14	A B C D E
15	A B C D E
16	A B C D E
17	A B C D E
18	A B C D E
19	A B C D E
20	A B C D E

Test IB

1	A B C D E
2	A B C D E
3	A B C D E
4	A B C D E
5	A B C D E
6	A B C D E
7	A B C D E
8	A B C D E

9.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

10.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

11.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

13.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

14.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

15.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

16.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

17.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

18.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Test IC

1	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
9	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
10	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
11	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
12	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
13	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
14	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
15	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
16	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

CUT HERE

Practice Test I

Practice Test IA: 25 Minutes, 20 Multiple-Choice Questions

Practice Test IB: 25 Minutes, 18 Questions (8 Multiple-Choice and 10 Grid-in)

Practice Test IC: 20 Minutes, 16 Multiple-Choice Questions

Practice Test IA

Time: 25 minutes

20 multiple-choice questions

Directions: Select the one correct answer of the five choices given and mark the corresponding circle on your answer sheet. Your scratch work should be done on any available space in the section.

Notes

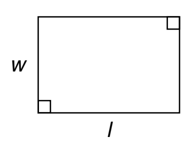
1. All numbers used are real numbers.
2. Calculators may be used.
3. Some problems may be accompanied by figures or diagrams. These figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that a figure is not drawn to scale. The figures and diagrams are meant to provide information useful in solving the problem or problems. Unless otherwise stated, all figures and diagrams lie in a plane.

Data That Can Be Used for Reference

Area

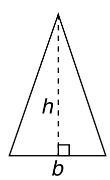
rectangle

$A = lw$



triangle

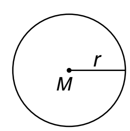
$A = \frac{1}{2} bh$



circle

$A = \pi r^2$

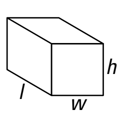
$C = 2\pi r$



Volume

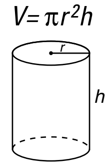
rectangular solid

$V = lwh$



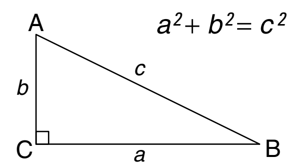
right circular cylinder

$V = \pi r^2 h$



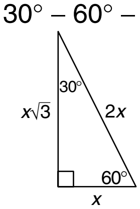
Pythagorean Relationship

$a^2 + b^2 = c^2$

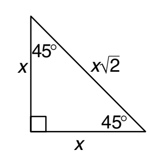


Special Triangles

30° – 60° – 90°



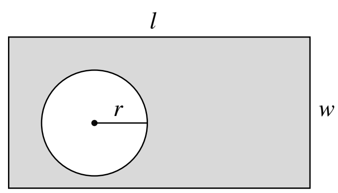
45° – 45° – 90°



A circle is composed of 360°

A straight angle measures 180°

The sum of the angles of a triangle is 180°



1. In the figure above, a circle of radius r is inside a rectangle with dimensions as shown. In terms of l, w, and r, what is the area of the shaded region?

A. $lw - 2\pi r$
B. $2l + 2w - \pi r^2$
C. $2l + 2w - \pi r$
D. $lw - \pi r^2$
E. $lw - r^2$

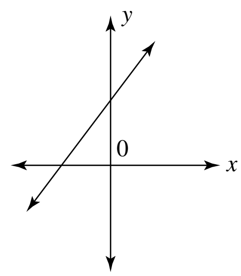
2. How many two-digit positive integers are multiples of both 3 and 5?

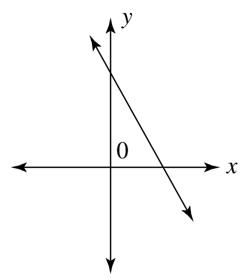
A. 6
B. 19
C. 33
D. 48
E. 52

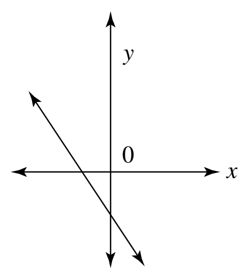
3. If 18% of n is 60, what is 6% of n?

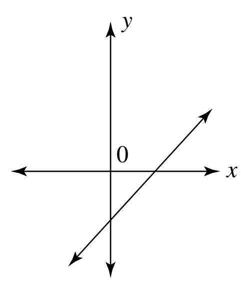
A. 3.6
B. 10.8
C. 20
D. 111.11
E. 333.33

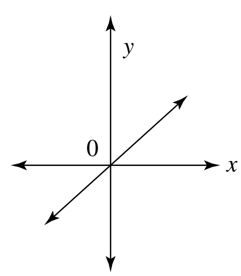
4. The linear function f is given by $f(x) = mx + b$, where m is a positive constant and b is a negative constant. Which of the following could be the graph of f?

A.


B.


C.


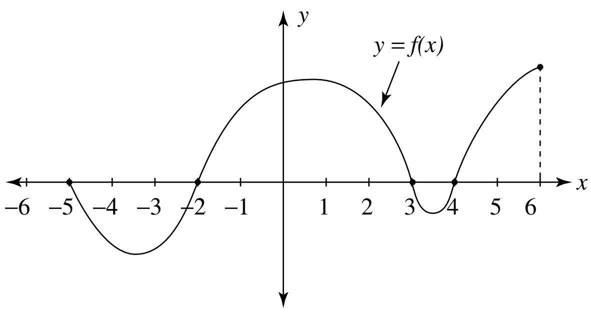
D.


E.


GO ON TO THE NEXT PAGE

5. If $3x - 2 = 7$, what is the value of $5x + 4$?

A. 3
B. 13
C. $\frac{37}{3}$
D. 19
E. 21



6. Shown above is the graph of $f(x)$ for $-5 \leq x \leq 6$. For what values of x is $f(x)$ positive?

A. $0 < x < 60$
B. $-2 < x < 3$ and $4 < x < 6$
C. $-5 < x < 2$ and $3 < x < 4$
D. $0 \leq x \leq 6$
E. $-2 \leq x \leq 3$ and $4 \leq x \leq 6$

7. In a class of 35 students, 25 girls averaged 92% on the last test, while the 10 boys averaged 85% on the same test. What is the average for the entire class on this test?

A. 87%
B. 88.5%
C. 89.5%
D. 90%
E. 92%

8. In the xy-plane, what is the x-coordinate of the point of intersection of the lines whose equations are $y = \frac{2}{3}x - 1$ and $y = \frac{-3}{2}x + 12$?

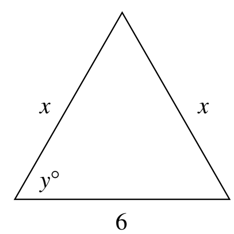
A. $\frac{-66}{5}$
B. $\frac{11}{5}$
C. 6
D. 12
E. 18

9. If $|2x - 3| < 9$, which of the following is a possible value of x?

A. –10
B. –5
C. 0
D. 5
E. 10

10. If x, y, and z are consecutive integers with $x < y < z$, which of the following could be equal to $x + y + z$?

A. $3z + 3$
B. $3z - 6$
C. $3y - 3$
D. $3x + 6$
E. $3y$



11. If the perimeter of the triangle above is 18, what is the value of y?

A. 6
B. 30
C. 45
D. 60
E. 90

12. A rectangle and a square have equal perimeters. If the rectangle has width 8 and length 12, what is the area of the square?

A. 10
B. 40
C. 48
D. 96
E. 100

13. How many positive four-digit integers are multiples of 5 and less than 4,000?

A. 270
B. 300
C. 540
D. 600
E. 800

14. Starting from point *A,* Milo rides his bicycle 5 miles east, then 3 miles south, followed by a ride of 13 miles east, and finally 9 miles north, ending up at point *B*. After resting for a while, Milo rides directly from point *B* back to point *A*. How far does he ride on his return trip?

A. 30
B. 13
C. 10
D. 9
E. 5

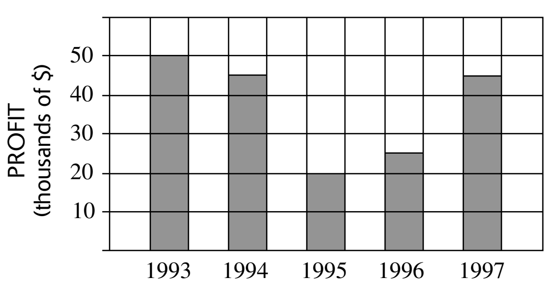
x	-2	-1	0	1
$f(x)$	5	2	1	2

15. The table above gives values of the quadratic function *f* for some values of *x*. Which of the following represents *f?*

A. $f(x) = 3x^2 - 7$
B. $f(x) = 2x^2 + 1$
C. $f(x) = x^2 - 1$
D. $f(x) = x^2 + x + 1$
E. $f(x) = x^2 + 1$

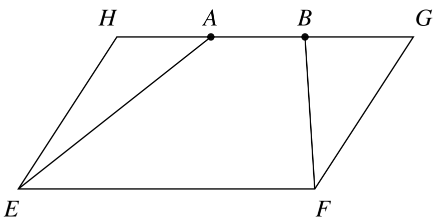
16. If 5 less than some number is 3 more than twice the number, what is the number?

A. -8
B. $\frac{2}{3}$
C. $\frac{3}{2}$
D. $\frac{8}{3}$
E. 3



17. Using the data from the graph above, what was the percent increase in profits from 1996 to 1997?

A. 20%
B. 25%
C. 40%
D. 45%
E. 80%



18. In parallelogram *EFGH,* points *A* and *B* trisect $\overline{HG}$. What is the ratio of the area of *ABFE* to the area of *EFGH?*

A. $\frac{1}{6}$
B. $\frac{1}{3}$
C. $\frac{1}{2}$
D. $\frac{2}{3}$
E. $\frac{5}{6}$

19. At Smallville High School, the probability that its boys' basketball, wrestling, and soccer teams win their league titles this year are 80%, 75%, and 60%, respectively. What is the probability that, for these three boys' teams, basketball and soccer will win league titles but wrestling will NOT?

A. $\frac{3}{25}$
B. $\frac{3}{20}$
C. $\frac{9}{50}$
D. $\frac{9}{25}$
E. $\frac{2}{5}$

20. On a shopping trip, Amal spent $\frac{1}{5}$ of his money to buy a hat, and then used $\frac{1}{2}$ of his remaining money to purchase a coat. After spending $10.00 on a belt, he then had $12.00 left. How much money did Amal start out with before he went shopping?

A. $110.00
B. $100.00
C. $60.00
D. $55.00
E. $50.00

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Practice Test IB

Time: 25 minutes

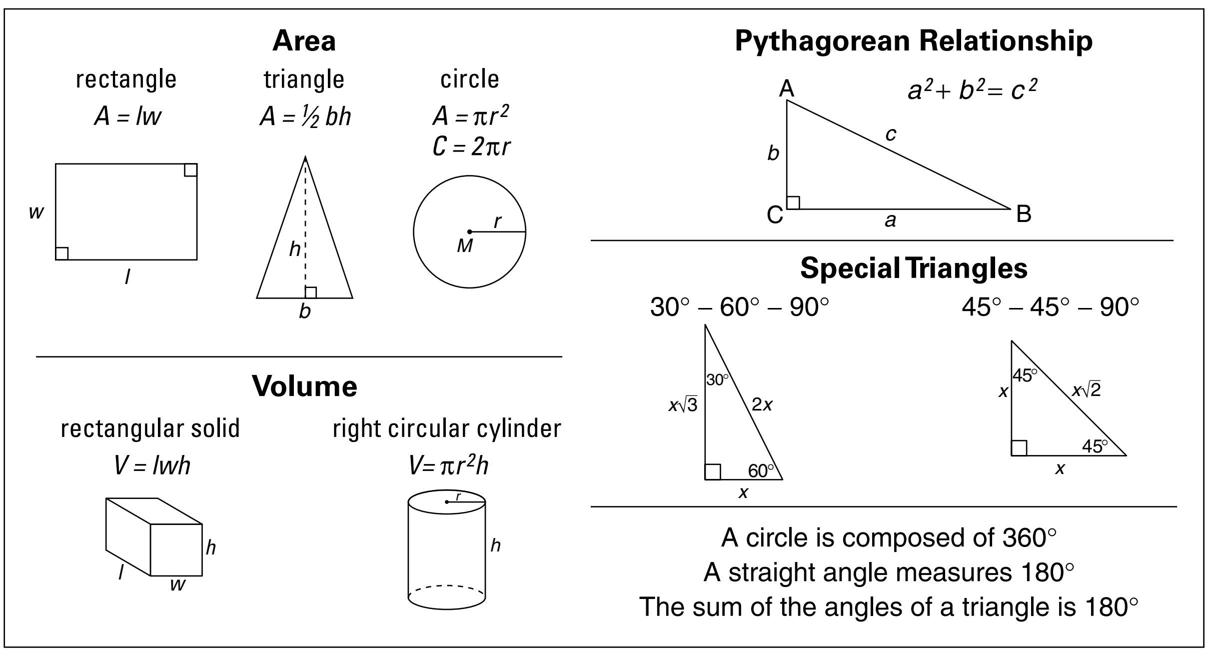
18 questions: (8 multiple choice and 10 grid-in)

Directions: This section is composed of two types of questions. Use the 25 minutes allotted to answer both question types. For Questions 1–8, select the one correct answer of the five choices given and mark the corresponding circle on your answer sheet. Your scratch work should be done on any available space in the section.

Notes

1. All numbers used are real numbers.
2. Calculators may be used.
3. Some problems may be accompanied by figures or diagrams. These figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that a figure is not drawn to scale. The figures and diagrams are meant to provide information useful in solving the problem or problems. Unless otherwise stated, all figures and diagrams lie in a plane.

Data That Can Be Used for Reference

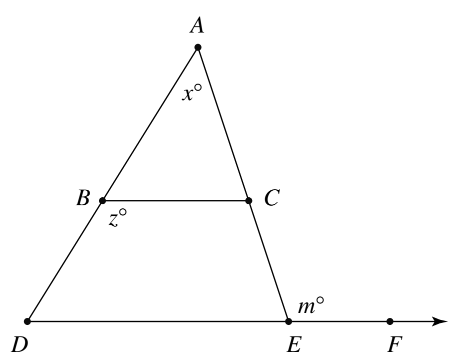


1. If $4x + 3 > 27$, which of the following CANNOT be the value of x?

A. 3
B. 7
C. 11
D. 12
E. 17

2. On a map, the scale is 2 inches equals 120 miles. How many miles apart are two towns if the distance between them on the map is $7\frac{3}{4}$ inches?

A. 420.75 miles
B. 465 miles
C. 930 miles
D. 1,860 miles
E. 4,650 miles



3. In the figure above, $\overline{BC}$ and $\overline{DF}$ are parallel. If $m = 100$ and $x = 20$, what is the value of $x + z$?

A. 20
B. 80
C. 100
D. 120
E. 160

4. If $m + 3 = x(2 - x)$, then in terms of x, $m - 2 =$

A. $-x^2 + 2x + 5$
B. $x^2 - 2x - 5$
C. $-x^2 + 2x - 5$
D. $-x^2 + 2x + 1$
E. $-x^2 - 2x - 1$

5. The midpoint of $\overline{AB}$ is point M. If point A has coordinates $(3, -2)$ and point M has coordinates $(-1, 2)$, what are the coordinates of point B?

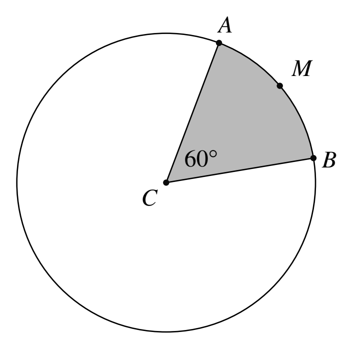
A. (1, 0)
B. (–5, 6)
C. (2, –2)
D. (1, 2)
E. (5, –6)

6. A taxicab charges \$.90 for the first $\frac{1}{5}$ mile and \$.60 for each additional $\frac{1}{5}$ mile. If a customer rides in this cab for m miles $\left(m > \frac{1}{5}\right)$, which of the following functions describes the cost, $c(m)$, of this taxi ride in dollars?

A. $c(m) = .90 + .60(m - 1)$
B. $c(m) = .90 + .60m - 1$
C. $c(m) = .90 + .60[5(m - 1)]$
D. $c(m) = .90 + .60(5m - 1)$
E. $c(m) = .90 + .60(5m - \frac{1}{5})$

7. If $\frac{x^m}{x^5} = x^7$ and $(x^r)^3 = x^{12}$, then the value of $m - r =$

A. 3
B. 4
C. 8
D. 12
E. 20



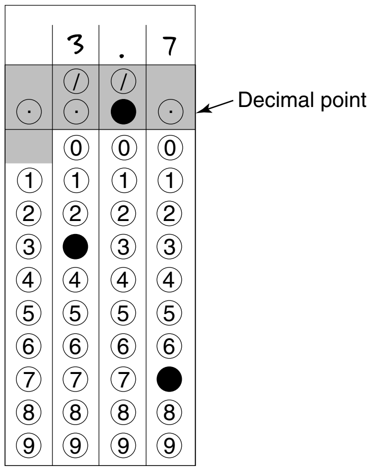
8. For circle C above, the area of the shaded sector CAB is 6π. What is the length of arc AMB?

A. 2
B. π
C. 2π
D. 12
E. 12π

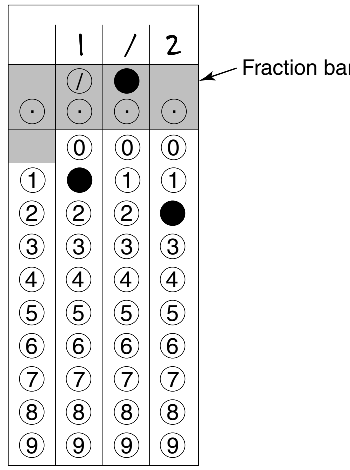
GO ON TO THE NEXT PAGE

Directions for Student-Produced Response Questions (Grid-ins): Questions 9–18 require you to solve the problem and enter your answer by carefully marking the circles on the special grid. Examples of the appropriate way to mark the grid follow.

Answer 3.7

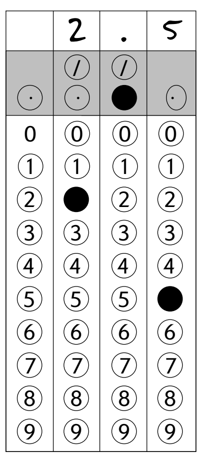


Answer $\frac{1}{2}$

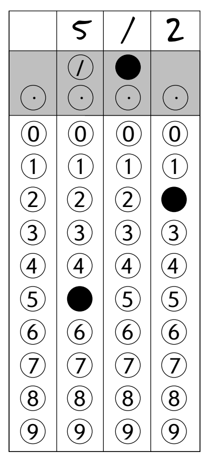


Answer $2\frac{1}{2}$

Do not grid-in mixed numbers in the form of mixed numbers. Always change mixed numbers to improper fractions or decimals.

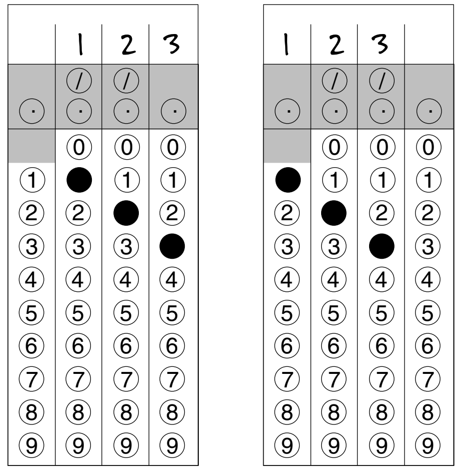


or



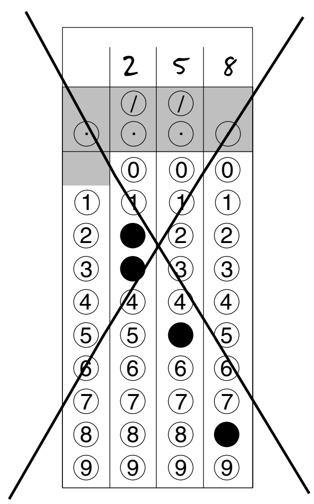
Answer 123

Space permitting, answers may start in any column. Each grid-in answer below is correct.



Note: Circles must be filled in correctly to receive credit. Mark only one circle in each column. No credit will be given if more than one circle in a column is marked. Example:

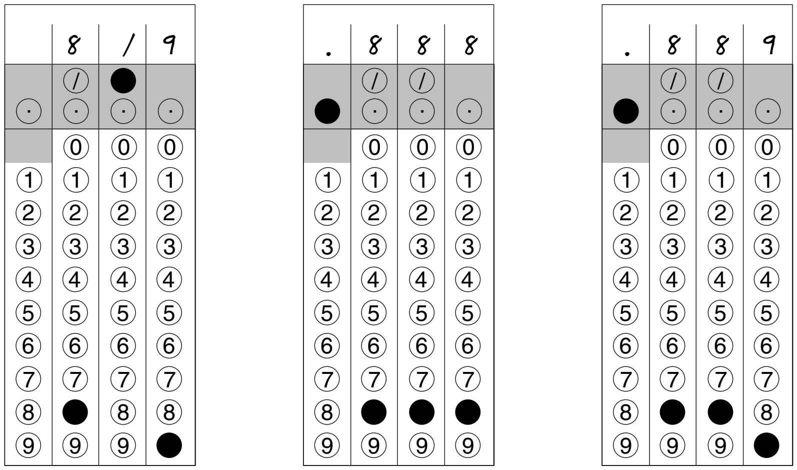
Answer 258 (no credit)



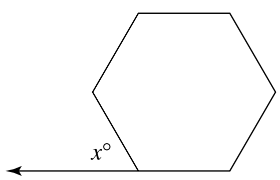
GO ON TO THE NEXT PAGE

Answer 8/9

Accuracy of decimals: Always enter the most accurate decimal value that the grid will accommodate. For example: An answer such as .8888. . . can be gridded as .888 or .889. Gridding this value as .8, .88, or .89 is considered inaccurate and therefore not acceptable. The acceptable grid-ins of 8/9 are:

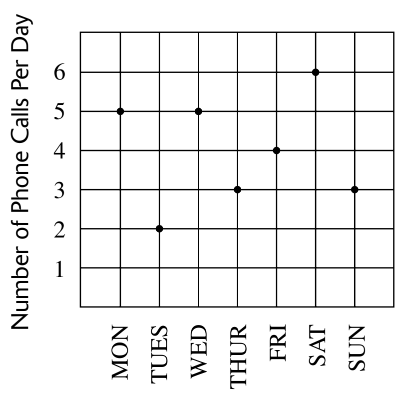


Be sure to write your answers in the boxes at the top of the circles before doing your gridding. Although writing out the answers above the columns is not required, it is very important to ensure accuracy. Even though some problems may have more than one correct answer, grid only one answer. Grid-in questions contain no negative answers.



9. The polygon above is a regular hexagon. What is the value of *x?*

10. Five students are entered in a contest that will award prizes of first, second, and third place. If a given student can win only one prize, in how many different ways can the three prizes be given away in this contest?



11. According to the data in the graph above, what is the average (arithmetic mean) of the number of phone calls per day for the week Monday through Sunday?

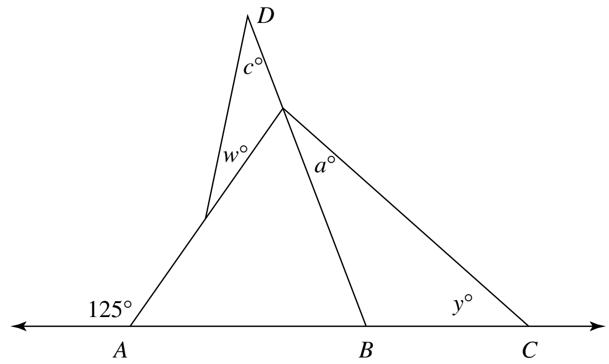
12. If $m^2 - n^2 = 24$ and $m - n = 6$, what is the value of $m + n$?

13. If $a \oplus b = ab - \frac{a}{b}$, then what is the value of $12 \oplus 3$?

14. In a school having 300 students, 180 take Spanish and 150 take French. What percent of the students in this school took Spanish but not French? (Do not include the % symbol in your answer.)

15. The sum of four consecutive odd integers is 456. What is the sum of the smallest and largest of these integers?

16. A sack contains three red, four blue, and five green marbles. The marbles are all the same size and the same weight. If a blindfolded person were to reach in the sack and draw one marble on each of three successive tries (with no replacement), what is the probability that he/she would draw a red, a blue, and then a red marble in exactly that order?



Note: Figure not drawn to scale.

17. In the figure above with $\overline{AC}$ and $\overline{BD}$, what is the value of $a + c + w + y$?

18. If $2x + 3y$ is equal to 80% of $5y$, what is the value of $\frac{x}{y}$?

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Practice Test IC

Time: 20 minutes

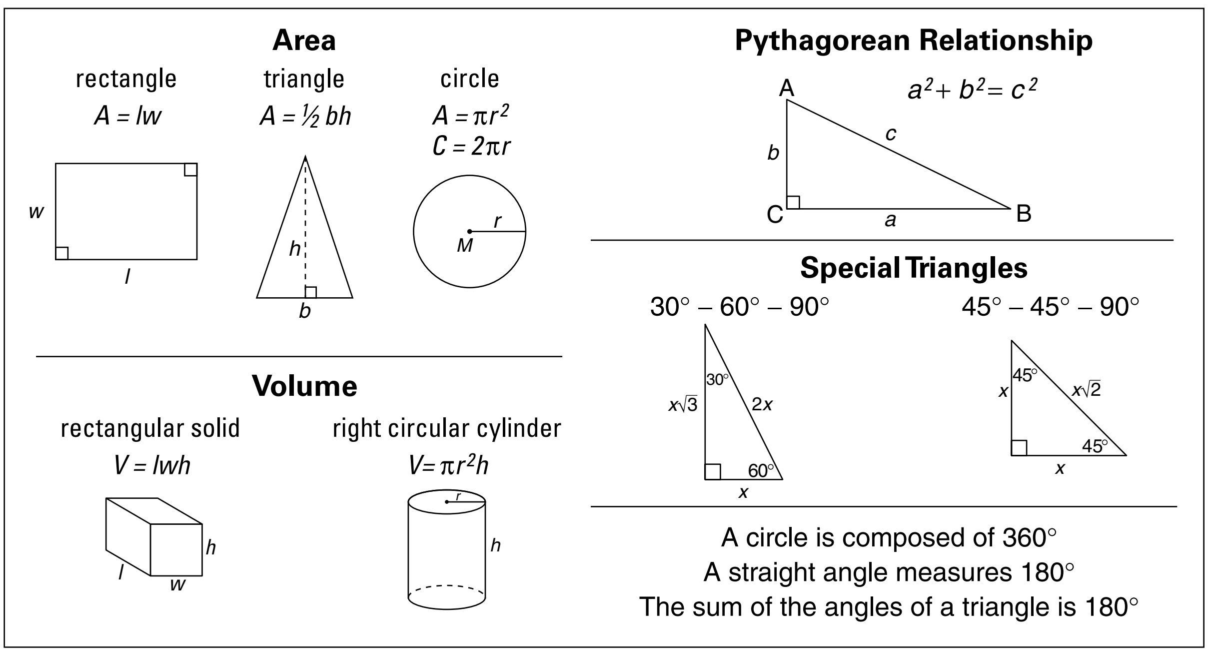
16 multiple-choice questions

Directions: Select the one correct answer of the five choices given and mark the corresponding circle on your answer sheet. Your scratch work should be done on any available space in the section.

Notes

1. All numbers used are real numbers.
2. Calculators may be used.
3. Some problems may be accompanied by figures or diagrams. These figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that a figure is not drawn to scale. The figures and diagrams are meant to provide information useful in solving the problem or problems. Unless otherwise stated, all figures and diagrams lie in a plane.

Data That Can Be Used for Reference

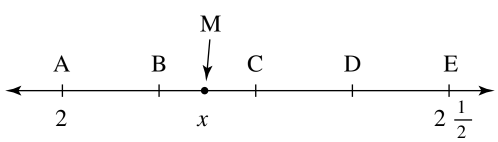


1. If $3x + 15 = 36$, then $x + 5 =$

A. 4
B. 12
C. 15
D. 22
E. 23

2. Xena bought a new refrigerator costing \$625. She paid \$100 as a down payment and will pay the remainder \$75 per month. How many months will it take for Xena to pay off her new refrigerator?

A. 4
B. 5
C. 6
D. 7
E. 9



3. Points *B, C,* and *D* are equally spaced between *A* and *E* on $\overleftrightarrow{AE}$. If *M* is the midpoint of $\overline{BC}$, what is the value *of x?*

A. $2\frac{1}{16}$
B. $2\frac{3}{32}$
C. $2\frac{3}{16}$
D. $2\frac{1}{4}$
E. $2\frac{3}{8}$

4. The equation $2\sqrt{x} - 3 = 5$ is true for which value of *x?*

A. 0
B. 4
C. 9
D. 16
E. 25

5. The average (arithmetic mean) of the ages of 12 girls is *g* and the average of the ages of 17 boys is *b*. What is the total of all the girls' and boys' ages combined?

A. $\frac{12g + 17b}{g + b}$
B. $\frac{g + b}{29}$
C. $\frac{12g + 17b}{29}$
D. $12g + 17b$
E. $\frac{12g + 17b}{2}$

6. If $f(x) = x^2 - 3x + 5$, then $f(2x) =$

A. $4x^2 - 6x + 5$
B. $2x^2 - 6x + 10$
C. $2x^2 - 6x + 5$
D. $x^2 - 6x + 5$
E. $4x^2 - 6x + 5$

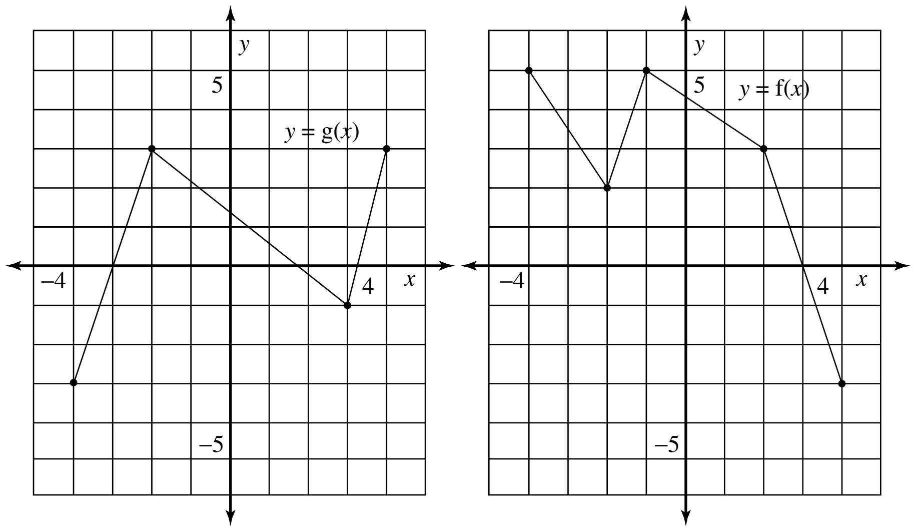
7. The length of one side of triangle is 35. The length of another side of the same triangle is 10. Which of the following could NOT be the value of the third side of this triangle?

A. 24
B. 27
C. 31
D. 39
E. 43

8. If $2a + 6b = 16$ and $a + 3b + c = 12$, what is the value of *c?*

A. –4
B. 4
C. 8
D. 28
E. It cannot be determined from the given information.

GO ON TO THE NEXT PAGE



9. The graphs of g and f are shown above. What is the value of $f(g(3))$?

A. 0
B. 1
C. 3
D. 4
E. 5

10. What is the radius of a circle for which its area and circumference are numerically equivalent?

A. $\frac{1}{2}$
B. 1
C. 2
D. 3
E. 4

11. If $\frac{2x - y}{c} = 5m$, what is x in terms of y and m?

A. $\frac{5m + 3 + y}{2}$
B. $\frac{15}{2}m + y$
C. $\frac{5m + 3}{2} + y$
D. $\frac{12m + y}{2}$
E. $\frac{15m + y}{2}$

12. In the xy-plane, the endpoints of a diameter of a given circle are (–2, 3) and (4, 7). What is the radius of the circle?

A. $\sqrt{13}$
B. $\sqrt{15}$
C. $2\sqrt{13}$
D. $2\sqrt{26}$
E. 13

13. If x is an odd integer and y is an even integer, which of the following must be an even integer?

I. xy
II. $(x + 2)(y - 5)$
III. y^x

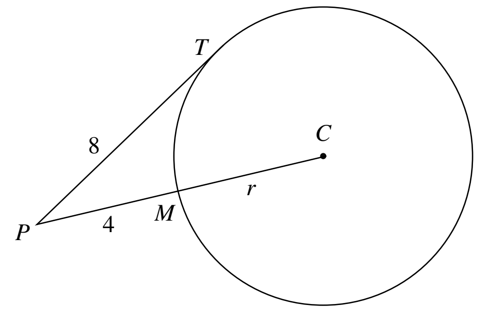
A. I only
B. II only
C. III only
D. I and II only
E. I and III only

14. If $f(x) = 3x^2 - 5$, what is an integral value of r for which $f(2r) = 7r$?

A. -3
B. -1
C. 1
D. 2
E. 4

15. Maria hiked a distance of 15 miles at a rate of 5 mph, and then returned the same distance at a rate of 3 mph. What is her average rate for the entire hike?

A. 3.25
B. 3.50
C. 3.75
D. 4.25
E. 4.50



16. In the diagram above, $\overline{PT}$ is tangent to circle C at point T. If $PT = 8$ and $PM = 4$, what is the radius of circle C?

A. 4
B. 6
C. 8
D. 10
E. 12

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Scoring Practice Test I

Answer Key for Practice Test I

Practice Test IA

1. D
2. A
3. C
4. D
5. D
6. B
7. D
8. C
9. D
10. E
11. D
12. E
13. D
14. C
15. E
16. B
17. E
18. D
19. A
20. D

Practice Test IB

1. A
2. B
3. D
4. C
5. B
6. D
7. C
8. C
9. 60
10. 60
11. 4
12. 4
13. 32
14. 50
15. 228
16. $\frac{1}{55}$ or .018
17. 125
18. $\frac{1}{2}$ or .5

Practice Test IC

1. B
2. D
3. C
4. D
5. D
6. A
7. A
8. B
9. E
10. C
11. E
12. A
13. E
14. C
15. C
16. B

Analyzing Your Test Results

The charts on the following pages should be used to carefully analyze your results and spot your strengths and weaknesses. The complete process of analyzing each subject area and each individual problem should be completed for each practice test. These results should then be reexamined for trends in types of errors (repeated errors) or poor results in specific subject areas. This reexamination and analysis is of tremendous importance to you in assuring maximum test preparation benefit.

Mathematics Analysis Sheet

Section A	***Possible***	***Completed***	***Right***	***Wrong***
Multiple Choice	20			
Subtotal	20			
Section B	***Possible***	***Completed***	***Right***	***Wrong***
Multiple Choice	8			
Grid-Ins	10			
Subtotal	18			
Section C	***Possible***	***Completed***	***Right***	***Wrong***
Multiple Choice	16			
Subtotal	16			
Overall Math Totals	54			

Analysis/Tally Sheet for Problems Missed

One of the most important parts of test preparation is analyzing why you missed a problem so that you can reduce the number of mistakes. Now that you have taken the practice test and checked your answers, carefully tally your mistakes by marking them in the proper column.

Reason for Mistakes					
	Total Missed	***Simple Mistake***	***Misread Problem***	***Lack of Knowledge***	***Lack of Time***
Section A : Math					
Section B : Math					
Section C : Math					
Total Math					

Reviewing the preceding data should help you determine why you are missing certain problems. Now that you've pinpointed the type of error, compare it to other practice tests to spot other common mistakes.

Complete Answers and Explanations for Practice Test I

Practice Test IA Explanations

1. **D.** $Area_{shaded} = Area_{rect.} - Area_{circle}$
$= lw - \pi r^2$

2. **A.** If the integer is a multiple of both 3 and 5, it must also be a multiple of 15. The two-digit multiples of 15 are: 15, 30, 45, 60, 75, 90. So there are 6 such integers.

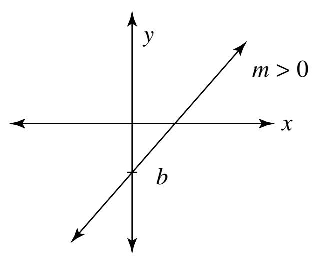
3. **C.** Noting that 6% is just 1/3 of 18%, 6% should be 1/3 of 60, or just 20.

   Another way to look at it is: 18% of n = 60

   $\frac{18\% \text{ of } n}{3} = \frac{60}{3}$ Divide both sides by 3.

   $= 20$

4. **D.** The equation $f(x) = mx +$ b can also be written as just $y = mx + b$, which is known as the slope-intercept form of a linear equation, with m being the slope of the line and b the y-intercept. You are told that m is positive, so the graph of the line will rise uphill from left to right, as in choices **A, D,** or **C.** But you are also told that b is negative, so the graph of the line should cross the y-axis below the x-axis (where y-intercept will be negative). Only Choice **D** fits both conditions.



5. **D.** To solve this equation:

   $3x - 2 = 7$

   $3x - 2 + 2 = 7 + 2$ Add 2 to each side to isolate the variable.

   $3x = 9$

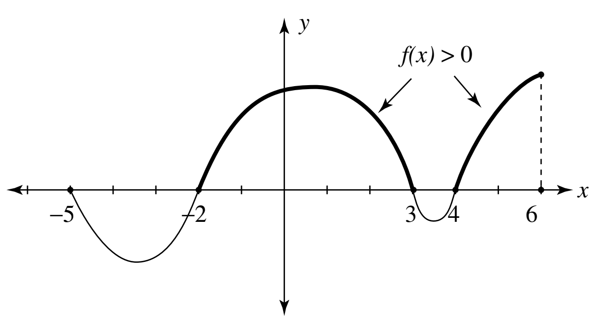
   $\frac{3x}{3} = \frac{9}{3}$ Divide both sides by 3.

   $x = 3$

   You are not done yet; you still need to find the value of $5x + 4$.

   $5x + 4 = 5(3) + 4 = 19$

6. **B.**



   In the figure above, the heavy, dark portions of the graph of $y = f(x)$ are above the x-axis. This is where $f(x)$ is positive. These regions correspond to x values in the intervals $-2 < x < 3$ and $4 < x < 6$.

7. D.

$$\begin{aligned}\text{class average} &= \frac{\text{total of girls' scores} + \text{total of boys' scores}}{\text{total number of students}}\\ &= \frac{25(92)+10(85)}{25+10}\\ &= \frac{2300+850}{35}\\ &= \frac{3150}{35}\\ &= 90\end{aligned}$$

8. C. To find the *x*-coordinate of the point of intersection of the graphs of the two lines, you need to set the equations equal to each other and then solve for *x*.

$\frac{2}{3}x - 1 = \frac{-3}{2}x + 12$	
$6\left(\frac{2}{3}x - 1\right) = 6\left(\frac{-3}{2}x + 12\right)$	Multiply both sides by 6 to get rid of fractions.
$4x - 6 = -9x + 72$	Distribute the 6 through the parentheses.
$4x - 6 + 9x = -9x + 72 + 9x$	Add 9*x* to get the variable on just one side.
$13x - 6 = 72$	
$13x - 6 + 6 = 72 + 6$	Add 6 to both sides to isolate the variable.
$13x = 78$	
$\frac{13x}{13} = \frac{78}{13}$	Divide both sides by 13.
$x = 6$	

9. D. To solve an absolute value inequality of the form $|\text{"stuff"}| < N$, you must break it up into the multiple inequality: $-N <$ "stuff" $< N$

So the given absolute inequality $|2x - 3| < 9$ breaks up into the multiple inequality:

$-9 < 2x - 3 < 9$	
$-9 + 3 < 2x - 3 + 3 < 9 + 3$	Add 3 to all parts of the inequality.
$-6 < 2x < 12$	
$\frac{-6}{2} < \frac{2x}{2} < \frac{12}{2}$	Divide all parts of the inequality by 2.
$-3 < x < 6$	

So your answer must be between –3 and 6. The only answer that fits is 5, Choice **D.**

Notice that you could also have solved this by just trying the choices **A** through **E** until you found a number that made the inequality true; again that would have been $x = 5$, in Choice **D.**

10. E. Consecutive integers are one apart from each other—as with 7, 8, and 9 or as with 23, 24, and 25. You have three possibilities with which to work:

i. If you use *x* for the smallest integer, you will have:

x = the first integer

then $x + 1$ = the second integer

and $x + 2$ = the third integer

The sum of the 3 consecutive integers would then be: $x + (x + 1) + (x + 2) = 3x + 3$

This is not one of your answer choices.

ii. If you use y for the middle integer, you will have:

$y - 1$ = the first integer

then y = the second integer

and $y + 1$ = the third integer

The sum of the 3 consecutive integers would then be: $(y - 1) + y + (y + 1) = 3y$

This IS one of your answer choices, Choice **E.**

iii. Just for the record, if you had used z for the largest integer, you would have had:

$z - 2$ = the first integer

then $z - 1$ = the second integer

and z = the third integer

The sum of the 3 consecutive integers would then be: $(z - 2) + (z - 1) + z = 3z - 3$

This was not one of your answer choices.

11. D. The perimeter of the triangle is found by adding the lengths of its three sides; since you are told that the perimeter is 18, you have the following equation:

$2x + 6 = 18$

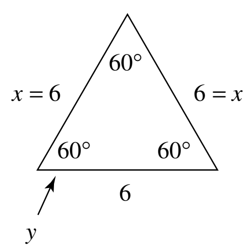
$2x + 6 - 6 = 18 - 6$ Subtract 6 from both sides to isolate the variable

$2x = 12$

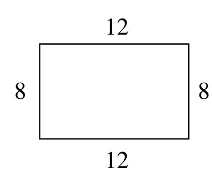
$\frac{2x}{2} = \frac{12}{2}$ Divide both sides by 2.

$x = 6$

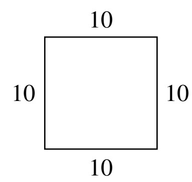
Since each side of the triangle is 6, the triangle is equilateral; therefore, each angle is 60°. In particular, $y = 60°$, as in the figure below.



12. E.



In the figure above, you can find the perimeter of the rectangle by adding the lengths of its sides to get $8 + 12 + 8 + 12 = 40$. You are told that the perimeters of the rectangle and the square are equal, so the perimeter of the square is also 40. With all sides of a square being equal, one side of the square is just $\frac{40}{4} = 10$. So your square looks like the one in the next figure.



The area of the square above is just 10(10) = 100.

13. **D.** You must form four-digit integers that satisfy two conditions:

I. They must be multiples of 5.

II. They must be less than 4,000.

You set up four blanks, each representing one digit of your 4 digit number:

__ __ __ __

Using condition #1, your number must end in a 0 or 5 if it is to be a multiple of 5. So you can choose the last digit in only 2 ways. Your series of blanks now looks like:

__ __ __ 2, where 2 represents the number of ways to choose the last digit (0 or 5)

Next, using condition #2, your number must begin with the digit 1, 2, or 3 if the number is to be less than 4,000. So you can choose your first digit in only 3 ways. Your series of blanks now looks like:

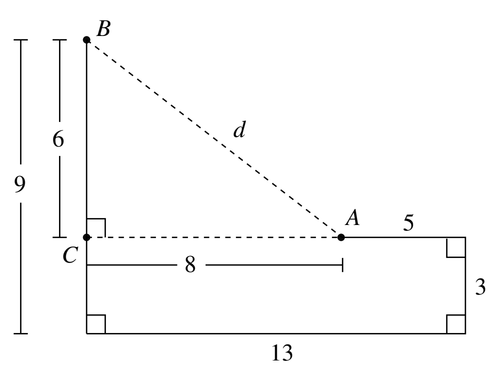
3 __ __ 2, where 3 now represents the number of ways to choose the first digit (1, 2, 3)

The remaining digits can be any one of 10 digits, 0 through 9. So your completed series of blanks becomes:

3 10 10 2 Your answer is just the product of these four numbers. (3)(10)(10)(2) = 600.

So you can create 600 different four-digit integers that satisfy conditions I and II above.

14. **C.** After "sketching" Milo's bike ride with its many right angle turns, the figure looks like the one below.



A right triangle is chosen having $\overline{AB}$ as its hypotenuse. The legs of this right triangle are found by subtracting the east and west distances (13 – 5 = 8) and by subtracting the south and north distances (9 – 3 = 6). You then have a multiple of a 3-4-5 right triangle: one side is 6, the other side is 8, so the hypotenuse must be 10. Thus $AB = 10$.

15. **E.** Substituting a few of the given ordered pairs $(x, f(x))$ into each answer choice is probably the most efficient way to arrive at the correct answer choice. So here goes!

A. $f(x) = 3x^2 - 7$

Use (–2, 5):

Does $5 = 3(-2)^2 + 7$?

$5 = 12 + 7$

false

B. $f(x) = 2x^2 + 1$

Use (–2, 5):

Does $5 = 2(-2)^2 + 1$?

$5 = 9$

false

C. $f(x) = x^2 - 1$

Use (–2, 5):

Does $5 = (-2)^2 - 1$?

$5 = 3$

false

D. $f(x) = x^2 + x + 1$

Use (–2, 5):

Does $5 = (-2)^2 + (-2) + 1$?

$5 = 3$

false

E. By default, Choice **E** must be the correct function.

Just for the record: If one of choices **A** through **D** had worked for the point (–2, 5), you would then have had to try it for the other 3 points to be sure the function worked for all 4 points in the table.

16. B. Using n for the number mentioned in the problem, you can translate the given problem into the following equation:

$5 - n = 2n + 3$

$5 - n + n = 2n + 3 + n$ Add n to both sides to get the variable on just one side.

$5 = 3n + 3$

$5 - 3 = 3n + 3 - 3$ Subtract 3 from both sides to isolate the variable.

$2 = 3n$

$\frac{2}{3} = \frac{3n}{3}$ Divide both sides by 3.

$\frac{2}{3} = n$

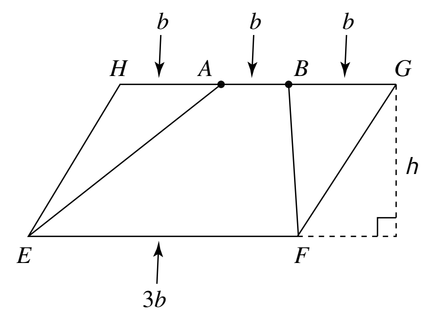
17. E. From the table, you find the profits for the years 1997 and 1996, and then proceed as follows:

$$\frac{\begin{array}{r}\text{profits in 1997}\\ -\text{profits in 1996}\end{array}}{\text{increase in profits}} = \frac{\begin{array}{r}45{,}000\\ -25{,}000\end{array}}{20{,}000}$$

$$\begin{aligned}\%\ \text{increase} &= \frac{\text{amount of increase}}{\text{amount original}} \times 100\\ &= \frac{20{,}000}{25{,}000} \times 100\\ &= \frac{20}{25} \times 100\\ &= \frac{20}{25} \times 100\\ &= .80 \times 100\\ &= 80\%\end{aligned}$$

You can reduce $\frac{20}{25}$ to $\frac{4}{5}$, which equals .80

18. **D.** Suppose the base of parallelogram EFGH is $3b$ and its height is h. Since points A and B trisect $\overline{HG}$, you can make each of the segments $\overline{HA}$, $\overline{AB}$, and $\overline{BG}$ have length b. So your figure appears like the one below:



You now find the area of each requested figure and then determine their ratio.

$ABFE$ is a trapezoid. Its area is: $A = \frac{1}{2}h\left(b_1 + b_2\right)$.

So with height h, $b_1 = 3b$, and $b_2 = b$, $AREA_{ABFE} = \frac{1}{2}h(3b + b) = \frac{1}{2}h(4b) = 2hb$.

$EFGH$ is a parallelogram. Its area is: $A = bh$.

So with height h, and base $3b$, $Area_{EFGH} = (3b) = 3bh$.

Thus the ratio of their areas is: $\frac{AREA_{ABFE}}{AREA_{EFGH}} = \frac{2hb}{3bh} = \frac{2}{3}$

19. **A.** Probability of basketball winning is 80%, or $\frac{4}{5}$.

Probability of wrestling losing is 25%, or $\frac{1}{4}$. (Note: 100% – 75% = 25%)

Probability of soccer winning is 60%, or $\frac{3}{5}$.

To find the requested combined probability, you just multiply the probabilities of the individual events:

Basketball wins		Wrestling loses		Soccer wins		
$\frac{4}{5}$	$\times$	$\frac{1}{4}$	$\times$	$\frac{3}{5}$	$=$	$\frac{3}{25}$

20. **D.** Letting M be the amount of money that Amal had before he spent any money on his shopping trip, you have:

Amal spent $\frac{1}{5}$ of his money: He spent $\frac{1}{5}M$, so he had $\frac{4}{5}M$ left.

He spent $\frac{1}{2}$ of his remaining money: $\frac{1}{2}\left(\frac{4}{5}M\right) = \frac{2}{5}M$, so he now has $\frac{2}{5}M$ left.

After spending \$10, he has \$12 left: $\frac{2}{5}M - 10 = 12$, you then solve this equation

$$\frac{2}{5}M - 10 = 12$$

$$+10 + 10$$ Add 10 to each side to isolate the variable term.

$$\frac{2}{5}M = 22$$

$$\frac{5}{2}\left(\frac{2}{5}M\right) = \frac{5}{2}(22)$$ Multiply both sides by $\frac{5}{2}$, the reciprocal of $\frac{2}{5}$.

$$M = 55$$

So Amal had \$55 before he started his shopping trip.

Practice Test IB Explanations

1. **A.** To solve this inequality:

$4x + 3 > 27$

$-3 \quad -3$ Subtract 3 from each side to isolate the variable term.

$4x > 24$

$\frac{4x}{4} > \frac{24}{4}$ Divide both sides by 4.

$x > 6$

Since x can be any number greater than 6, Choice **A** CANNOT be one of the values of x.

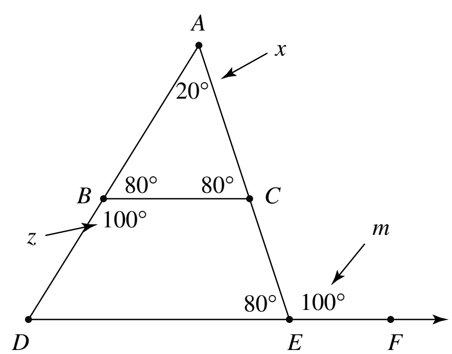
2. **B.** You can set up and solve a proportion to answer this question.

$$\frac{120 \text{ miles}}{2 \text{ inches}} = \frac{\text{M miles}}{7\frac{3}{4} \text{ inches}}$$

If you note that on the left, the 120 on top is just 60 times the 2 on the bottom, then on the left side, M on top should be $60 \times 7\frac{3}{4}$ on the bottom.

So, $60\left(7\frac{3}{4}\right) = 60(7.75) = 465$ miles.

3. **D.** Referring to the figure in the problem, since $m = 100$, the measure of $\angle CED$ is 80. Since $\overline{BC}$ and $\overline{DF}$ are parallel, the measure of $\angle ACB$ is also 80. In $\triangle ABC$, with $x = 20$, and measure of $\angle ACB$ being 80, the measure of $\angle ABC$ must be 80. Finally, z is just the supplement of $\angle ABC$, so $z = 100$.



To finish the problem, you note that:

$x + z = 20 + 100 = 120$

4. **C.** You will solve the given equation for m, and then find $m - 2$.

$m + 3 = x(2 - x)$

$m + 3 = 2x - x^2$ Distribute the x on the right side of the equation.

$-3 \quad -3$ Subtract 3 from both sides to isolate the m.

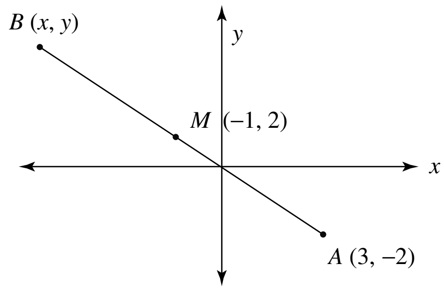
$m = 2x - x^2 - 3$

$m = -x^2 + 2x - 3$ Write the terms in descending order.

$-2 \quad -2$ Subtract 2 since you want to get $m - 2$.

$m - 2 = x^2 + 2x - 5$

5. B. Using the given data and drawing a rough sketch in the xy-plane, your figure looks like the one below:



To find the coordinates (x, y) of the endpoint B, you use the midpoint formula to solve for x and y as follows:

$\frac{x+3}{2} = -1$ and $\frac{y+-2}{2} = 2$

Then $x + 3 = -2$ $\quad y + -2 = 4$

So, $x = -5$ and $\quad y = 6$

Therefore, the coordinates of point B are (–5, 6).

6. D.

*Cost for first $\frac{1}{5}$ mile: .90

Number of $\frac{1}{5}$ miles in m miles: $5m$

Number of $\frac{1}{5}$ miles beyond the first $\frac{1}{5}$ mile: $(5m - 1)$

**Cost of the additional $\frac{1}{5}$ miles beyond first $\frac{1}{5}$ mile : $.60(5m - 1)$

Cost of total trip: $c(m) = .90 + .60(5m - 1)$ (just the sum of * and ** above)

7. C. You will solve the first equation for m; solve the second equation for r; and then find the value of $m - r$.

$\frac{x^m}{x^5} = x^7$

$x^5\left(\frac{x^m}{x^5}\right) = x^5\left(x^7\right)$ Multiply both sides by x^5.

$x^m = x^{12}$ Add the exponents on the right.

So $m = 12$

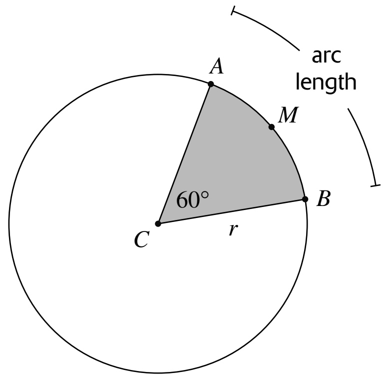
$(x^r)^3 = x^{12}$ Multiply the exponents on the left.

$x^{3r} = x^{12}$

$3r = 12$ Set the exponents equal and then solve for r.

$r = 4$ Therefore, $m - r = 12 - 4 = 8$.

8. C.



You are given the area of the shaded sector in the diagram above. From the area, you will find the radius of the circle and use that to determine the length of the requested arc.

$AREA_{sec\ tor} = \frac{x}{360}(\pi r^2)$, where x is the measure of the central angle and r is the radius.

So you have $6\pi = \frac{60}{360}(\pi r^2)$

$6\pi = \frac{1}{6}(\pi r^2)$ Reduced $\frac{60}{360}$ to $\frac{1}{6}$

$6(6\pi) = 6\left[\frac{1}{6}(\pi r^2)\right]$ Multiplied both sides by 6.

$36\pi = \pi r^2$

$\frac{36}{\pi} = \frac{\pi r^2}{\pi}$ Divided both sides by π.

$36 = r^2$

$6 = r$

So the radius of the circle is 6. Next, find the length of arc *AMB*.

$AREA_{length} = \frac{x}{360}(2\pi r)$, where x is the central angle and r is the radius.

Then you have: $AREA_{length} = \frac{60}{360}(2\pi r)$

$= \frac{1}{6}(12\pi)$

$= 2\pi$

9. 60. For any regular polygon (all sides are equal and all interior angles are equal), the measure of an exterior angle $= \frac{360}{n}$, where n is the number of sides. So, for your hexagon, with $n = 6$, you have:

measure of ext. $\angle = \frac{360}{6} = 60$.

10. 60. To help you determine the number of ways to give away the three prizes among the five people, you set up three blanks, one for each prize:

$\overline{1^{st}}\ \ \overline{2^{nd}}\ \ \overline{3^{rd}}$

Since you have five people, any one of these five could win first place; therefore, your blanks look like:

5

$\overline{1^{st}}\ \ \overline{2^{nd}}\ \ \overline{3^{rd}}$

After one person has been chosen to win first place, there are only four people left. Now, you have:

$\underline{\ 5\ }\ \ \underline{\ 4\ }\ \ \underline{\quad}$
$1^{st}\ \ 2^{nd}\ \ 3^{rd}$

Since you selected one person for first and one more for second place, you only have three people left from which to win third place—your blanks are now filled out as follows:

$\underline{\ 5\ }\ \ \underline{\ 4\ }\ \ \underline{\ 3\ }$
$1^{st}\ \ 2^{nd}\ \ 3^{rd}$

Your answer is just the product of these numbers—so (5)(4)(3) = 60.

11. **4.** Using the data from the graph, the average can be computed as follows:

$$average = \frac{5+2+5+3+4+6+3}{7} = \frac{28}{7} = 4$$

12. **4.** Notice that you can factor the expression $m^2 - n^2$

$m^2 - n^2$

$m^2 - n^2 = (m - n)(m + n)$

$24 = 6(m + n)$ You are given that $m^2 - n^2 = 24$ and $m - n = 6$.

$\frac{24}{6} = \frac{6(m+n)}{6}$ Divide both sides by 6.

$4 = m + n$

13. **32.** The operation, "⊕," is defined by the given equation:

$a \oplus b = ab - \frac{a}{b}$ To find $12 \oplus 3$, you let $a = 12$ and $b = 3$ in the given equation.

$12 \oplus 3 = (12)(3) - \frac{12}{3}$

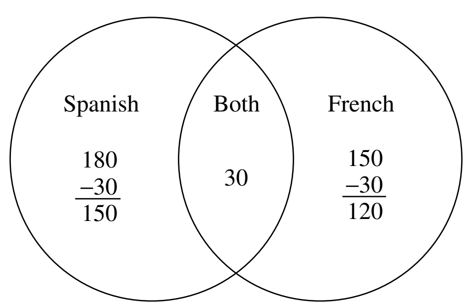
$= 36 - 4 = 32$

14. **50.** If you add the 180 students taking Spanish to the 150 students taking French, you end up with 330 students. Since there are only 300 students in the school, there must be 30 students taking BOTH languages (330 – 300 = 30). You can then sort out the rest of the data as follows:

$$\begin{array}{l} 180 \text{ taking Spanish} \\ \underline{-30 \text{ taking both}} \\ 150 \text{ taking JUST Spanish} \end{array}$$

Therefore, $\frac{150}{300} = 50\%$ of the students are taking Spanish but not French.

You could also have used the Venn Diagram, shown below, to sort out the necessary data.



15. **228.** Let n = the first odd integer,

then $n + 2$ = the second odd integer,

and $n + 4$ = the third odd integer,

while $n + 6$ = the fourth odd integer.

Since their sum is 456, your equation becomes:

$n + (n + 2) + (n + 4) + (n + 6) = 456$

$4n + 12 = 456$ Combine like terms of the left side of equation.

$-12 \quad -12$ Subtract 12 from each side.

$4n = 444$

$\frac{4n}{4} = \frac{444}{4}$ Divide both sides by 4.

$n = 111$

So our 4 consecutive odd integers are 111, 113, 115, and 117.

The sum of the smallest and largest is, therefore, $111 + 117 = 228$

Another method that works with any type of consecutive integer problem (consecutive integer, consecutive even integer, or consecutive odd integer) is to divide the sum of the integers by the number of integers, i.e., find the average. For our problem, $456 \div 4 = 114$. This is obviously not an odd integer, but you merely list the two odd integers before this, namely 111 and 113, as well as the two odd integers after this, namely 115 and 117. Then sum of smallest and largest is $111 + 117 = 228$.

16. $\mathbf{\frac{1}{55}}$ **or .018** You have $3 + 4 + 5 = 12$ marbles in the sack.

The probability of the first marble being a red is: $\frac{3}{12}$, since 3 of the 12 marbles are red.

The probability of the second marble being blue is: $\frac{4}{11}$, since 4 of the remaining 11 marbles are blue.

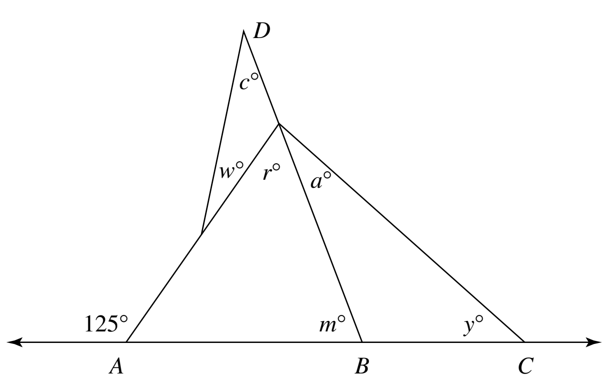
The probability of the third marble being red is: $\frac{2}{10}$, since only 2 of the remaining 10 marbles are red; remember you already drew one red marble on your first draw.

Your final answer is just the product of these individual probabilities:

Thus, $\frac{3}{12} \times \frac{4}{11} \times \frac{2}{10} = \frac{24}{1320} = \frac{1}{55}$ or .018.

17. **125.** In this problem you will use the following property three times:

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles of the triangle.



First: $125 = r + m$ by the property on the preceding page

Second: $r = w + c$ by the same property

Third: $m = a + r$ by the same property

If you take the first equation, $125 = r + m$, and then make substitutions using the second and third equations, you have:

$125 = (w + c) + (a + r)$

$125 = a + c + w + y$ re-order the terms

18. $\mathbf{\frac{1}{2}}$ **or .5** Translating the given sentence into an algebraic equation, you get:

$2x + 3y = .80(5y)$

$2x + 3y = 4y$

$-3y \quad -3y$ Subtract 3y from both sides of the equation.

$2x = y$

$\frac{2x}{y} = \frac{y}{y}$ Divide both sides by y since you are trying to get $\frac{x}{y}$.

$\frac{2x}{y} = 1$

$\frac{1}{2}\left(\frac{2x}{y}\right) = \frac{1}{2}(1)$ Multiply both sides by $\frac{1}{2}$ to get $\frac{x}{y}$ by itself.

$\frac{x}{y} = \frac{1}{2}$

Practice Test IC Explanations

1. B. To get from the expression $3x + 15$ to the expression $x + 5$, you can just divide the first expression by 3. So:

$3x + 15 = 36$

$\frac{3x+15}{3} = \frac{36}{3}$ Divide both sides by 3.

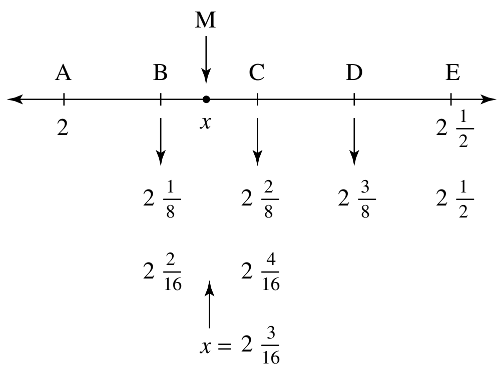
$x + 5 = 12$

2. D.

refrigerator cost	\$625
− down payment	− \$100
left to pay off	\$525

At \$75 per month it will take Xena $\frac{525}{75} = 7$ months to pay off her new refrigerator.

3. C. Since points *B, C,* and *D* are equally spaced between *A* and *E* on $\overleftrightarrow{AE}$, you divide the distance from point *A* to point *E* into 4 equal parts. Between *A* and *E* is a distance of $\frac{1}{2}$, which you will write as $\frac{4}{8}$ since you have to divide into 4 equal parts. This means point *B* will be at $2\frac{1}{8}$, point *C* will be at $2\frac{2}{8}$, and point *D* at $2\frac{3}{8}$. To find the coordinate *x* of point *M*, you need to find the number halfway between $2\frac{1}{8}$ and $2\frac{2}{8}$. If you change each of these fractions into sixteenths, you have $2\frac{2}{16}$ and $2\frac{4}{16}$. Therefore, *x* would be $2\frac{3}{16}$.



4. D. To solve this equation:

$2\sqrt{x} - 3 = 5$

$+3 \quad +3$ Add 3 to each side of the equation.

$2\sqrt{x} = 8$

$\frac{2\sqrt{x}}{2} = \frac{8}{2}$ Divide both sides by 2.

$\sqrt{x} = 4$

$\left(\sqrt{x}\right)^2 = 4^2$ Square both sides to get rid of square root on left.

$x = 16$

5. D. If the girls' average is g, then their total is $12g$.

Likewise, if the boys' average is b, then their total is $17b$.

So the total ages for both groups is: $12g + 17b$.

6. A. To find $f(2x)$, you substitute $2x$ for each x in the given function.

With $f(x) = x^2 - 3x + 5$, you then have

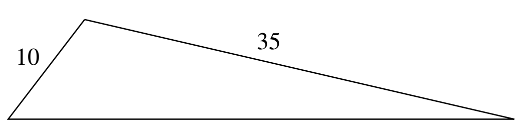
$f(2x) = (2x)^2 - 3(2x) + 5$

$= 4x^2 - 6x + 5$

7. A. The triangle inequality theorem in geometry relates the lengths of the sides of any triangle in the following way:

The difference of other two sides < one side of triangle < sum of other two sides.

For the triangle in this problem with its given side lengths, you find that the third side of this triangle is between 25 and 45. The only number choice not in this interval is 24, Choice **A.**



$35 - 10 < x < 35 + 10$

or

$25 < x < 45$

8. B.

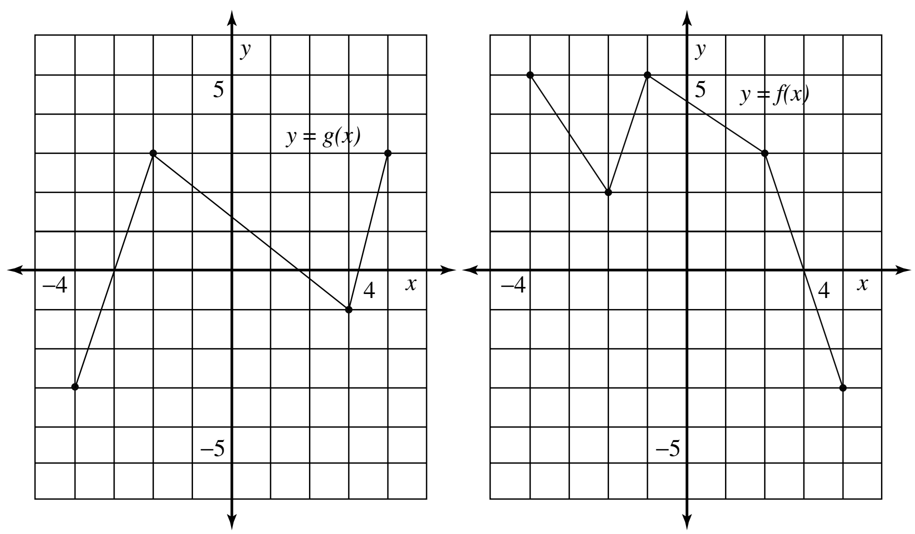
In the second given equation, $a + 3b + c = 12$, you notice that the first two terms, $a + 3b$, is exactly $\frac{1}{2}$ of the left side of the first equation: $2a + 6b = 16$. You proceed as follows:

$2a + 6b = 12$	This is the first given equation.
$\frac{2a+6b}{2} = \frac{16}{2}$	Divide both sides by 2; you are trying to get $a + 3b$.
$a + 3b = 8$	

You then substitute this into the second equation:

$(a + 3b) + c = 12$	Parentheses added for emphasis.
$8 + c = 12$	Replaced the $(a + 3b)$ with 8.
Then $c = 4$	

9. E. To find $f(g(3))$, you first find $g(3)$ and then find f of this value.



From the left graph: $g(3) = –1$	For an x-coordinate of 3, the y-coordinate is –1.
Using the right graph: $f(–1) = 5$	For an x-coordinate of –1, the y-coordinate is 5.

10. C. You are given that the area and circumference are equal—so you write and solve the following equation:

area = circumference

$\pi r^2 = 2\pi r$	Substitute appropriate formulas for area and circumference.
$\frac{\pi r^2}{\pi} = \frac{2\pi r}{\pi}$	Divide both sides by π.
$r^2 = 2r$	
$\frac{r^2}{r} = \frac{2r}{r}$	Divide both sides of equation by r.
$r = 2$	

11. E. To solve this equation, you isolate the x:

$$\frac{2x-y}{3}=5m$$

$$3\left(\frac{2x-y}{3}\right)=3(5m) \qquad \text{Multiply both sides by 3.}$$

$$2x-y=15m$$

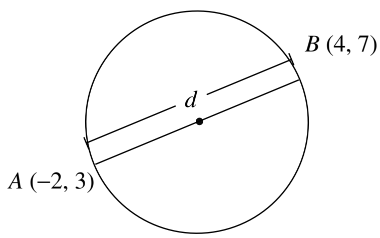
$$+y \quad +y \qquad \text{Add } y \text{ to both sides; you are solving for } x.$$

$$2x=15m+y$$

$$\frac{2x}{2}=\frac{15m+y}{2} \qquad \text{Divide both sides by 2.}$$

$$x=\frac{15m+y}{2}$$

12. A. The data given is shown in the figure below:



You first find the distance between points A and B, which gives you the circle's diameter; the radius will just be $\frac{1}{2}$ of this distance.

Using the distance formula: $d=\sqrt{(4--2)^2+(7-3)^2}$

$$=\sqrt{6^2+4^2}$$

$$=\sqrt{36+16}$$

$$d=\sqrt{52}=\sqrt{4\cdot13}=2\sqrt{13}$$

Remember this is the diameter; so half of this, just $\sqrt{13}$ is the radius.

13. E. Examining each of the choices I, II, and III you have:

I. $xy = odd \cdot even = even$, so our answer must include I.

II. $(x+2)(y-5) = (\text{odd}+2)(\text{even}-5) = odd \cdot odd = odd$, so Choice II is NOT an answer; so you can exclude answer choices **B** and **D** since they both involve II.

III. $y^x = (even)^{odd} = even$, so our answer must include III.

Therefore, the correct answer is I and III, Choice **E.**

14. C. With the function $f(x) = 3x^2 - 5$, you can solve the following equation:

$f(2r) = 7r$	
$3(2r)^2 - 5 = 7r$	Substituted $2r$ for x in the original function formula.
$3 \cdot 4r^2 - 5 = 7r$	
$12r^2 - 5 = 7r$	
$-7r \quad -7r$	Subtract $7r$ from both sides; since you have a quadratic equation, you will set it equal to 0 and then factor.
$12r^2 - 7r - 5 = 0$	
$(12r+5)(r-1) = 0$	Factored the left hand side.
$12r+5=0$ or $r-1=0$	Set each factor equal to 0.
$r = 1$	You are looking for an integral value of r, and the left factor will give you a fractional value.

15. **C.** To find the average rate for the entire trip you use the formula:

$$\text{average rate} = \frac{\text{total distance for trip}}{\text{total time for trip}}$$

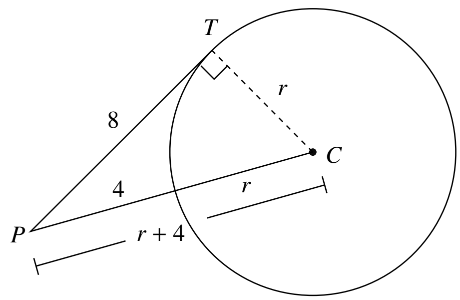
You first find the time for each part of the trip; remember $d = rt$, so that $t = \frac{d}{r}$

$$time_{going} = \frac{15\ miles}{5\ mph} = 3\ hours \text{ and } time_{returning} = \frac{15\ miles}{3\ mph} = 5\ hours$$

Now that you have all the necessary data, you find the average rate:

$$\text{average rate} = \frac{15+15}{3+5} = \frac{30}{8} = 3.75$$

16. **B.**



You have added in the radius from point C to point T. Since $\overleftrightarrow{PT}$ is tangent to the circle at point T, $\overline{PT} \perp \overline{TC}$. In right $\triangle PTC$, you use the Pythagorean theorem to create the following equation:

$8^2 + r^2 = (r+4)^2$	You can now solve this equation.
$64 + r^2 = r^2 + 8r + 16$	Expanded $(r+4)^2$ as $(r+4)(r+4)$ and used "FOIL."
$-r^2 \quad -r^2$	Subtracted r^2 from each side.
$64 = 8r + 16$	
$-16 \qquad -16$	Subtracted 16 from both sides of the equation.
$48 = 8r$	
$\frac{48}{8} = \frac{8r}{8}$	Divided both sides by 8.
$6 = r$	

CUT HERE

Test IIA

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E

9.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

10.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

11.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

13.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

14.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

15.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

16.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

17.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

18.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Test IIB

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
14	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
15	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
16	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
17	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
18	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
19	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
20	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

Test IIC

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
14	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
15	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
16	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

CUT HERE

Practice Test II

Practice Test IIA: 25 Minutes, 18 Questions (8 Multiple-Choice and 10 Grid-in)

Practice Test IIB: 25 Minutes, 20 Multiple-Choice Questions

Practice Test IIC: 20 Minutes, 16 Multiple-Choice Questions

Practice Test IIA

Time: 25 minutes

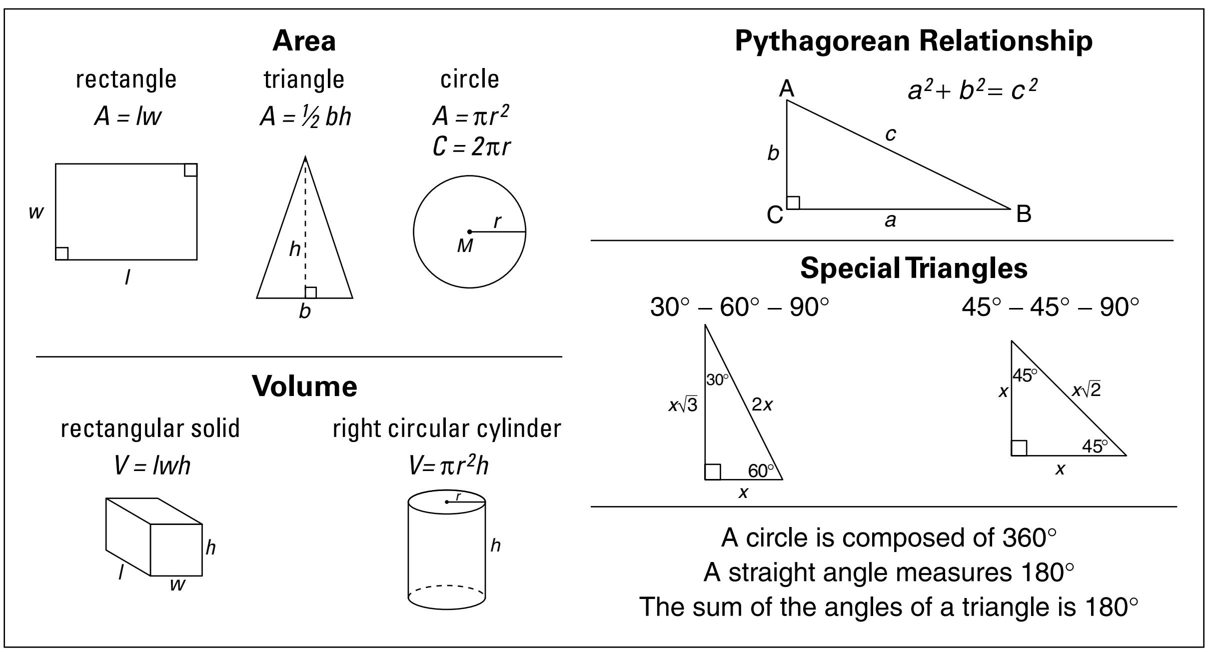
18 questions (8 multiple-choice and 10 grid-in)

Directions: This section is composed of two types of questions. Use the 25 minutes allotted to answer both question types. For Questions 1–8, select the one correct answer of the five choices given and mark the corresponding circle on your answer sheet. Your scratch work should be done on any available space in the section.

Notes

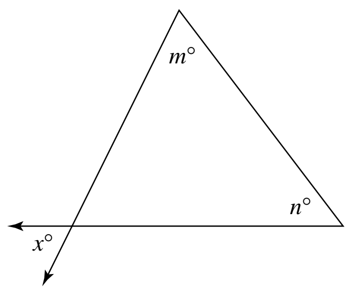
1. All numbers used are real numbers.
2. Calculators may be used.
3. Some problems may be accompanied by figures or diagrams. These figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that a figure is not drawn to scale. The figures and diagrams are meant to provide information useful in solving the problem or problems. Unless otherwise stated, all figures and diagrams lie in a plane.

Data That Can Be Used for Reference



1. If $7x - 5 = 4x - 17$, what is the value of x?

A. –6
B. –4
C. 1
D. 4
E. 5



Note: Figure not drawn to scale.

2. In the figure above, if $x = 50$, what is the value of $m + n$?

A. 50
B. 65
C. 75
D. 110
E. 130

3. In the xy-plane, the line with equation $y = 3x - 12$ crosses the x-axis at one point. What is the x-coordinate of this point?

A. –12
B. –4
C. 0
D. 3
E. 4

4. The average (arithmetic mean) of $5x$ and $3y$ is equal to 4 less than the product of x and y. Which of the following equations states the relationship given in the previous sentence?

A. $\dfrac{5x+3y}{2} = 4 - xy$

B. $\dfrac{5x+3y}{2} = \dfrac{xy}{4}$

C. $\dfrac{5x-3y}{2} = xy - 4$

D. $\dfrac{5x+3y}{2} = xy - 4$

E. $\dfrac{5x-3y}{2} = 4 - xy$

5. The number that results from 3^n, where n is a positive integer, CANNOT end in which of the following digits?

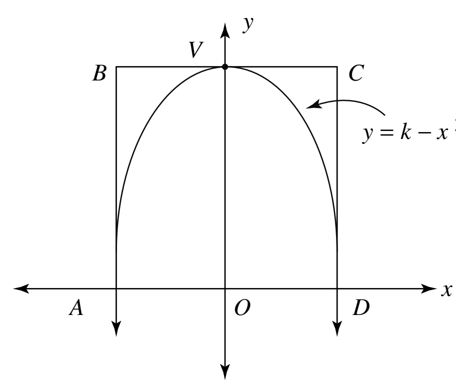
A. 1
B. 3
C. 5
D. 7
E. 9

6. A discount of 20%, followed by another discount of 25%, is equivalent to a single discount of what percent?

A. 33%
B. 40%
C. 45%
D. 55%
E. 60%

7. The equation $|2x - 3| = 7$ is true for which value(s) of x?

A. 2 only
B. 5 only
C. 2 and 5
D. –2 and 5
E. 2 and –5



Note: Figure not drawn to scale.

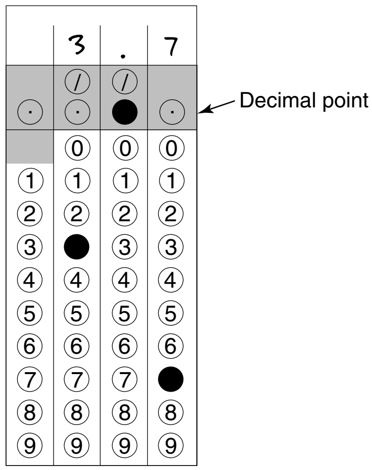
8. The figure above shows the graph of $y = k - x^2$, where k is some positive constant. If V is the midpoint of side BC of rectangle $ABCD$, and if the area of $ABCD$ is 16, what is the value of k?

A. 2
B. $2\sqrt{2}$
C. 4
D. $4\sqrt{2}$
E. 8

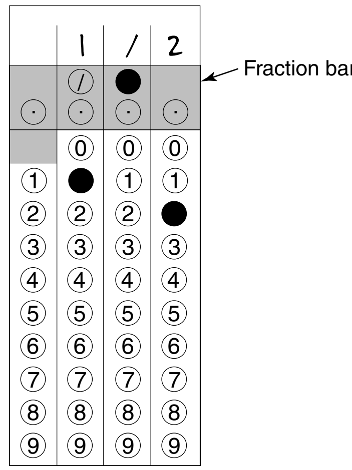
GO ON TO THE NEXT PAGE

Directions for Student-Produced Response Questions (Grid-ins): Questions 9–18 require you to solve the problem and enter your answer by carefully marking the circles on the special grid. Examples of the appropriate way to mark the grid follow.

Answer 3.7

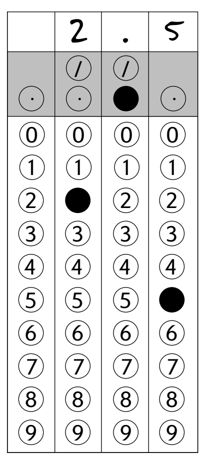


Answer 1/2

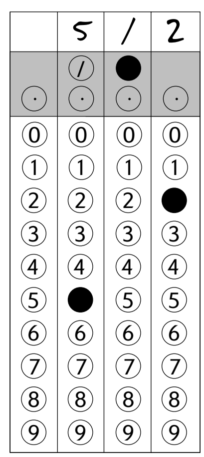


Answer 2 1/2

Do not grid-in mixed numbers in the form of mixed numbers. Always change mixed numbers to improper fractions or decimals.

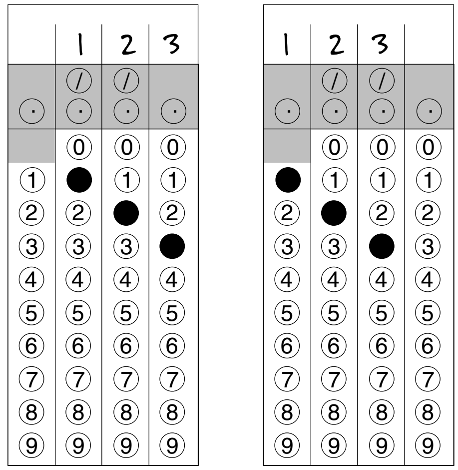


or



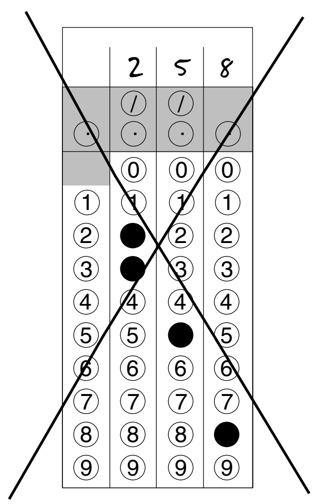
Answer 123

Space permitting, answers may start in any column. Each grid-in answer below is correct.



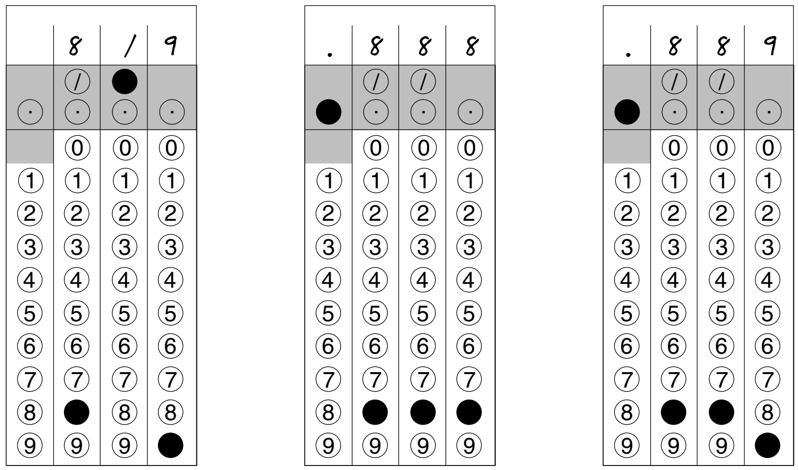
Note: Circles must be filled in correctly to receive credit. Mark only one circle in each column. No credit will be given if more than one circle in a column is marked. Example:

Answer 258 (no credit)



Answer 8/9

Accuracy of decimals: Always enter the most accurate decimal value that the grid will accommodate. For example: An answer such as .8888 . . . can be gridded as .888 or .889. Gridding this value as .8, .88, or .89 is considered inaccurate and therefore not acceptable. The acceptable grid-ins of 8/9 are:



Be sure to write your answers in the boxes at the top of the circles before doing your gridding. Although writing out the answers above the columns is not required, it is very important to ensure accuracy. Even though some problems may have more than one correct answer, grid only one answer. Grid-in questions contain no negative answers.

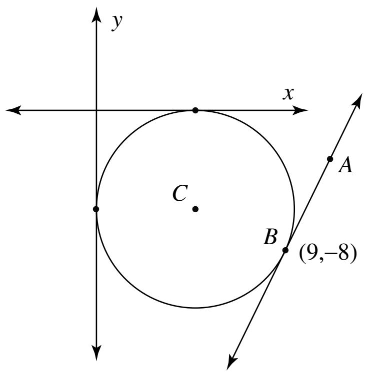
9. Dividing a number by $\frac{2}{5}$ and then multiplying the result by $\frac{4}{3}$ gives the same result as just dividing the original number by what fraction?

10. If $\frac{5}{2x} = \frac{2}{3y}$, what is the value of $\frac{x}{y}$?

11. If $m^2 + n^2 = 29$ and if $mn = 7$, then the value of $(m + n)^2 =$

12. If $x^{\frac{-1}{3}} = 2$ and if $y^{\frac{1}{2}} = 2$, what is the value of *xy?*

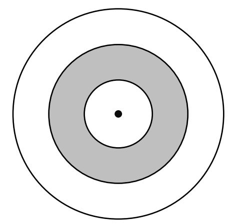
13. What is the largest of 5 consecutive even integers whose sum is 170?



14. In the figure above, the circle with center C and radius 5 is tangent to both the x and y axes. $\overleftrightarrow{AB}$ is tangent to the circle at point B. What is the slope of $\overleftrightarrow{AB}$?

15. $\triangle ABC$ in the xy-plane has vertices at A (5, 2), B (11, 2), and C (11, 6). What is the area of $\triangle ABC$?

16. If $m^2 + 2mn = 3n - 4w$, what is the value of n when $m = 3$ and $w = -5$?



17. The figure above consists of three circles having the same center. Their radii are 2, 3, and 4. The shaded area is what fraction of the area of the largest circle?

18. If $720 = a^m b^n c^r$, where a, b, and c are different positive prime integers, what is the value of $m + n + r$?

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Practice Test IIB

Time: 25 minutes

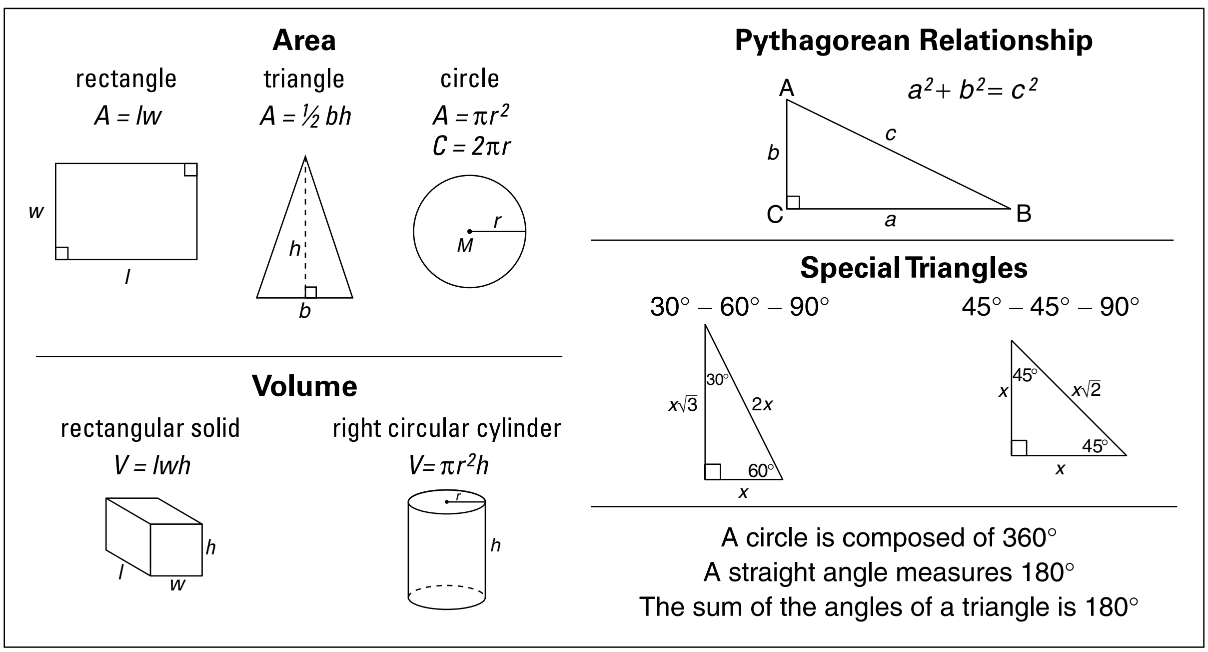
20 multiple-choice questions

Directions: Select the one correct answer of the five choices given and mark the corresponding circle on your answer sheet. Your scratch work should be done on any available space in the section.

Notes

1. All numbers used are real numbers.
2. Calculators may be used.
3. Some problems may be accompanied by figures or diagrams. These figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that a figure is not drawn to scale. The figures and diagrams are meant to provide information useful in solving the problem or problems. Unless otherwise stated, all figures and diagrams lie in a plane.

Data That Can Be Used for Reference



2, 3, 5, 8, 13, . . .

1. The first term in the sequence above is 2 and the second term is 3. For each term after the second term, its value represents the sum of the two terms preceding it. What is the tenth term of this sequence?

A. 34
B. 55
C. 89
D. 144
E. 233

2. In the *xy*-plane, the line *l* is perpendicular to the graph of the equation $3x + 4y = 12$. Which of the following is the slope of line *l*?

A. –4
B. –3
C. $\frac{-3}{4}$
D. $\frac{4}{3}$
E. 12

3. If $5(x + 3) = 17$, what is the value of x?

A. $\frac{2}{5}$
B. $\frac{5}{2}$
C. $\frac{14}{5}$
D. $\frac{22}{5}$
E. 9

4. If $3(2x + 4)(5 - x) = 0$, what are all the possible values of x?

A. 0 only
B. –2 and 5 only
C. 2 and –5 only
D. 0, –2, and 5
E. 0, 2, and –5

5. For which of the following functions does $f(-5) \neq f(5)$?

A. $f(x) = |x|$
B. $f(x) = 7$
C. $f(x) = x^3 + 5$
D. $f(x) = x^2 - 3$
E. $f(x) = 6 + x^4$

6. When $x = 2$, what is the value of $\frac{1}{x+1} + \frac{1}{x} + \frac{1}{x-1}$?

A. $\frac{1}{6}$
B. $1\frac{2}{5}$
C. $1\frac{5}{6}$
D. 5
E. 11

7. Mark is taller than Rebecca, and Charles is shorter than Rebecca but taller than Harriet. If *m, r, c*, and *h* represent the heights of Mark, Rebecca, Charles, and Harriet respectively, which of the following correctly represent the order of their heights?

A. $c < h < r < m$
B. $r < c < h < m$
C. $h < r < c < m$
D. $c < h < r < m$
E. $h < c < r < m$

8. The radius of one circle is $\frac{1}{3}$ the radius of a second circle. What is the ratio of the area of the smaller circle to the area of the larger circle?

A. 1:9
B. 1:3
C. 3:1
D. 6:1
E. 9:1

9. If $2 \le x \le 6$ and $6 \le xy \le 30$, which of the following gives the range of all possible values of *y?*

A. $\frac{1}{3} \le y \le \frac{1}{8}$
B. $\frac{1}{3} \le y \le \frac{1}{5}$
C. $4 \le y \le 24$
D. $3 < y < 5$
E. $3 \le y \le 5$

10. The quantity *m* varies inversely as the square of the quantity *r.* If $m = 9$ when $r = 4$, what is the value of *m* when $r = 6$?

A. 4
B. 6
C. 36
D. 64
E. 144

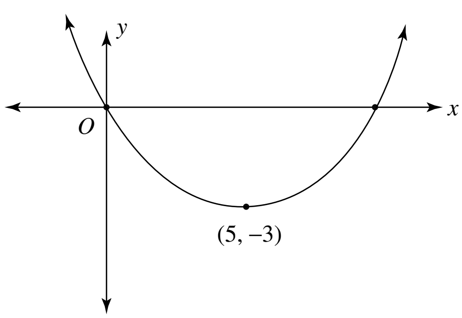
GO ON TO THE NEXT PAGE

11. If $x \triangle y = \frac{\sqrt{x}}{y}$, with $y \neq 0$, and if $4 \triangle m = 16 \triangle 2$, what is the value of *m?*

A. $\frac{1}{2}$
B. 1
C. 2
D. 8
E. 16

12. What is the measure of the largest angle of a triangle, in which the degree measure of its angles have a ratio of 5:6:7?

A. 10
B. 35
C. 50
D. 70
E. 75



Note: Figure not drawn to scale.

13. For the parabola above, (5, –3) is its vertex. Which of the following are the *x*-coordinates of two points on the graph of the parabola for which their *y*-coordinates are equal?

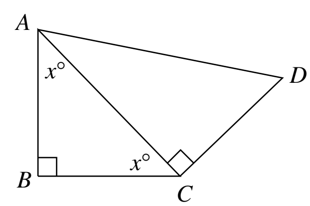
A. 4 and 7
B. 2 and 9
C. 0 and 8
D. –1 and 11
E. 1 and 10

14. Mr. and Mrs. Franklin took their three children to a theatre to watch a movie. They select a row on the side having only five seats. If each of the parents must sit at either end of the row, in how many different ways can the Franklin family be seated in the five seats?

A. 3
B. 6
C. 12
D. 24
E. 120

15. How many inches are there in a distance of *Y* yards, *F* feet, and *I* inches?

A. $3Y + F + 12I$
B. $36Y + F + 12I$
C. $3Y + 12F + I$
D. $36Y + 12F + I$
E. $3Y + 12F + 12I$

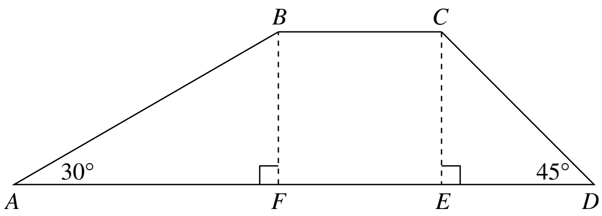


16. In the figure above, $\overline{AB} \perp \overline{BC}$ and $\overline{AC} \perp \overline{CD}$. If the length of $\overline{AB} = 2$ and the length of $\overline{BC} = 2$, what is the length of $\overline{AD}$?

A. $2\sqrt{2}$ (approximately 2.82)
B. $2\sqrt{3}$ (approximately 3.46)
C. 4
D. $3\sqrt{2}$ (approximately 4.24)
E. $2\sqrt{5}$ (approximately 4.47)

17. The perimeter of an equilateral triangle is 18. What is the area of the triangle?

A. $\frac{9}{2}$
B. $\frac{9\sqrt{2}}{2}$
C. $9\sqrt{2}$
D. $9\sqrt{3}$
E. 18



Note: Figure not drawn to scale.

18. In the figure above, $\overline{BC} \parallel \overline{AD}$. What is the ratio of the area of $\triangle ABF$ to the area of $\triangle CED$?

A. $\sqrt{3}:\sqrt{2}$
B. $\sqrt{2}:1$
C. 3:2
D. $\sqrt{3}:1$
E. 2:1

19. If m and r are constants and if $(x + m)(x + r) = x^2 + cx + w$, where c and w are constants, what is $\frac{c}{w}$ in terms of m and r?

A. $\frac{m}{r}$
B. $\frac{m+r}{m}$
C. $\frac{m+r}{mr}$
D. $\frac{mr}{m+r}$
E. It cannot be determined from the given information.

20. Let the function f be defined by $f(x) = x^2 - 2x$. For which values of m will $f(2m) = 15$?

A. –2 only
B. $\frac{-3}{2}$ and $\frac{5}{2}$
C. 3 and –5
D. –3 and 5
E. 3 only

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Practice Test IIC

Time: 20 minutes

16 multiple-choice questions

Directions: Select the one correct answer of the five choices given and mark the corresponding circle on your answer sheet. Your scratch work should be done on any available space in the section.

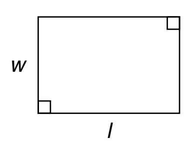
Notes

1. All numbers used are real numbers.
2. Calculators may be used.
3. Some problems may be accompanied by figures or diagrams. These figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that a figure is not drawn to scale. The figures and diagrams are meant to provide information useful in solving the problem or problems. Unless otherwise stated, all figures and diagrams lie in a plane.

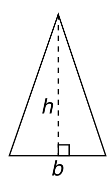
Data That Can Be Used for Reference

Area

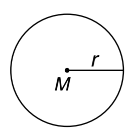
rectangle
$A = lw$



triangle
$A = \frac{1}{2} bh$

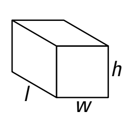


circle
$A = \pi r^2$
$C = 2\pi r$

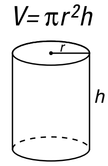


Volume

rectangular solid
$V = lwh$

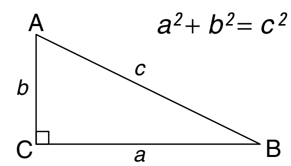


right circular cylinder
$V = \pi r^2 h$



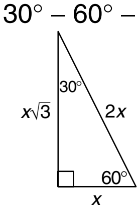
Pythagorean Relationship

$a^2 + b^2 = c^2$

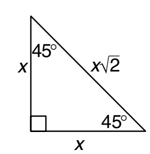


Special Triangles

30° – 60° – 90°



45° – 45° – 90°



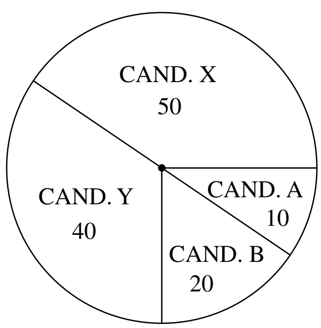
A circle is composed of 360°
A straight angle measures 180°
The sum of the angles of a triangle is 180°

1. Two positive integers are called "relatively prime" if their only common factor is 1. Which of the following pair of numbers is "relatively prime?"

A. 6 and 9
B. 10 and 5
C. 8 and 15
D. 12 and 4
E. 9 and 12

2. If the volume of a cube is 8, what is its total surface area?

A. 6
B. 8
C. 16
D. 18
E. 24

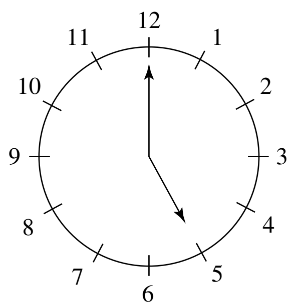


3. The graph above shows the number of votes received by each of four candidates in an election. What fraction of the votes did candidate *Y* receive?

A. $\frac{1}{6}$
B. $\frac{1}{4}$
C. $\frac{1}{3}$
D. $\frac{1}{2}$
E. $\frac{2}{5}$

4. If $f(x) = \frac{-3x^2}{7-x}$, then the value of $f(-2)$ =

A. $\frac{-12}{5}$
B. $\frac{-4}{3}$
C. $\frac{6}{7}$
D. $\frac{4}{3}$
E. $\frac{12}{5}$



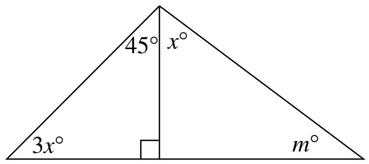
5. At 5:00pm, what is the degree measure of the angle between the hour and minute hands on the traditional clock above?

A. 75°
B. 115°
C. 120°
D. 150°
E. 165°

6. The denominator of a fraction is 3 more than its numerator. If the fraction is equal to $\frac{14}{35}$, what is the numerator of the fraction?

A. 2
B. 3
C. 5
D. 6
E. 7

GO ON TO THE NEXT PAGE



Note: Figure not drawn to scale.

7. In the figure above, what is the value of m?

A. 15
B. 30
C. 45
D. 60
E. 75

8. If $0 < x < 1$, which of the following statements must be true?

I. $\frac{1}{x} > x$
II. $x^2 < x$
III. $x^3 - x^2 > 0$

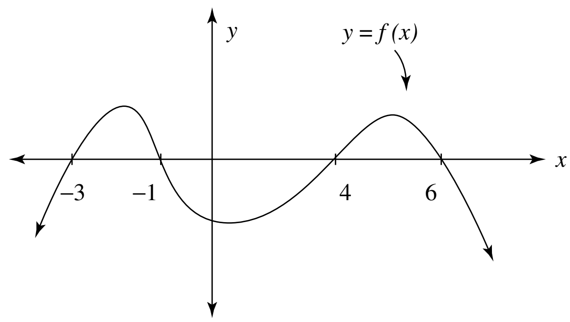
A. I only
B. II only
C. III only
D. I and II only
E. II and III only

9. The average of two numbers is x. If one of the numbers is $3m - 5$, then the other number, in terms of x and m, is

A. $\frac{x + 3m - 5}{2}$
B. $6m - 10 - x$
C. $6m - 5 - x$
D. $\frac{6m - 10}{x}$
E. $6m - 10 + x$

10. In the xy-plane, the graphs of $y = 2x + 7$ and $y = -3x + r$ intersect at the point $(-2, m)$. What is the value of r?

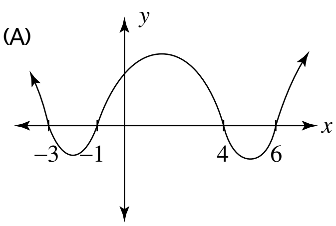
A. -3
B. -1
C. 2
D. 3
E. 4



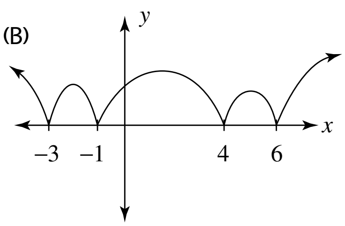
Note: Figure not drawn to scale.

11. The graph above is that of the equation $y = f(x)$. Which of the following could be the graph of $y = \left| f(x) \right|$?

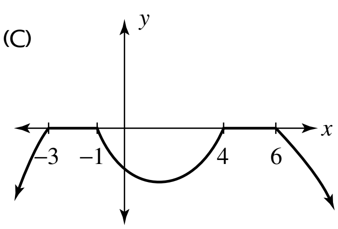
(A)



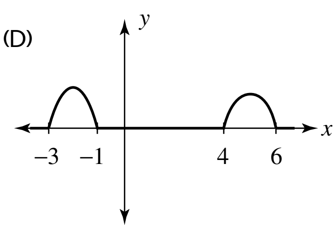
(B)



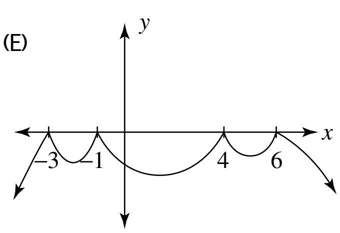
(C)



(D)



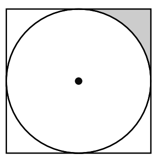
(E)



GO ON TO THE NEXT PAGE

12. The radius of a right circular cylinder is doubled, and its height is tripled. What is the ratio of the volume of the original cylinder to the volume of the new larger cylinder?

A. $\frac{1}{12}$
B. $\frac{1}{6}$
C. $\frac{1}{3}$
D. $\frac{1}{2}$
E. 6



13. In the figure above, a circle is inscribed within a square having a perimeter of 32. What is the area of the shaded region?

A. $64 - 16\pi$
B. $32 - 8\pi$
C. $16 - 4\pi$
D. $8 - 2\pi$
E. $4 - \pi$

14. In the *xy*-plane, how many different integer pairs (x, y) are in the solution set of $|x| + |y| \le 2$?

A. 4
B. 5
C. 8
D. 12
E. 13

15. The sum of three consecutive even integers is 72. If *x* represents the smallest, *y* the middle, and *z* the greatest of these even integers, which of the following equations could be used to represent the first sentence of this problem?

I. $3x + 6 = 72$
II. $3y = 72$
III. $3z - 6 = 72$

A. I only
B. II only
C. I and II only
D. I and III only
E. I, II, and III

16. If $\frac{x+4}{2}$ is an integer, and if $\frac{y-3}{2}$ is an integer, then xy must be

A. an even integer
B. a negative integer
C. an odd integer
D. a multiple of 12
E. a positive integer

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Scoring Practice Test II

Answer Key for Practice Test II

Practice Test IIA

1. B
2. E
3. E
4. D
5. C
6. B
7. D
8. C
9. $\frac{3}{10}$
10. $\frac{15}{14}$ or 1.07
11. 43
12. $\frac{1}{2}$ or .5
13. 38
14. $\frac{4}{3}$ or 1.33
15. 12
16. $\frac{11}{3}$ or 3.66 or 3.67
17. $\frac{5}{16}$ or .312 or .313
18. 7

Practice Test IIB

1. D
2. D
3. A
4. B
5. C
6. C
7. E
8. A
9. E
10. A
11. B
12. D
13. D
14. C
15. D
16. B
17. D
18. D
19. C
20. B

Practice Test IIC

1. C
2. E
3. C
4. B
5. D
6. A
7. E
8. D
9. B
10. A
11. B
12. A
13. C
14. E
15. E
16. A

Analyzing Your Test Results

The charts on the following pages should be used to carefully analyze your results and spot your strengths and weaknesses. The complete process of analyzing each subject area and each individual problem should be completed for each practice test. These results should then be reexamined for trends in types of errors (repeated errors) or poor results in specific subject areas. This reexamination and analysis is of tremendous importance to you in ensuring maximum test preparation benefit.

Mathematics Analysis Sheet

Section A	***Possible***	***Completed***	***Right***	***Wrong***
Multiple Choice	8			
Grid-Ins	10			
Subtotal	18			
Section B	***Possible***	***Completed***	***Right***	***Wrong***
Multiple Choice	20			
Subtotal	20			
Section C	***Possible***	***Completed***	***Right***	***Wrong***
Multiple Choice	16			
Subtotal	16			
Overall Math Totals	54			

Analysis/Tally Sheet for Problems Missed

One of the most important parts of test preparation is analyzing why you missed a problem so that you can reduce the number of mistakes. Now that you have taken the practice test and checked your answers, carefully tally your mistakes by marking them in the proper column.

Reason for Mistakes					
	Total Missed	***Simple Mistake***	***Misread Problem***	***Lack of Knowledge***	***Lack of Time***
Section A : Math					
Section B : Math					
Section C : Math					
Total Math					

Reviewing the preceding data should help you determine why you are missing certain problems. Now that you've pinpointed the type of error, compare it to other practice tests to spot other common mistakes.

Complete Answers and Explanations for Practice Test II

Practice Test IIA Explanations

1. **B.** To solve the equation:

$7x - 5 = 4x - 17$

$7x - 5 - 4x = 4x - 17 - 4x$ Subtract $4x$ from each side to get variable on just one side.

$3x - 5 = -17$

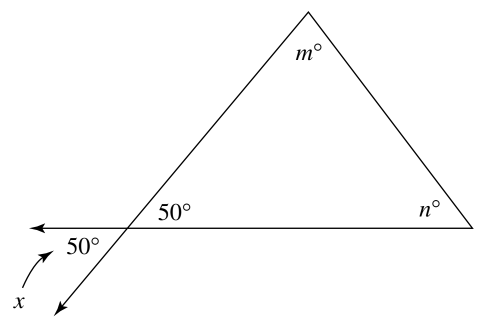
$3x - 5 + 5 = -17 + 5$ Add 5 to each side to isolate variable term.

$3x = -12$

$\frac{3x}{3} = \frac{-12}{3}$ Divide both sides by 3.

$x = -4$

2. **E.**



In the figure, the two angles marked 50° are vertical angles, so they are the same measure. The sum of the angles in the triangle must be 180°, so you have:

$50 + m + n = 180$

$50 + m + n - 50 = 180 - 50$ Subtract 50 from each side.

$m + n = 130$

3. **E.** When the graph of the equation $y = 3x - 12$ crosses the x-axis, its y-coordinate will be 0. So you then have:

$0 = 3x - 12$

$0 + 12 = 3x - 12 + 12$ Add 12 to both sides of the equation.

$12 = 3x$

$\frac{12}{3} = \frac{3x}{3}$

$4 = x$

4. **D.** Our given relationship reads something like the following: "average of $5x$ and $3y$ is equal to product of x and y, then 4 less than this." Translating this into an algebraic equation, you get:

$$\frac{5x + 3y}{2} = xy - 4$$

5. C. Just try different values of n and see what you get for the last digit of the answer.

For $n = 1$: $3^1 = 3$; number ends in a digit of 3

For $n = 2$: $3^2 = 9$; number ends in a digit of 9

For $n = 3$: $3^3 = 27$; number ends in a digit of 7

For $n = 4$: $3^4 = 81$; number ends in a digit of 1

So the number cannot end in a digit of 5, Choice **C.**

6. B. 20% off means that $\frac{4}{5}$ of the price remains.

25% off means that $\frac{3}{4}$ of that price remains.

So a discount of 20% followed by a discount of 25% would leave $\frac{4}{5} \cdot \frac{3}{4} = \frac{3}{5}$ of the original price, meaning $\frac{2}{5}$, or 40% has been discounted.

7. D. To solve an equation of the form $\left|\text{"}stuff\text{"}\right| = N$, break it into two parts:

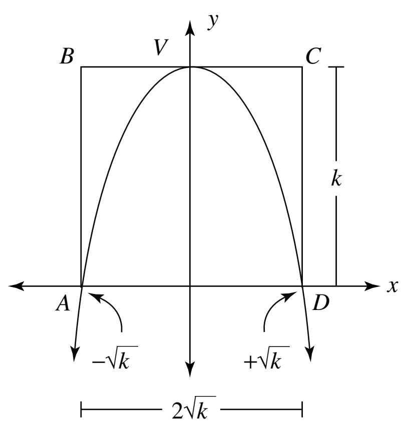
"stuff" = $-N$ or "stuff" = N

So the given equation $|2x - 3| = 7$ turns into the 2 parts:

$2x - 3 = -7$	or	$2x - 3 = 7$	
$2x - 3 + 3 = -7 + 3$	or	$2x - 3 = 7 + 3$	Add 3 to both sides.
$2x = -4$		$2x = 10$	
$\frac{2x}{2} = \frac{-4}{2}$		$\frac{2x}{2} = \frac{10}{2}$	Divide both sides by 2.

$x = -2$ or $x = 5$

8. C. By setting $x = 0$ in the given equation $y = k - x^2$, you get that $y = k$ is the y-intercept of the graph of this equation. So the parabola crosses the y-axis at k. Thus the height of the rectangle $ABCD$ is also just k, as shown in the figure below.



By setting $y = 0$ in the given equation, you can find where the graph of the parabola crosses the x-axis.

$$y = k - x^2$$

$$0 = k - x^2$$

$$0 + x^2 = k - x^2 + x^2$$ Add x^2 to both sides of the equation.

$$x^2 = k$$

therefore $x = \pm\sqrt{k}$

So you know that the parabola crosses the x-axis at $-\sqrt{k}$ on the left and $+\sqrt{k}$ on the right.

Thus the base of the rectangle is $2\sqrt{k}$.

The area of the rectangle ABCD is given as 16, so you have:

$\text{Area}_{\text{rect.}} = \text{base} \times \text{height}$

$16 = 2\sqrt{k} \cdot k$ Substitute the base and height.

$\frac{16}{2} = \frac{2\sqrt{k} \cdot k}{2}$ Divide both sides by 2.

$8 = \sqrt{k} \cdot k$

$8^2 = \left(\sqrt{k} \cdot k\right)^2$ Square both sides to get rid of the radical.

$64 = k \times k^2$

$64 = k^3$

$4 = k$

9. $\mathbf{\frac{3}{10}}$

Letting N be the given number, you have:

$$\frac{N}{\frac{2}{5}} \cdot \frac{4}{3} = N \cdot \frac{5}{2} \cdot \frac{4}{3} = N \cdot \frac{10}{3}$$

So after dividing your number N by $\frac{2}{5}$ and then multiplying by $\frac{4}{3}$, you end with just the number N multiplied by $\frac{10}{3}$.

The question asks what you should divide the original number N by to get $N \cdot \frac{10}{3}$.

If you divide N by $\frac{3}{10}$, you will get $\frac{N}{\frac{3}{10}} = N \cdot \frac{10}{3}$. So divide by $\frac{3}{10}$.

10. $\mathbf{\frac{15}{14}}$ **or 1.07**

Starting with the original proportion:

$\frac{5}{7x} = \frac{2}{3y}$

$5 \cdot 3y = 7x \cdot 2$ Take the diagonal products (cross-multiply).

$15y = 14x$

$\frac{15y}{y} = \frac{14x}{y}$ Divide by y, since you want to get $\frac{x}{y}$.

$15 = \frac{14x}{y}$

$\frac{1}{14} \cdot 15 = \frac{1}{14} \cdot \frac{14x}{y}$ Multiply both sides by $\frac{1}{14}$.

$\frac{15}{14} = \frac{x}{y}$

You could also write the answer as a decimal: $\frac{x}{y} = 1.07$

11. 43

First, expand the term $(m + n)^2$:

$(m + n) = (m + n)(m + n)$

$= m^2 + 2mn + n^2$

$= m^2 + n^2 + 2(mn)$ Reordered terms for purpose of next step.

$= 29 + 2(7)$ It is given that $m^2 + n^2 = 29$ and $mn = 7$.

$= 29 + 14 = 43$

12. $\mathbf{\frac{1}{2}}$ **or .5**

First, solve the given equations for x and y; then you can find the value of xy.

$$x^{\frac{-1}{3}} = 2$$

$$\left(x^{\frac{-1}{3}}\right)^{-3} = 2^{-3}.$$ You have taken the –3 power of both sides so that the exponent of x on the left will then be just 1.

$$x^1 = 2^{-3}$$

$$x = \frac{1}{2^3}.$$ Remember that, in general, $(number)^{negative} = \frac{1}{(number)^{positive}}$.

$$x = \frac{1}{8}$$

Next, solve for y.

$$y^{\frac{1}{2}} = 2$$

$$\left(y^{\frac{1}{2}}\right)^2 = 2^2.$$ Take the second power of both sides; you want just y^1 on the left.

$$y = 4$$

Finally, $xy = xy = \frac{1}{8} \bullet 4 = \frac{1}{2}$

You could also write the decimal .5

13. 38

Let n = first even integer.

Then $n + 2$ = second even integer,

so $n + 4$ = third even integer,

and $n + 6$ = fourth even integer,

with $n + 8$ = fifth even integer.

Since their sum is 170, you have the equation:

$$n + (n + 2) + (n + 4) + (n + 6) + (n + 8) = 170$$

$$5n + 20 = 170$$ Combine like terms on the left side.

$$5n + 20 - 20 = 170 - 20$$ Subtract 20 from both sides.

$$5n = 150$$

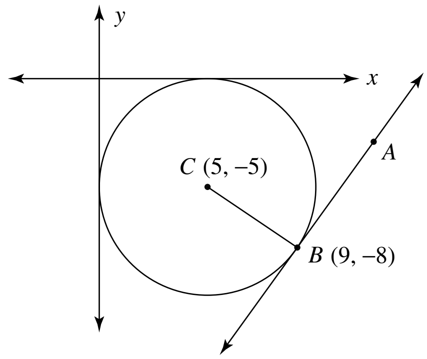
$$\frac{5n}{5} = \frac{150}{5}$$

$$n = 30$$

So your five consecutive even integers are 30, 32, 34, 36, and the largest one 38.

Another shorter approach works as follows: Divide the sum of the integers by the number of integers . . . so 170 ÷ 5 = 34, which gives the middle of our list of numbers. So the two above this are 36, and the largest 38.

14. $\mathbf{\frac{4}{3}}$ **or 1.33**

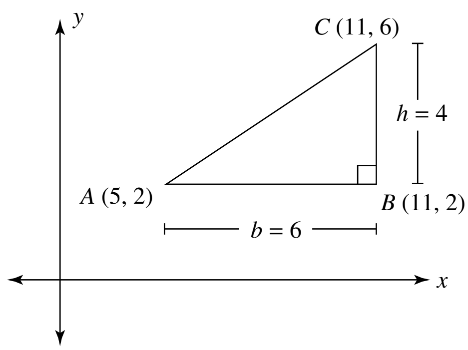


Since the circle is tangent to both axes, located in the fourth quadrant, and it has a radius of 5, you know that its center has coordinates (5, –5) as in the figure above. Since $\overleftrightarrow{AB}$ is tangent to the circle at point *B,* you also know that $\overleftrightarrow{AB} \perp \overleftrightarrow{CB}$. Remembering that the slopes of perpendicular lines have opposite reciprocal slopes (for example, $\frac{2}{3}$ and $\frac{-3}{2}$); you have to find only the slope of $\overline{CB}$ and take its opposite reciprocal.

$slope_{\overline{BC}} = \frac{-8 - -5}{9 - 5} = \frac{-3}{4}$ So the slope of $\overleftrightarrow{AB}$ is just $\frac{4}{3}$ or 1.33

15. 12

Below is a rough sketch of the points in the *xy*-plane.



Since points *A* and *B* have the same *y*-coordinate, they lie on a horizontal line segment, and since points *B* and *C* have the same *x*-coordinate, they lie on a vertical line segment as shown in the figure above.

You can easily see that the base of the triangle is just 6 and height is just 4.

Therefore, its area = $\frac{1}{2}bh = \frac{1}{2} \cdot 6 \cdot 4 = 12$.

16. $\mathbf{\frac{11}{3}}$ **or 3.66 or 3.67**

$m^2 + 2mn = 3n - 4w$

$3^2 + (2 \cdot 3 \cdot n) = 3n - (4 \cdot -5)$ Substituted given values of $m = 3$ and $w = -5$.

$9 + 6n = 3n + 20$

$9 + 6n - 3n = 3n + 20 - 3n$ Subtract $3n$ to get the variable on just one side.

$9 + 3n = 20$

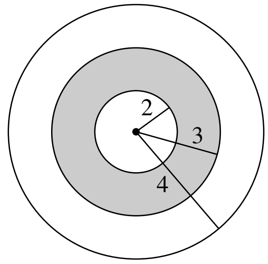
$9 + 3n - 9 = 20 - 9$ Subtract 9 from each side of the equation.

$3n = 11$

$\frac{3n}{3} = \frac{11}{3}$ Divide both sides by 3.

$n = \frac{11}{3}$ or 3.66 or 3.67

17. $\frac{5}{16}$ **or .312 or .313**



Using the figure above, you see that the shaded area is just the difference between the areas of the medium and small circles:

$\text{Area}_{\text{shaded}} = \text{Area}_{\text{med. circle}} - \text{Area}_{\text{small circle}}$

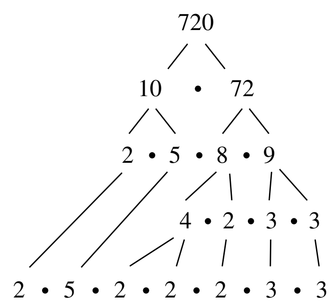
$= \pi(3)^2 - \pi(2)^2 = 9\pi - 4\pi = 5\pi$

Also, $\text{Area}_{\text{big circle}} = \pi(4)^2 = 16\pi$

The requested ratio is: $\frac{AREA_{big.circle}}{AREA_{shaded}} = \frac{5\pi}{16\pi} = \frac{5}{16}$ or .312 or .313

18. 7

Using a "factor tree," you find the prime factorization of 720 in the form of $a^m \times b^n \times c^r$.



Since $720 = 2^4 \times 3^2 \times 5^1$, the desired numbers *m, n*, and *r* are just the exponents 4, 2, and 1.

If you wrote the factored form as $720 = 3^2 \times 2^4 \times 5^1$ instead, the desired *m, n*, and *r* would still be the exponents 2, 4, and 1.

Since you are finding the sum of *m, n*, and *r,* the order of the exponents makes no difference.

So, in any order, $m + n + r = 4 + 2 + 1 = 7$ always.

Practice Test IIB Explanations

1. D. The first few terms of the sequence are 2, 3, 5, 8, 13, . . .

The third term, 5, is the sum of 2 and 3, the two terms preceding it.

The fourth term, 8, is the sum of 3 and 5, the two terms preceding it.

The fifth term, 13, is the sum of 5 and 8.

Continuing in this manner, you get:

The sixth term = 8 + 13 = 21. The sequence is now 2, 3, 5, 8, 13, 21.

The seventh term = 13 + 21 = 34. The sequence is now 2, 3, 5, 8, 13, 21, 34.

The eighth term = 21 + 34 = 55. The sequence is now 2, 3, 5, 8, 13, 21, 34, 55.

The ninth term = 34 + 55 = 89. The sequence is now 2, 3, 5, 8, 13, 21, 34, 55, 89.

The tenth term = 55 + 89 = 144. The sequence is now 2, 3, 5, 8, 13, 21, 34, 55, 89, 144.

2. **D.** To find the slope of $3x + 4y = 12$, solve for *y,* expressing the equation in slope-intercept form:

$3x + 4y = 12$

$3x + 4y - 3x = 12 - 3x$ Subtract $3x$ from each side to isolate the *y* term.

$4y = 12 - 3x$

$\frac{4y}{4} = \frac{12}{4} - \frac{3x}{4}$ Divide all terms by 4 to get *y* by itself.

$y = 3 - \frac{3}{4}x$, or in slope-intercept form, $y = \frac{-3}{4}x + 3$.

The slope of the line above is $\frac{-3}{4}$. Since perpendicular lines have opposite reciprocal slopes, the slope of line *l* is $\frac{4}{3}$.

3. **A.** To solve this equation:

$5(x + 3) = 17$

$5x + 15 = 17$ Distribute the 5 on the left.

$5x + 15 - 15 = 17 - 15$ Subtract 15 from each side to isolate the variable term.

$5x = 2$

$\frac{5x}{5} = \frac{2}{5}$ Divide both sides of equation by 5.

$x = \frac{2}{5}$

4. **B.** If $3(2x + 4)(5 - x) = 0$, then one of the two factors containing *x* could be 0.

So either $2x + 4 = 0$ or $5 - x = 0$

$2x = -4$ or $5 = x$

$x = -2$ or $x = 5$

5. **C.** To determine if $f(-5)$ does NOT equal $f(5)$, substitute –5 for *x* and 5 for *x* to see if the answers are the same.

A. For $f(x) = |x|$, You can easily see that when either –5 or 5 are put in for *x,* the answer is 5 both times.

B. For $f(x) = 7$, the answer is 7 no matter what the value of *x;* so once again the answers are the same: 7 both times.

C. For $f(x) = x^3 + 5$, you get:

$f(-5) = (-5)^3 + 5$ and $f(5) = (5)^3 + 5$

$= -125 + 5 = 125 + 5$

$= -120 \neq 130$

So in this case, $f(-5) \neq f(5)$; thus Choice **C** is the answer.

6. **C.**

$\frac{1}{x+1}+\frac{1}{x}+\frac{1}{x-1}$

$=\frac{1}{2+1}+\frac{1}{2}+\frac{1}{2-1}$ Substitute 2 for each x in the expression in the first line.

$=\frac{1}{3}+\frac{1}{2}+1$

$=\frac{2}{6}+\frac{3}{6}+\frac{6}{6}$ Change all fractions to common denominator of 6.

$=\frac{11}{6}=1\frac{5}{6}$

7. **E.** Mark is taller than Rebecca $\to m > r$ or can also be written as $r < m$ (note that all answer choices involve <, not >).

Charles is shorter than Rebecca $\to c < r$.

but Charles is taller than Harriet $\to h < c$.

Putting these three inequalities together: $h < c$ and $c < r$ and $r < m$, you end up with the compound inequality: $h < c < r < m$, which is Choice **E.**

8. **A.** If you let the radius of the larger circle be r, then the radius of the smaller circle is $\frac{1}{3}r$. The requested ratio is then:

$$\frac{AREA_{small}}{AREA_{large}}=\frac{\pi\left(\frac{1}{3}r^2\right)}{\pi r^2}=\frac{\pi\left(\frac{1}{9}r^2\right)}{\pi r^2}=\frac{1}{9}$$

You could also have used 3 for the radius of the large circle and 1 for the radius of the small circle and obtained the same result.

9. **E.** The second inequality is: $6 \le xy \le 30$

The first inequality is: $2 \le x \le 6$

If you divide these inequalities, term by term, you obtain: $3 \le y \le 5$, which is Choice **E.**

10. **A.**

m varies inversely as square of $r \to m=\frac{k}{r^2}$, where k is some constant.

Substituting the given values $m = 9$ and $r = 4$, you then get: $9=\frac{k}{4^2}$.

$9=\frac{k}{16}$

$16 \cdot 9 = 16 \cdot \frac{k}{16}$ Multiply both sides by 16.

$144 = k$

Therefore, the relationship between m and r can be expressed as: $m=\frac{144}{r^2}$.

With $r = 6$, you have: $m=\frac{144}{6^2}=\frac{144}{36}=4$

11. **B.** With the operation $\triangle$ defined by the equation $x\triangle y=\frac{\sqrt{x}}{y}$, you can solve the equation:

$4\triangle m = 16\triangle 2$

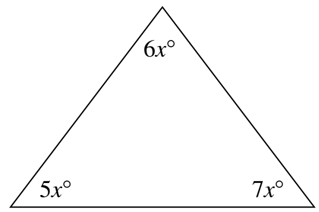
$\frac{\sqrt{4}}{m}=\frac{\sqrt{16}}{2}$

$\frac{2}{m}=\frac{4}{2}$ Simplified each square root in previous equation.

$\frac{2}{m}=2$

$m = 1$

12. **D.** Since the ratio of the angle measures is 5:6:7, you can label the angle measures of the triangle with $5x$, $6x$, and $7x$ as in the figure below.



Then, you have the equation: $5x + 6x + 7x = 180$

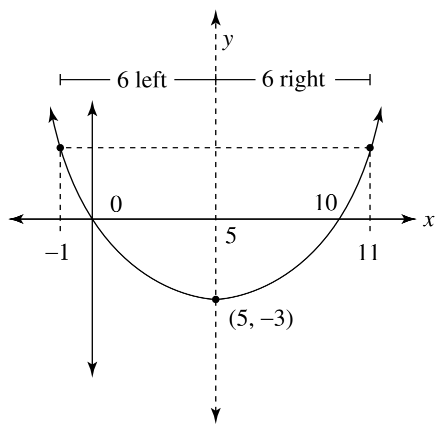
$18x = 180$ Combined like terms on left.

$\frac{18x}{18} = \frac{180}{18}$ Divide both sides by 18.

$x = 10$

But you want the measure of the largest angle of the triangle. So you find the value of $7x = 7(10) = 70$.

13. **D.**



In the figure above, the parabola has vertex at the point (5, –3). The dashed vertical line passing through 5 on the x-axis is called the "axis of symmetry" for the parabola. The points on the parabola to the left of the axis of symmetry are just mirror images of the corresponding points on the parabola to the right of the axis of symmetry. For example, you see that the parabola passes through the origin, which is 5 units left of the axis of symmetry; therefore, the parabola must cross the x-axis at 10 also, since that is just 5 units to the right of the axis of symmetry. So to answer the given question, you are looking for two numbers that are same distance—left and right—from 5. Choice **D** has just such a pair of numbers: –1 and 11. –1 is 6 units left, and 11 is 6 units right of the axis of symmetry as shown in the figure above.

14. **C.** Below is a series of five blanks, one for each seat in our row of five seats: $\overline{P}\ \overline{C}\ \overline{C}\ \overline{C}\ \overline{P}$

Under each blank is the letter P for parent, or C for child. The person to sit at the left end of the row can be chosen two ways (one of the two parents) and the person to sit at the other end can be chosen only one way (the remaining parent must sit there). So your series of blanks now looks like:

$\frac{2}{P}\ \frac{}{C}\ \frac{}{C}\ \frac{}{C}\ \frac{1}{P}$

Next you need to seat the three children. The first child seat can be filled three ways (any one of the three children), then the next seat only two ways (one of the remaining two children), and finally the last child seat can only be filled one way (the last un-chosen child must sit there). Your series of seating choices now is complete:

$\frac{2}{P}\ \frac{3}{C}\ \frac{2}{C}\ \frac{1}{C}\ \frac{1}{P}$. Your final answer is just the product of these numbers; $2 \times 3 \times 2 \times 1 \times 1 = 12$

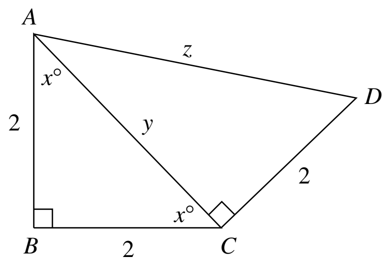
15. D. To change yards to inches, multiply by 36 (1 yard = 36 inches)

To change feet to inches, multiply by 12 (1 foot = 12 inches)

So Y yards = $36Y$ inches, F feet = $12F$ inches.

Therefore a distance of Y yards, F feet, and I inches = $36Y + 12F + I$ inches

16. B.



In the figure above, $\triangle ABC$ is an isosceles right triangle. Since $AB = 2$, then $BC = 2$ also. Using the pattern for an isosceles right triangle, x, x, $x\sqrt{2}$, you know that $AB = 2\sqrt{2}$, labeled y in the diagram. $\triangle ACD$ is a right triangle, so you use the Pythagorean theorem to find the length of $\overline{AD}$:

$$y^2 + 2^2 = z^2$$

$$\left(2\sqrt{2}\right)^2 + 2^2 = z^2 \qquad \text{Replace } y \text{ with } 2\sqrt{2}.$$

$$8 + 4 = z^2$$

$$12 = z^2$$

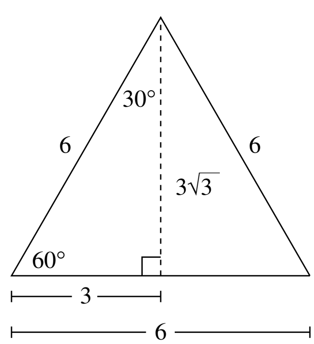
$$\sqrt{12} = z \qquad \text{Take the square root of both sides.}$$

$$\sqrt{4 \cdot 3} = z$$

$$2\sqrt{3} = z$$

So the length of $\overline{AD}$ is $2\sqrt{3}$.

17. D. Since the triangle is equilateral, each angle has a measure of 60°; since the equilateral triangle has a perimeter of 18, the length of each side is 6.

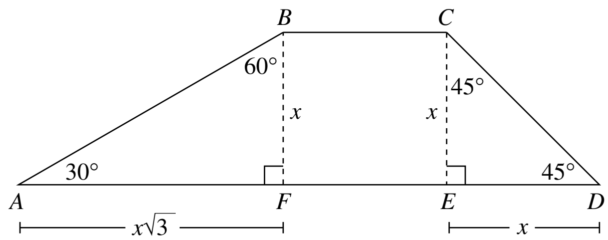


Referring to the figure above, the altitude drawn to the base of the triangle will bisect the base and bisect the angle at the top of the triangle, forming two 30°-60°-90° triangles.

Using your $\sqrt{3}$ pattern for this type of triangle, you can find the height of the equilateral triangle to be $3\sqrt{3}$; the base of the equilateral triangle is obviously 6.

The area of the triangle is then $\frac{1}{2}\, b \bullet h = \frac{1}{2} \bullet 6 \bullet 3\sqrt{3} = 9\sqrt{3}$.

18. D.



In the figure above, let the height $\overline{CE}$ of the isosceles right $\triangle CED$ be x. Then the base $\overline{ED}$ will also be x. Since $\overline{CE}$ is x, $\overline{BF}$ will also be x and this will be the height of $\triangle AFB$. Notice that $\triangle AFB$ is a 30°-60°-90° triangle, so using the $\sqrt{3}$ pattern, you find the base $\overline{AF}$ of this triangle is just $x\sqrt{3}$. Now that you have the base and height for each triangle, you can find the requested ratio using the area of triangle formula $\frac{1}{2}bh$.

$$\frac{AREA._{ABF}}{AREA._{CED}} = \frac{\frac{1}{2}\bullet x\sqrt{3}\bullet x}{\frac{1}{2}\bullet x\bullet x} = \frac{\sqrt{3}}{1} \text{ or } \sqrt{3}:1$$

19. C. You are told that the given expressions are equal, so you have:

$x^2 + cx + w = (x + m)(x + r)$

$x^2 + cx + w = x^2 + rx + mx + mr$	Expand the right side (FOIL).
$x^2 + cx + w = x^2 + (r + m)x + mr$	Factor x out of the 2 middle terms.
$x^2 + cx + w = x^2 + (r + m)x + mr$	Just write in left side of the equation.
$x^2 + \underline{cx} + \underline{\underline{w}} = x^2 + \underline{(r + m)}x + \underline{\underline{mr}}$	Add underlining for the next step.

For the left side and right side polynomials to be equal, the coefficients of their respective terms must be equal. So you have:

$c = r + m$; coefficient of the x term on left = coefficient of x term on right

$w = mr$; constant term on left = constant term on right

You can now find the requested ratio: $\frac{c}{w} = \frac{r+m}{mr}$, which is the same as $\frac{m+r}{mr}$, Choice **C.**

20. B. Using the given function $f(x) = x^2 - 2x$, you have the following equation to solve:

$f(2m) = 15$	
$(2m)^2 - 2(2m) = 15$	Put $2m$ in for x in the given function.
$4m^2 - 4m = 15$	
$4m^2 - 4m - 15 = 15 - 15$	Subtract 15 from each side; you want to set the equation equal to 0 and then factor it later.
$4m^2 - 4m - 15 = 0$	
$(2m + 3)(2m - 5) = 0$	Factor the left hand side.
$2m + 3 = 0$ and $2m - 5 = 0$	Set each factor equal to 0.
$2m = -3$ and $2m = 5$	
$m = \frac{-3}{2}$ and $m = \frac{5}{2}$	

Practice Test II

Practice Test IIC Explanations

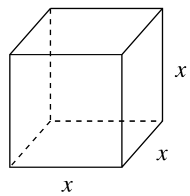
1. C. Checking each of the answer choices, you are looking for a pair of numbers whose only common factor is 1.

A. 6 and 9 have common factors of 1 and 3. So they are NOT relatively prime.

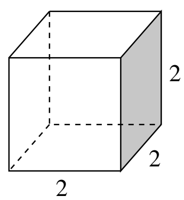
B. 10 and 5 have common factors of 1 and 5. So they are NOT relatively prime.

C. 8 and 15 only have a common factor of 1. So these ARE relatively prime.

2. E.

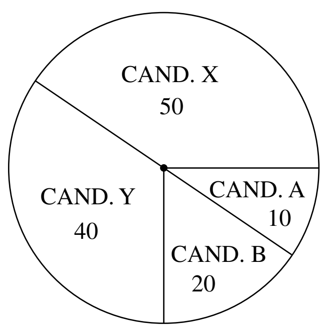


To find the volume of the cube in the figure above, you find the product of the length, width, and height, getting volume equals x^3. Since you are told the volume is 8, you have $x^3 = 9$, so $x = 2$.



In the figure above, the total surface area will be the area of the shaded side square multiplied by 6, since there are 6 equal faces to the cube. The area of the shaded square is $2 \times 2 = 4$, so the total area is $6 \times 4 = 24$.

3. C.

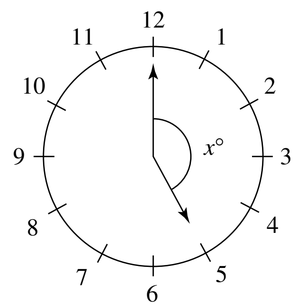


From the circle graph above, the total number of votes cast was $50 + 40 + 20 + 10 = 120$. Candidate Y received 40 of those votes, so candidate Y received $\frac{40}{120} = \frac{1}{3}$ of the votes.

4. B. Using the given function $f(x) = \frac{-3x^2}{7-x}$, you find $f(-2)$ by putting -2 in place of x:

$$f(-2) = \frac{-3(-2)^2}{7-(-2)} = \frac{-3 \cdot 4}{9} = \frac{-12}{9} = \frac{-4}{3}$$

5. **D.**



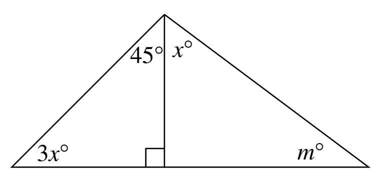
In the clock above, each hour represents $\frac{1}{12}$ of the clock. So 5:00 would represent $\frac{5}{12}$ of the clock. The clock, a circle, has 360°s, so $\frac{5}{12} \cdot 360° = 150°$.

6. **A.** If you let n = the numerator of the fraction, then $n + 3$ = the denominator of the fraction.

The problem says your fraction, which is now $\frac{n}{n+3}$, is equal to $\frac{14}{35}$. You then solve:

$\frac{n}{n+3} = \frac{14}{35}$	
$35n = 14(n + 3)$	Cross multiply to solve the given proportion.
$35n = 14n + 42$	Distribute the 14 on the right side.
$35n - 14n = 14n + 42 - 14n$	Subtract $14n$ from each side to isolate the variable term.
$21n = 42$	
$\frac{21n}{21} = \frac{42}{21}$	Divide both sides by 21.
$n = 2$	

7. **E.**



In the left right triangle in the figure above: $3x + 45 = 90$

$3x + 45 - 45 = 90 - 45$	Subtract 45 from each side.
$3x = 45$	
$\frac{3x}{3} = \frac{45}{3}$	Divide both sides by 3.
$x = 15$	

In the right triangle in the right side of the figure above, you know that:

$x + m = 90$	
$15 + m = 90$	Substitute $x = 15$ from above.
$15 + m - 15 = 90 - 15$	Subtract 15 from each side.
$m = 75$	

8. D. Since x is between 0 and 1, use $x = \frac{1}{2}$ and try each choice, I, II, and III.

I. $\frac{1}{x} > x \rightarrow$ with $x = \frac{1}{2}$, you have $\frac{1}{\frac{1}{2}} > \frac{1}{2}$, which simplifies into $2 > \frac{1}{2}$: TRUE

So your answer must include I, thus eliminating Choice **E.**

II. $x^2 < x \rightarrow$ with $x = \frac{1}{2}$, you have $\left(\frac{1}{2}\right)^2 < \frac{1}{2}$, which simplifies into $\frac{1}{4} < \frac{1}{2}$: TRUE

So your answer must now include both I and II. The only viable choice is **D.** You don't even need to try III.

9. B. If you let the other number be N, you then have:

$\frac{x+N}{2} = 3m - 5$	On the left is the average of x and N.
$2\left(\frac{x+N}{2}\right) = 2(3m-5)$	Multiply both sides by 2.
$x + N = 6m - 10$	Distribute the 2 on the right side.
$x + N - x = 6m - 10 - x$	Subtract x from both sides—you are solving for N.
$N = 6m - 10 - x$	

10. A. Since the graphs of the two equations intersect at the point $(-2, m)$, this point must lie on the graph of EACH of the equations. In particular, when substituted into the first equation, you must get a true statement, so you have:

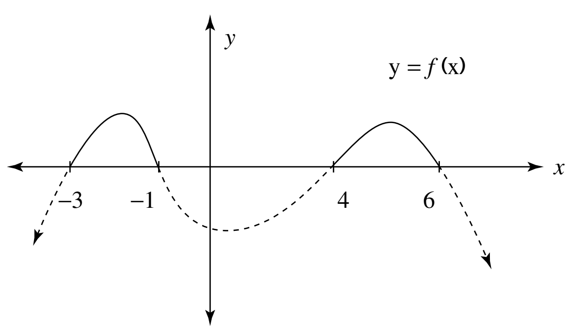
$y = 2x + 7$ and use the point $(-2, m)$

$m = 2(-2) + 7$	Substitute m for y and -2 for x.
$m = -4 + 7$	
$m = 3$	So the point of intersection is now $(-2, 3)$. When substituted into the second equation, you must get a true statement, so you now have:

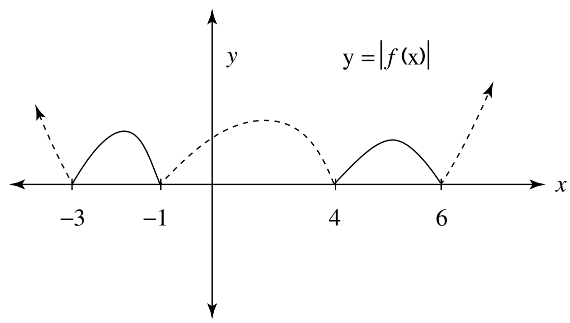
$y = -3x + r$ and use the point $(-2, 3)$

$3 = -3(-2) + r$	Substitute 3 for y and -2 for x.
$3 = 6 + r$	
$-3 = r$	

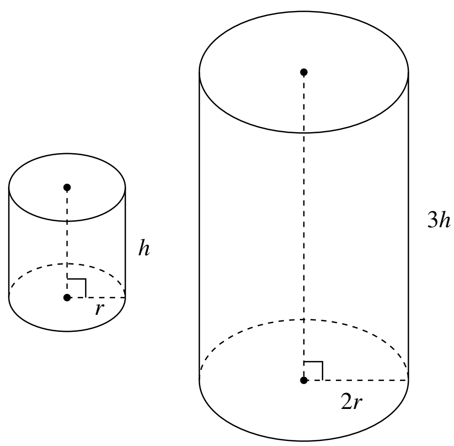
11. B.



Above is the graph of $y = f(x)$ with portions of the graph below the x-axis dotted, rather than solid. These are the regions in which $f(x)$ is negative. If you took the absolute value of these negative numbers, they would turn into positive numbers instead. So the portions of the graph of $y = f(x)$ that are below the x-axis would end up above the x-axis for the graph of $y = |f(x)|$, as in the figure below. Note that the positive regions of $y = f(x)$ would change their location since their absolute value would still be positive for the graph of $y = |f(x)|$.



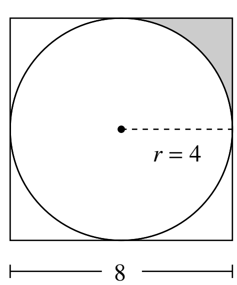
12. **A.** With the radius of our cylinder doubled and its height tripled, you end up with the original and new cylinders as shown below.



$$\frac{VOL_{original}}{VOL_{new}} = \frac{\pi r^2 h}{\pi(2r)^2(3h)} = \frac{\pi r^2 h}{\pi \cdot 4r^2 \cdot 3h} = \frac{\pi r^2 h}{12\pi r^2 h} = \frac{1}{12}$$

13. **C.** To find the area of the shaded region, you find the difference in areas of the square and circle, and then find $\frac{1}{4}$ of this result.

If the perimeter of the square is 32, the length of one of its sides is just $\frac{32}{4} = 8$. As can be seen in the figure below, the radius of the circle is just $\frac{1}{2}$ of this, so $r = 4$.



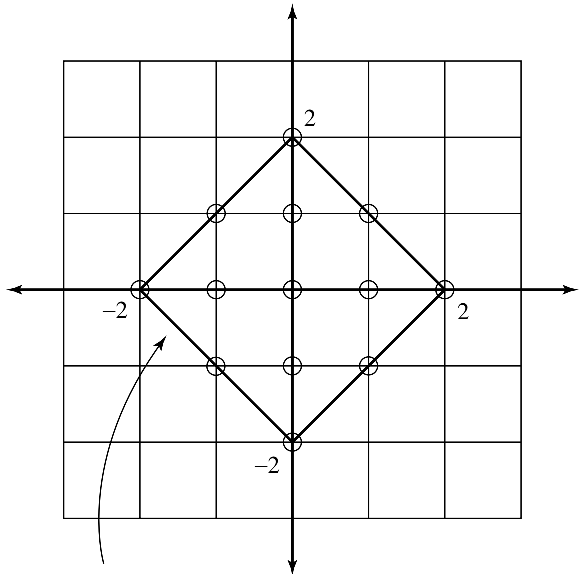
Then $AREA_{shaded} = \dfrac{AREA_{square} - AREA_{circle}}{4}$

$= \dfrac{8^2 - \pi \cdot 4^2}{4}$ Area square = $(\text{side})^2$ and area circle = πr^2

$= \dfrac{64 - 16\pi}{4}$

$= 16 - 4\pi$ Divide each term in expression above by 4.

14. E. In the figure below, the four dark segments forming the diamond represent the graph of the equation $|x| + |y| = 2$.



Graph of $|x| + |y| = 2$
(solid line segments)

The integer pairs *(x, y)* that satisfy the inequality $|x| + |y| \leq 2$ are those on the figure above with an open circle around them—count them all and you count 13 such points.

15. E. With x = smallest even integer,

then $x + 2$ = medium even integer,

and $x + 4$ = largest even integer.

Their sum would be: $x + (x + 2) + (x + 4) = 3x + 6$. So $3x + 6 = 72$ works, as in I.

With $y - 2$ = smallest even integer,

then y = medium even integer,

and $y + 2$ = largest even integer.

Their sum would be: $(y - 2) + y + (y + 2) = 3y$. So $3y = 72$, also works, as in II.

With $z - 4$ = smallest even integer,

then $z - 2$ = medium even integer,

and z = largest even integer.

Their sum would be: $(z - 4) + (z - 2) + z = 3z - 6$. So $3z - 6 = 72$ also works, as in III.

Therefore I, II, and III work, so the correct answer choice is **E.**

16. A. If $\frac{x+4}{2}$ is an integer, that means $x + 4$ must be even, so x is even.

If $\frac{y-3}{2}$ is an integer, that means $y - 3$ must be even, so y is odd.

Then with x even and y odd, their product xy will be even, as in Choice A.

CUT HERE

Test IIIA

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
14	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
15	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
16	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
17	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
18	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
19	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
20	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

Test IIIB

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

9.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

10.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

11.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

13.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

14.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

15.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

16.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

17.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

18.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Test IIIC

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
14	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
15	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
16	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

CUT HERE

Practice Test III

Practice Test IIIA: 25 Minutes, 20 Multiple-Choice Questions

Practice Test IIIB: 25 Minutes, 18 Questions (8 Multiple-Choice and 10 Grid-in)

Practice Test IIIC: 20 Minutes, 16 Multiple-Choice Questions

Practice Test IIIA

Time: 25 minutes

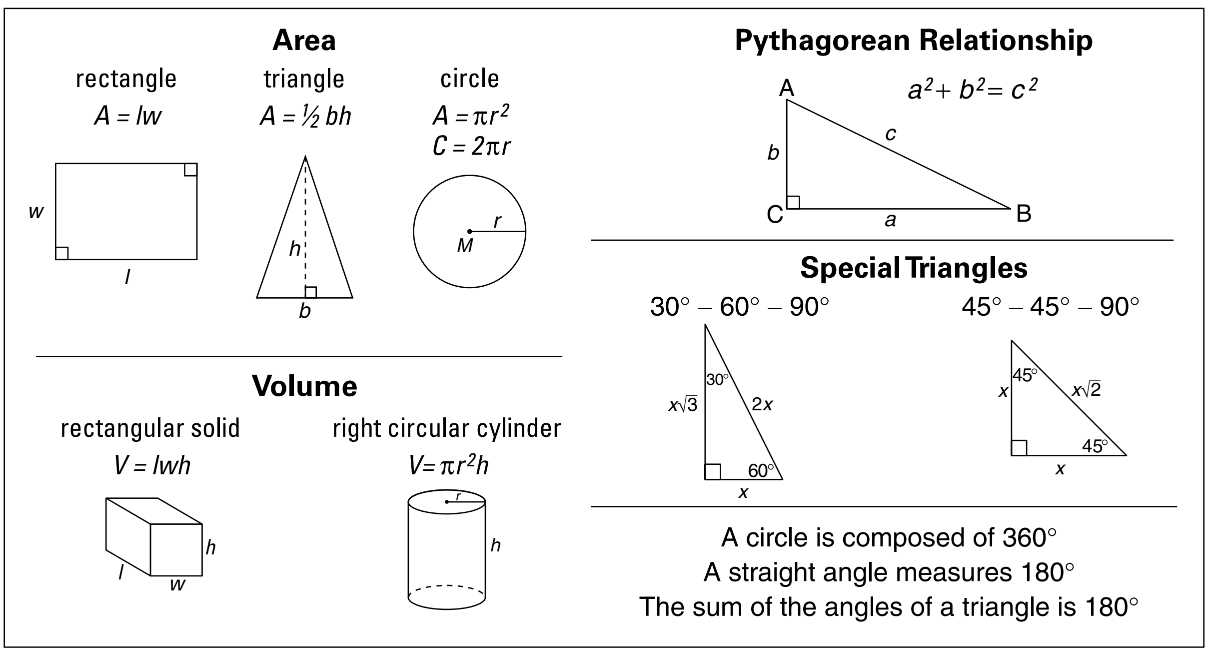
20 multiple-choice questions

Directions: Select the one correct answer of the five choices given and mark the corresponding circle on your answer sheet. Your scratch work should be done on any available space in the section.

Notes

1. All numbers used are real numbers.
2. Calculators may be used.
3. Some problems may be accompanied by figures or diagrams. These figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that a figure is not drawn to scale. The figures and diagrams are meant to provide information useful in solving the problem or problems. Unless otherwise stated, all figures and diagrams lie in a plane.

Data That Can Be Used for Reference

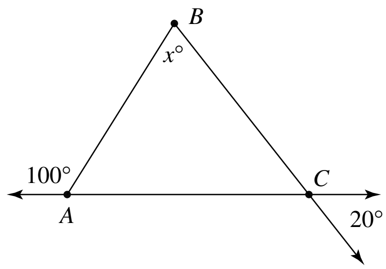


1. If $3x - 5 = 16$, then $x =$

A. -7
B. $\frac{11}{3}$
C. 4
D. 7
E. 18

2. If $\frac{x+4}{x} = \frac{9}{7}$, what is the value of x?

A. $3\frac{1}{2}$
B. 7
C. 14
D. 21
E. 35



Note: Figure not drawn to scale.

3. In the figure above, what is the value of x?

A. 20
B. 50
C. 60
D. 80
E. 140

4. At a flooring center, you can purchase carpet in one of five styles and in one of 10 colors. How many different style-color combinations are possible at this flooring center?

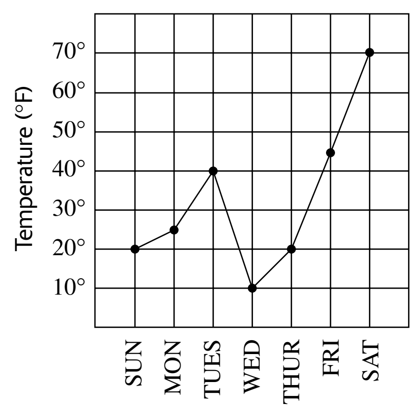
A. 15
B. 36
C. 40
D. 45
E. 50

5. In the xy-plane, a circle has its center at point C (0, 0) and contains the point P (–3, 4). Under the translation that moves point C to (–2, 3), where will point P be located after this translation?

A. (–1, 1)
B. (1, –1)
C. (–3, 2)
D. (–4, 5)
E. (–5, 7)

6. If $f(x) = \frac{(2-x)^2}{x} + \frac{3}{5}$, what is the value of $f(5)$?

A. 0
B. $1\frac{1}{5}$
C. $2\frac{2}{5}$
D. $3\frac{3}{5}$
E. 4



7. According to the data in the graph above, between which two consecutive days of the week was the percent increase in temperature the greatest?

A. Sunday to Monday
B. Monday to Tuesday
C. Wednesday to Thursday
D. Thursday to Friday
E. Friday to Saturday

Practice Test III

GO ON TO THE NEXT PAGE

8. An employee had to take a pay cut of 20%. A while later, she received a pay raise of 25%. Her new salary after the pay raise was what percent of her salary before the pay cut?

A. 45%
B. 55%
C. 80%
D. 95%
E. 100%

9. If the average (arithmetic mean) of x, 6, 10, 12, $5x$, and 10 is equal to $3x$, what is the value of x?

A. 2
B. 3
C. 4
D. 5
E. 6

10. $3x - 2y = -12$ and $x + 3y = 7$

For what values of x and y are the equations above both true?

A. $x = -6, y = -3$
B. $x = 4, y = 1$
C. $x = -1, y = \frac{9}{2}$
D. $x = -2, y = 3$
E. $x = 10, y = -1$

11. It takes Lisa h hours to walk a distance of m miles. At this same rate, how many hours will it take her to walk a distance of d miles?

A. $\frac{dm}{h}$
B. $\frac{dh}{m}$
C. $\frac{mh}{d}$
D. $\frac{d}{mh}$
E. $\frac{m}{hd}$

12. If the ratio of the number of boys to the number of girls in a class is $\frac{3}{4}$, which of the following could NOT be the number of students in this class?

A. 21
B. 28
C. 33
D. 42
E. 49

13. The ratio of the length of a rectangle to its width is 5:3. If the length is 10, what is the area of the rectangle?

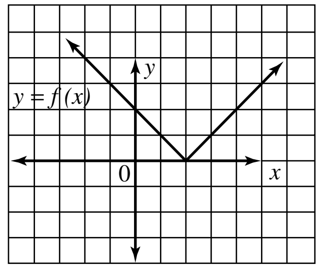
A. 6
B. 15
C. 16
D. 32
E. 60

14. If $x = \frac{-b + \sqrt{c}}{m}$, what is c in terms of x, b, and m?

A. $m^2x^2 - b^2$
B. $(x - m + b)^2$
C. $\sqrt{xm + b}$
D. $(mx + b)^2$
E. $\frac{(x - m)^2}{b^2}$

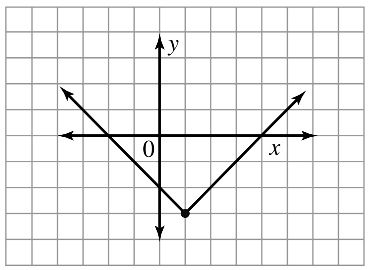
15. The minimum speed on a given stretch of freeway is 45 mph, and the maximum speed is 65 mph. Which of the following inequalities could be used to decided if a motorist driving at a rate of r mph is within legal limits?

A. $|r - 45| \leq 10$
B. $|r - 55| \leq 10$
C. $|r - 65| \leq 10$
D. $|r - 55| < 10$
E. $|r - 10| \leq 55$

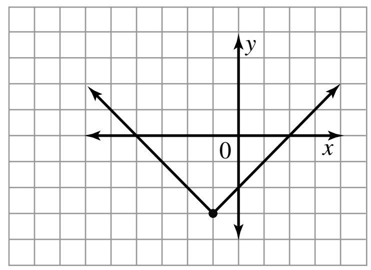


16. Above is the graph of $y = f(x)$. Which of the graphs below is that of $y = f(x + 1) + 3$?

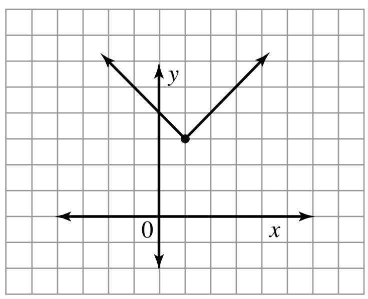
A.



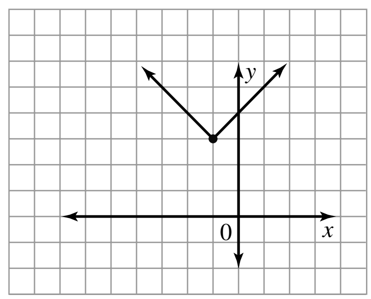
B.



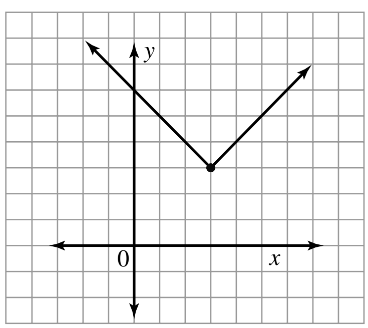
C.



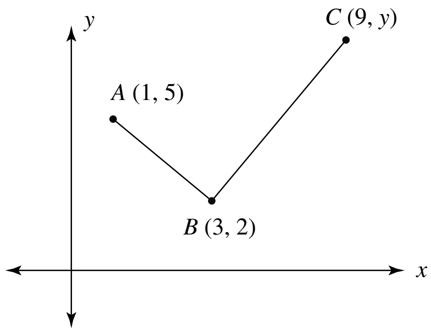
D.



E.



GO ON TO THE NEXT PAGE



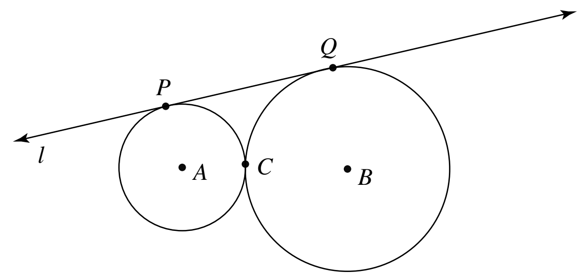
Note: Figure not drawn to scale.

17. In the figure above, $\overline{AB} \perp \overline{BC}$. What is the value of *y?*

A. $\frac{-3}{2}$
B. $\frac{2}{3}$
C. 2
D. 6
E. 12

18. If $8^{2x} = 4^{4x-3}$, what is the value of *x?*

A. $\frac{1}{4}$
B. 1
C. 3
D. 4
E. 6



Note: Figure not drawn to scale.

19. In the figure above, circles *A* and *B* are tangent at *C,* and line *l* is tangent to circles *A* and *B* at points *P* and *Q* respectively. If the radius of circle *A* is 2 and the radius of circle *B* is 6, what is the length of $\overline{PQ}$?

A. 6
B. $4\sqrt{3}$
C. $6\sqrt{2}$
D. 8
E. 10

20. A small rocket is fired into the air from the top of a tower. Its height *h,* in feet, after *t* seconds, is given by: $h(t) = -16t^2 + 64t + 32$. The rocket is 80 feet above the ground twice—once on the way up and then again on the way down. How many seconds pass between these two times?

A. 1
B. 2
C. 3
D. 4
E. 5

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Practice Test IIIB

Time: 25 minutes

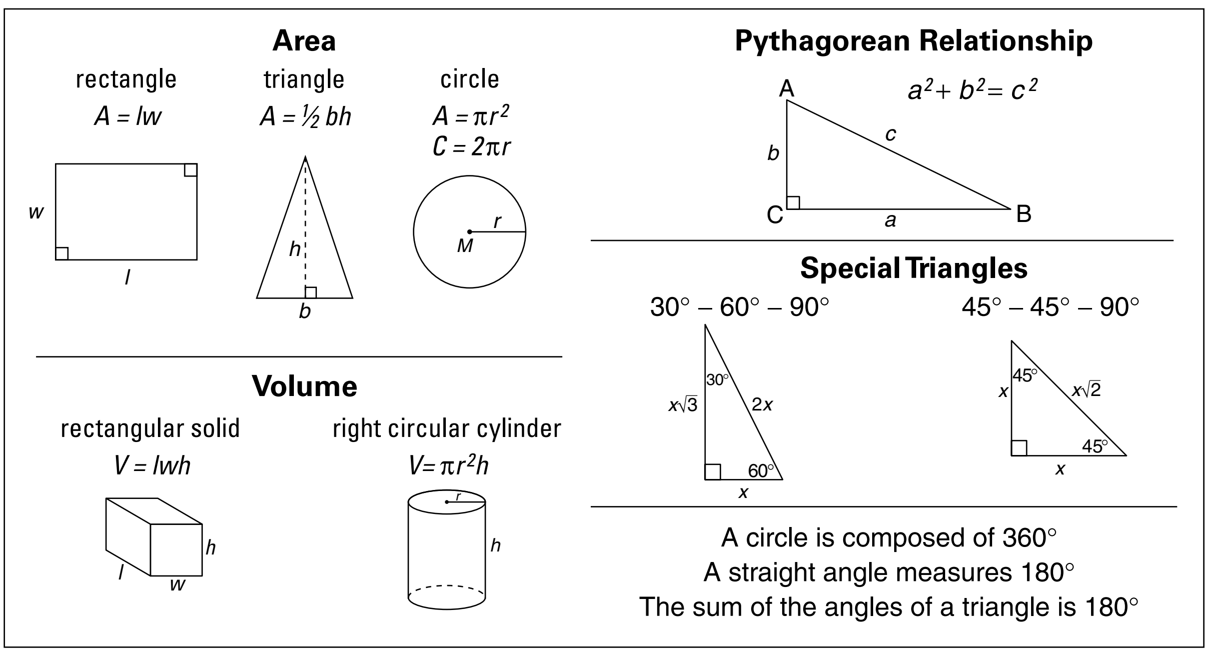
18 questions (8 multiple-choice and 10 grid-in)

Directions: This section is composed of two types of questions. Use the 25 minutes allotted to answer both question types. For Questions 1–8, select the one correct answer of the five choices given and mark the corresponding circle on your answer sheet. Your scratch work should be done on any available space in the section.

Notes

1. All numbers used are real numbers.
2. Calculators may be used.
3. Some problems may be accompanied by figures or diagrams. These figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that a figure is not drawn to scale. The figures and diagrams are meant to provide information useful in solving the problem or problems. Unless otherwise stated, all figures and diagrams lie in a plane.

Data That Can Be Used for Reference



GO ON TO THE NEXT PAGE

1. What is the average (arithmetic mean) of the numbers $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$?

A. $\frac{1}{22}$

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

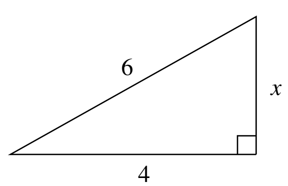
E. $\frac{5}{12}$

2. $\sqrt{12} + \sqrt{75} =$

A. $\sqrt{87}$
B. $7\sqrt{6}$
C. $7\sqrt{3}$
D. $10\sqrt{3}$
E. $29\sqrt{3}$

3. If n represents an even integer, which of the following represents an odd integer?

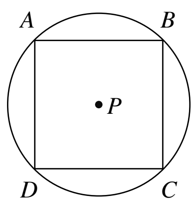
A. $3n + 2$
B. $n^2 + 3$
C. $(n + 4)^2$
D. $5n - 4$
E. $7n + n^2$



Note: Figure not drawn to scale.

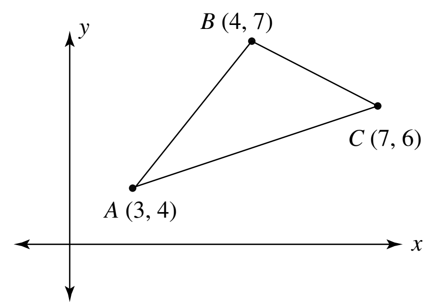
4. In the right triangle above, what is the value of x?

A. 2
B. $2\sqrt{5}$ (approximately 4.47)
C. 5
D. $5\sqrt{2}$ (approximately 7.07)
E. $2\sqrt{13}$ (approximately 7.21)



5. In the figure above, square $ABCD$ is inscribed within circle P. If the area of the circle is 8π, what is the area of the square?

A. $2\sqrt{2}$
B. 8
C. 16
D. $16\sqrt{2}$
E. 32



Note: Figure not drawn to scale.

6. In $\triangle ABC$ above, $\overline{AB} \perp \overline{BC}$. What is the product of the slopes of the sides of $\triangle ABC$?

A. -2
B. -1
C. $\frac{-1}{2}$
D. 1
E. 2

7. If $(x + m)(x + n) = x^2 - 5x - 36$, what is the value of $\frac{m+n}{mn}$?

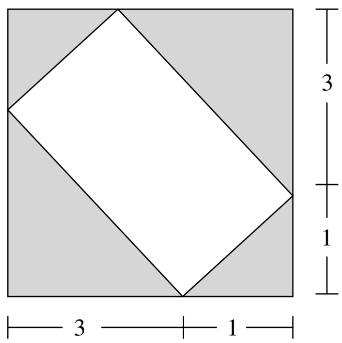
A. $\frac{-4}{9}$

B. $\frac{5}{36}$

C. $\frac{9}{4}$

D. $\frac{36}{5}$

E. It cannot be determined from the given information.

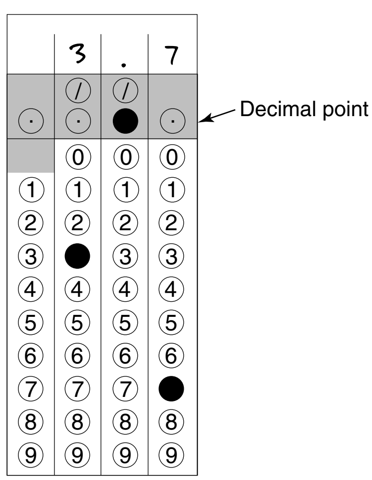


8. In the figure above, a rectangle is inscribed within a square of side length 4. What is the area of the shaded region?

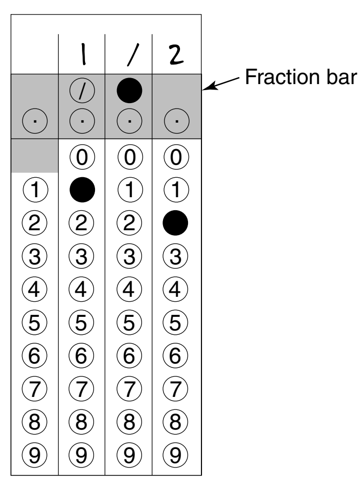
A. 16
B. 10
C. 6
D. $3\sqrt{2}$
E. $\sqrt{2}$

Directions for Student-Produced Response Questions (Grid-ins): Questions 9–18 require you to solve the problem and enter your answer by carefully marking the circles on the special grid. Examples of the appropriate way to mark the grid follow.

Answer: 3.7

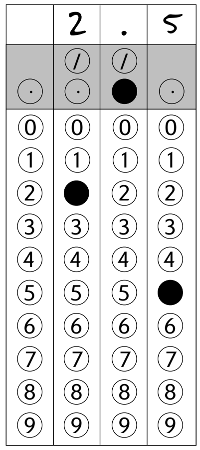


Answer 1/2

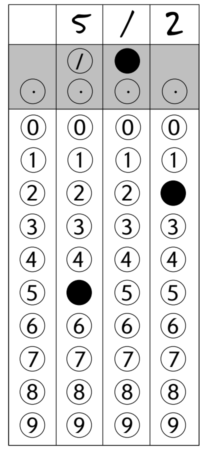


Answer 2 1/2

Do not grid-in mixed numbers in the form of mixed numbers. Always change mixed numbers to improper fractions or decimals.

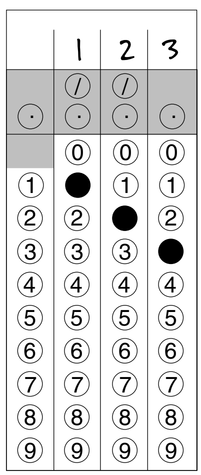


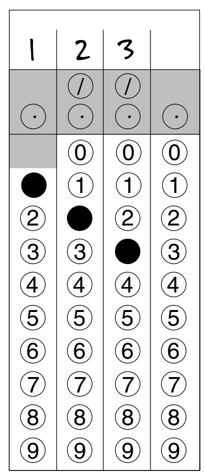
or



Answer 123

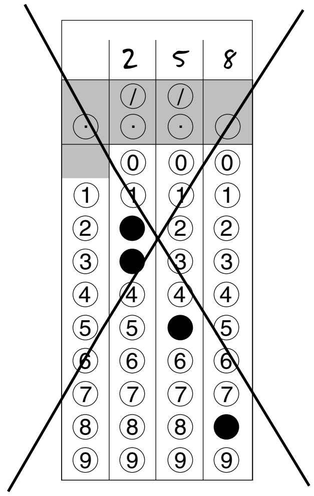
Space permitting, answers may start in any column. Each grid-in answer below is correct.





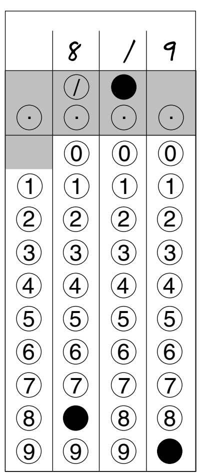
Answer 258 (no credit)

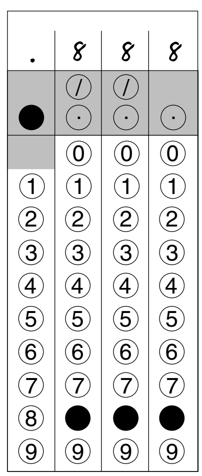
Note: Circles must be filled in correctly to receive credit. Mark only one circle in each column. No credit will be given if more than one circle in a column is marked. Example:

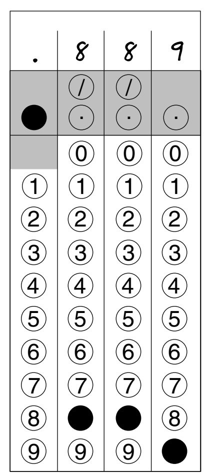


Answer 8/9

Accuracy of decimals: Always enter the most accurate decimal value that the grid will accommodate. For example: An answer such as .8888 . . . can be gridded as .888 or .889. Gridding this value as .8, .88, or .89 is considered inaccurate and therefore not acceptable. The acceptable grid-ins of 8/9 are:







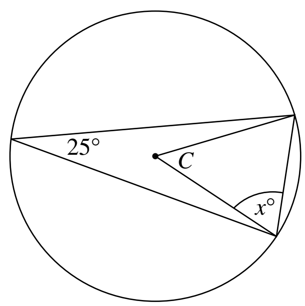
Be sure to write your answers in the boxes at the top of the circles before doing your gridding. Although writing out the answers above the columns is not required, it is very important to ensure accuracy. Even though some problems may have more than one correct answer, grid only one answer. Grid-in questions contain no negative answers.

Practice Test III

GO ON TO THE NEXT PAGE

9. The length of a rectangle is decreased by 25% while its width is decreased by 20%. The area of the new rectangle is what fraction of the area of the original rectangle?

10. If $x = 4$, what is the value of $\frac{3xy^2}{x^2y^2 - 5y^2}$?



Note: Figure not drawn to scale.

11. In the figure above, point C is the center of the circle. What is the value of x?

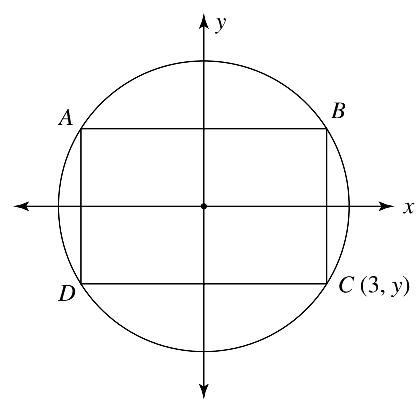
12. If $\frac{10x + 15m}{x + m} = 12$, what is the value of $\frac{m}{x}$?

13. If $12(m + n)(c - r) = 72$ and $3(m + n) = 9$, what is the value of $c - r$?

14. The population of bacteria in a culture doubles every 10 minutes. If the current bacteria population is 1,024, how many minutes ago was the bacteria population only 32?

15. If the median of a list of seven consecutive odd integers is 23, what is the difference between the largest and smallest of these integers?

16. The digits 2, 3, 4, 5, and 6 are used to create five-digit numbers that begin and end with an even digit. If each digit can be used only once in each of the numbers, how many of these five-digit numbers can be created?



Note: Figure not drawn to scale.

17. In the figure above, rectangle $ABCD$ is inscribed in the circle with center at the origin in the xy-plane. Point C with coordinates $(3, y)$ is located on this circle. If the radius of the circle is 5, what is the area of rectangle $ABCD$?

18. If the average (arithmetic mean) of five different two-digit positive integers is 20, what is the greatest possible difference between the largest and the smallest of these five integers?

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Practice Test IIIC

Time: 20 minutes

16 multiple-choice questions

Directions: Select the one correct answer of the five choices given and mark the corresponding circle on your answer sheet. Your scratch work should be done on any available space in the section.

Notes

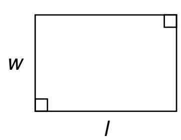
1. All numbers used are real numbers.
2. Calculators may be used.
3. Some problems may be accompanied by figures or diagrams. These figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that a figure is not drawn to scale. The figures and diagrams are meant to provide information useful in solving the problem or problems. Unless otherwise stated, all figures and diagrams lie in a plane.

Data That Can Be Used for Reference

Area

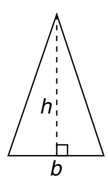
rectangle

$A = lw$



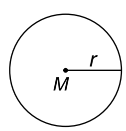
triangle

$A = \frac{1}{2}bh$



circle

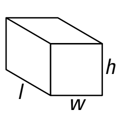
$A = \pi r^2$
$C = 2\pi r$



Volume

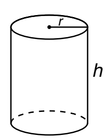
rectangular solid

$V = lwh$



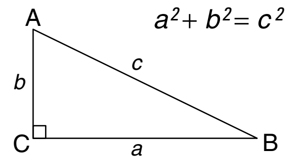
right circular cylinder

$V = \pi r^2 h$



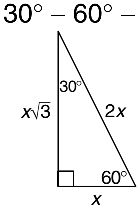
Pythagorean Relationship

$a^2 + b^2 = c^2$

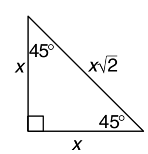


Special Triangles

30° – 60° – 90°



45° – 45° – 90°



A circle is composed of 360°

A straight angle measures 180°

The sum of the angles of a triangle is 180°

Practice Test III

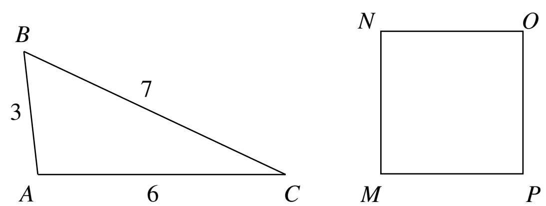
GO ON TO THE NEXT PAGE

1. If $m - n = 11$, what is the value of $2m - 2n - 7$?

A. 4
B. $7\frac{1}{2}$
C. 15
D. 22
E. 29

2. If $\frac{2m}{3r} = \frac{8}{5}$, what is the value of $\frac{r}{m}$?

A. $\frac{5}{12}$
B. $\frac{5}{8}$
C. $\frac{12}{5}$
D. $\frac{5}{8}$
E. $\frac{3}{2}$



Note: Figures not drawn to scale.

3. In the figures above, $\triangle ABC$ and square $MNOP$ have equal perimeters. What is the length of $\overline{MP}$?

A. 2
B. 4
C. 5
D. 8
E. 16

4. The quantity $5x - 7$ is how much larger than the quantity $-10 + 5x$?

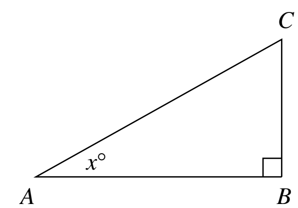
A. -17
B. -3
C. 3
D. 7
E. 17

5. If $0 < x < y < 1$, which of the following must be positive?

A. $x - y$
B. $x^2 - y^2$
C. $\frac{1}{x} - \frac{1}{y}$
D. $-(y - x)$
E. $\frac{-3y}{2x}$

6. When a given number is decreased by 5 and this result is tripled, the number obtained is 24. What is the given number?

A. $\frac{11}{3}$
B. $\frac{29}{3}$
C. 7
D. 12
E. 13



Note: Figure not drawn to scale.

7. In $\triangle ABC$ above, $\overline{AB} \perp \overline{BC}$, $x = 30$, and the length of $\overline{AC}$ is $8\sqrt{3}$. What is the area of $\triangle ABC$?

A. 16
B. $24\sqrt{2}$
C. $24\sqrt{3}$
D. $48\sqrt{3}$
E. 144

8. If the area of a circle is $\frac{9\pi}{4}$, what is its circumference?

A. $\frac{81\pi}{256}$
B. $\frac{81\pi}{16}$
C. $\frac{3}{2}$
D. $\frac{3\pi}{2}$
E. 3π

9. In the xy-plane, a given circle with center at the origin contains the point (–3, 4). Which of the following points is NOT on the given circle?

A. (3, –4)
B. (0, –5)
C. (3, 4)
D. $(0, \sqrt{7})$
E. None of the above points are on the given circle.

10. If $0 < x < 1$, which of the following gives the correct order of $\sqrt[3]{x}$, $\sqrt{x}$, and $\frac{1}{x}$?

A. $\frac{1}{x} < \sqrt{x} < \sqrt[3]{x}$
B. $\sqrt[3]{x} < \frac{1}{x} < \sqrt{x}$
C. $\sqrt{x} < \sqrt[3]{x} < \frac{1}{x}$
D. $\sqrt[3]{x} < \sqrt{x} < \frac{1}{x}$
E. $\frac{1}{x} < \sqrt[3]{x} < \sqrt{x}$

11. In the xy-plane, $\left(\sqrt{7}, 5\right)$ is a point on the graph of the equation $y = x^2 + k$. For what positive value of x will $y = 7$?

A. 2
B. 3
C. 4
D. 5
E. 6

12. If $x^{3/4} = y^{3/8}$, what is x in terms of y?

A. $\frac{1}{y}$
B. $\sqrt{y}$
C. y^2
D. y^3
E. y^6

13. If $\sqrt{xy} = 6$, which of the following could NOT be the value of $x + y$?

A. 37
B. 20
C. 18
D. 15
E. 13

14. In the xy-plane, the set of points 5 units from the point A (6, 3) lie on a circle with center A and radius 5. What are all the possible points on this circle having an x-coordinate of 9?

A. (9, 8) only
B. (9, 7) and (9, –1)
C. (9, 8) and (9, –2)
D. (9, –2) only
E. (9, 7) only

15. Rachelle invested \$1,000 in an IRA paying 6% per year. The accumulated value of her investment t years later is given by the function A, where $A(t) = 1{,}000(1.06)^t$. In how many years will Rachelle's investment be worth \$1,191.02?

A. 1
B. 2
C. 3
D. 4
E. 5

16. The length of the longer leg of a right triangle is one less than twice the length of the shorter leg, and the length of the hypotenuse is one more than twice the length of the shorter leg. If the length of the shorter leg is x, which of the following equations could be used to find the value of x?

A. $x + (2x - 1) = 2x + 1$
B. $x^2 + (2x - 1)^2 = (2x - 1)^2$
C. $x^2 = (2x - 1)^2 + (2x + 1)^2$
D. $x^2 + (2x - 1)^2 = (2x + 1)^2$
E. $x(2x - 1) = 2x + 1$

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

STOP

Scoring Practice Test III

Answer Key for Practice Test III

Practice Test IIIA

1. D
2. C
3. D
4. E
5. E
6. C
7. D
8. E
9. C
10. D
11. B
12. C
13. E
14. D
15. B
16. C
17. D
18. C
19. B
20. B

Practice Test IIIB

1. C
2. C
3. B
4. B
5. C
6. C
7. B
8. B
9. $\frac{3}{5}$
10. $\frac{12}{11}$ or 1.09
11. 65
12. $\frac{2}{3}$ or .666 or .667
13. 2
14. 50
15. 12
16. 36
17. 48
18. 44

Practice Test IIIC

1. C
2. A
3. B
4. C
5. C
6. E
7. C
8. E
9. D
10. C
11. B
12. B
13. C
14. B
15. C
16. D

Analyzing Your Test Results

The charts on the following pages should be used to carefully analyze your results and spot your strengths and weaknesses. The complete process of analyzing each subject area and each individual problem should be completed for each practice test. These results should then be reexamined for trends in types of errors (repeated errors) or poor results in specific subject areas. This reexamination and analysis is of tremendous importance to you in ensuring maximum test preparation benefit.

Mathematics Analysis Sheet

Section A	***Possible***	***Completed***	***Right***	***Wrong***
Multiple Choice	20			
Subtotal	20			
Section B	***Possible***	***Completed***	***Right***	***Wrong***
Multiple Choice	8			
Grid-Ins	10			
Subtotal	18			
Section C	***Possible***	***Completed***	***Right***	***Wrong***
Multiple Choice	16			
Subtotal	16			
Overall Math Totals	54			

Analysis/Tally Sheet for Problems Missed

One of the most important parts of test preparation is analyzing why you missed a problem so that you can reduce the number of mistakes. Now that you have taken the practice test and checked your answers, carefully tally your mistakes by marking them in the proper column.

	Reason for Mistakes				
	Total Missed	***Simple Mistake***	***Misread Problem***	***Lack of Knowledge***	***Lack of Time***
Section A : Math					
Section B : Math					
Section C : Math					
Total Math					

Reviewing the preceding data should help you determine why you are missing certain problems. Now that you've pinpointed the type of error, compare it to other practice tests to spot other common mistakes.

Complete Answers and Explanations for Practice Test III

Practice Test IIIA Explanations

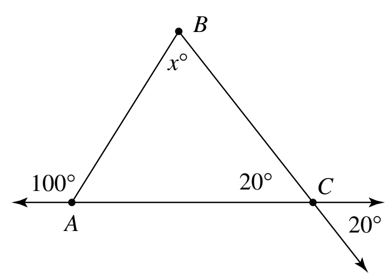
1. D. To solve the equation:

$3x - 5 = 16$	
$3x - 5 + 5 = 16 + 5$	Add 5 to each side to isolate the variable term.
$3x = 21$	
$\frac{3x}{3} = \frac{21}{3}$	Divide each side by 3.
$x = 7$	

2. C. To solve this equation:

$\frac{x+4}{x} = \frac{9}{7}$	
$7(x + 4) = 9x$	Cross multiply to solve the proportion.
$7x + 28 = 9x$	Distribute the 7 on the left side.
$7x + 28 - 7x = 9x - 7x$	Subtract $7x$ from each side to the variable on just one side.
$28 = 2x$	
$\frac{28}{2} = \frac{2x}{2}$	Divide both sides by 2.
$14 = x$	

3. D.



In the figure above, the two angles marked 20° are equal because they are vertical angles.

Then you know that $100 = 20 + x$, since the measure of an exterior angle of a triangle equals the sum of the measures of the two nonadjacent interior angles. So by simple subtraction, you see that $x = 80$.

4. E. You are making two types of choices: style and color of carpet. Using blanks to help solve this problem, you have:

$\frac{}{\text{style}} \quad \frac{}{\text{color}}$

Since you have five styles and 10 colors from which to choose, you can make a style-color choice in the following manner:

$\frac{5}{\text{style}} \cdot \frac{10}{\text{color}}$ Your answer is just the product of 5 and 10, which is 50.

5. E. The translation that moves point C from (0, 0) to (–2, 3) moves the point 2 units left (0 – 2 = –2, the new x-coordinate), and then 3 units up (0 + 3 = 3, the new y-coordinate.). So the point P (–3, 4), after being moved 2 units left (–3 + –2 = –5), and then 3 units up (4 + 3 = 7) will end up at the point (–5, 7).

6. C. To find the value of $f(5)$, you need to substitute 5 for x in the $f(x)$ formula:

$$f(x) = \frac{(2-x)^2}{x} + \frac{3}{5}$$

so $f(5) = \frac{(2-5)^2}{5} + \frac{3}{5} = \frac{(-3)^2}{5} + \frac{3}{5} = \frac{9}{5} + \frac{3}{5} = \frac{12}{5} = 2\frac{2}{5}$.

7. D. You need to compare the temperature change between the two days to the temperature on the first day of the two-day pair.

A. Sunday to Monday: $\frac{5}{20}$

B. Monday to Tuesday: $\frac{15}{25}$

C. Wednesday to Thursday: $\frac{10}{10}$

D. Thursday to Friday: $\frac{25}{20}$

E. Friday to Saturday: $\frac{25}{45}$

Notice that the fractions for choices **A, B, C,** and **E** are all less than or equal to 1, but greater than 1 for Choice **D.** Therefore Choice **D** will have the greatest percent increase in temperature.

8. E. Suppose that the employee was originally paid P dollars.

After a 20% (or $\frac{1}{5}$) pay cut, she will make just $\frac{4}{5}P$ dollars.

After the next 25% (or $\frac{1}{4}$) pay raise, she will make $\frac{5}{4}\left(\frac{4}{5}P\right) = P$ dollars.

Therefore, her new salary after the pay cut and then the pay raise will be exactly what is was before the cut and raise. So she will have 100% of her original salary.

Another way to do the problem is to pick a salary, say $100.

Original salary	100
–20% pay cut	–20
New salary	80
+ 25% pay raise	+20
	100

Final salary, $100, is the same as the original, so it's 100% of original.

9. C. Writing the average of the numbers equals $3x$, you have:

$$\frac{x+6+10+12+5x+20}{6} = 3x$$

$$6\left(\frac{x+6+10+12+5x+20}{6}\right) = 6(3x) \qquad \text{Multiply both sides by 6.}$$

$$x+6+10+12+5x+20 = 18x$$

$$6x+48 = 18x \qquad \text{Combine like terms on the left side.}$$

$$6x+48-6x = 18x-6x \qquad \text{Subtract } 6x \text{ to get all the variables on one side.}$$

$$48 = 12x$$

$$\frac{48}{12} = \frac{12x}{12}$$

$$4 = x$$

Practice Test III

10. D. To solve the system of linear equations, you must first try to eliminate one of the variables, as follows:

$3x - 2y = -12$

$x + 3y = 7$ You will multiply the bottom equation by –3 to make the coefficients of x opposites in the top and bottom equations.

$3x - 2y = -12$

$-3(x + 3y) = -3(7)$

$$\begin{array}{r} 3x - 2y = -12 \\ \underline{-3x - 9y = -21} \end{array}$$ Distribute the –3 through the bottom equation.

$-11y = -33$ This is the sum of the pair of equations in previous step. Notice that the x's cancelled out ($3x + -3x = 0$).

$\frac{-11y}{-11} = \frac{-33}{-11}$ Divide both sides by –11.

$y = 3$

To find the corresponding value of x, substitute $y = 3$ into either of the original equations, and then solve that for x. The original second equation looks like less work:

$x + 3y = 7$

$x + 3(3) = 7$ Put 3 in place of y.

$x + 9 = 7$

$x = -2$

Therefore the point of intersection is (–2, 3).

11. B. Lisa walked a distance of m miles in a time of h hours. So her rate was:

$\frac{m \text{ miles}}{h \text{ hours}}$. To find how many hours it took her to walk a distance of d miles, use the formula $t = \frac{d}{r}$ to get the following:

$$t = \frac{d \text{ miles}}{\frac{m \text{ miles}}{h \text{ hours}}} = (d \text{ miles}) \times \frac{h \text{ hours}}{m \text{ miles}} = \frac{dh}{m} \text{ hours}$$

Notice in the work in the line above that the "miles" labels will cancel out just like numbers, leaving you with just an "hours" label for the final answer.

12. C. Since the ratio of boys to girls is $\frac{3}{4}$, you must have:

3 parts girls + 4 parts boys = 7 parts total. So the number of students in the class must be a multiple of 7. Choice **C,** 33, is NOT a multiple of 7.

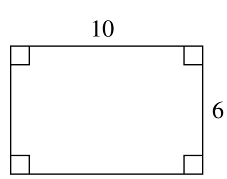
13. E. With a length to width ratio of 5:3 and a length of 10, you can set up and solve the following proportion:

$\frac{5}{3} = \frac{10}{w}$

$5w = 30$ Cross-multiply to solve the proportion.

$w = 6$

Your rectangle now looks like the figure below.



The area of the rectangle above is now $6 \times 10 = 60$

14. D. You solve the following equation for *c,* in terms of *x, b,* and *m.*

$$x = \frac{-b+\sqrt{c}}{m}$$

$$m \cdot x = m \cdot \left(\frac{-b+\sqrt{c}}{m}\right)$$ Multiply both sides of the equation by *m.*

$$mx = -b + \sqrt{c}$$
$$+b \quad +b$$ Add *b* to both sides to isolate the *c* term.

$$mx + b = \sqrt{c}$$

$$(mx+b)^2 = \left(\sqrt{c}\right)^2$$ Square both sides to get rid of the radical.

$$(mx + b)^2 = c$$

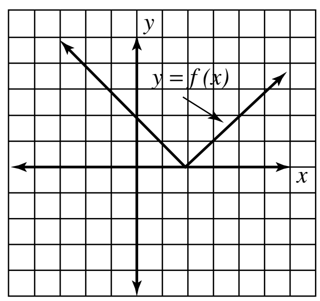
15. B. With a minimum speed of 45 mph and a maximum speed of 65 mph, you can write an inequality that represents the range of legal speed limits *r:*

$45 \le r \le 65$. Notice that half-way between these two numbers is 55.

So each of these rates, 45 and 65, is just 10 units either left of right of 55.

This relationship can be expressed with an absolute value inequality: $|r - 55| \le 10$, which can be translated as: "the distance between *r* and 55 is less than or equal to 10."

16. C.



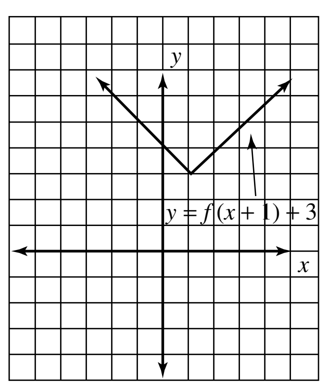
Above is the graph of $y = f(x)$. You want to find the graph of $y = f(x+1) + 3$.

In general, the graphs of any equation $y = f(x)$ and $y = f(x+h) + k$, where *h* and *k* are constants, have exactly the same shape. The only difference is where they are located in the *xy*-plane. The constants *h* and *k* affect the graph of $y = f(x)$ as follows:

i. If $h < 0$, move graph of $y = f(x)$ RIGHT *h* units; if $h > 0$, move graph of $y = f(x)$ LEFT *h* units.

ii. If $k < 0$, move graph of $y = f(x)$ DOWN *k* units; if $k > 0$, move graph of $y = f(x)$ UP *k* units.

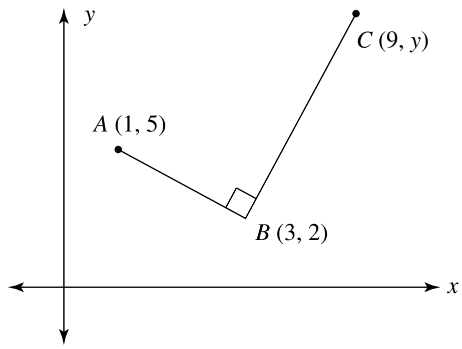
So with $y = f(x + 1) + 3$, you move the graph of $y = f(x)$ 1 unit LEFT and 3 units UP to arrive at the graph below.



Practice Test III

Note that an easy check is to see that the "vertex" was moved left 1 and then up 3 with the graph in the same vertical orientation.

17. **D.**



Since $\overline{AB} \perp \overline{BC}$, you know that their slopes are just opposite reciprocals. First, find the slope of $\overline{AB}$ and use that to help you find the value of *y*.

Slope of $\overline{AB} = \frac{2-5}{3-1} = \frac{-3}{2}$

Then the slope of $\overline{BC}$ has to be $\frac{2}{3}$ (the opposite reciprocal of $\frac{-3}{2}$).

Next, find the slope of $\overline{BC}$ in terms of *y*, set it equal to $\frac{2}{3}$, and then solve for *y*.

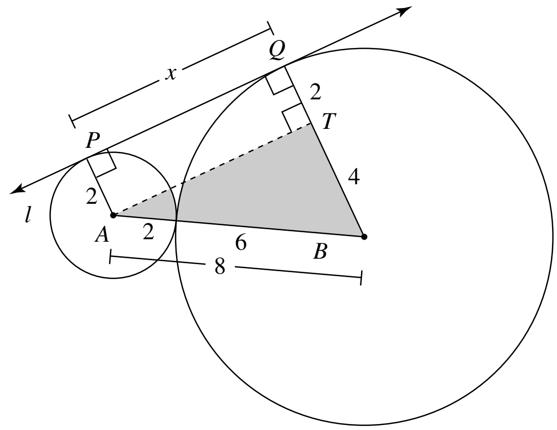
Slope of $\overline{BC} = \frac{y-2}{9-3} = \frac{2}{3}$

$\frac{y-2}{6} = \frac{2}{3}$	
$3(y-2) = 12$	Cross-multiply to solve the proportion.
$3y - 6 = 12$	Distribute the 3 on the left side.
$3y - 6 + 6 = 12 + 6$	Add 6 to each side to isolate the variable.
$3y = 18$	
$\frac{3y}{3} = \frac{18}{3}$	
$y = 6$	

18. **C.** To solve the equation $8^{2x} = 4^{4x-3}$, notice that both bases, 8 and 4, are powers of 2. So you will first change both sides of the equation to the same base, and then solve the resulting equation.

$8^{2x} = 4x^{4x-3}$	
$(2^3)^{2x} = (2^2)^{4x-3}$	Replace 8 with 2^3 and 4 with 2^2.
$2^{6x} = 2^{8x-6}$	On each side, multiply the exponents.
$6x = 8x - 6$	In the line above, since the bases are equal, the exponents are equal.
$6x - 8x = 8x - 6 - 8x$	Subtract 8*x* from each side to get variables on just one side.
$-2x = -6$	
$\frac{-2x}{-2} = \frac{-6}{-2}$	Divide both sides by –2.
$x = 3$	

19. **B.**



In the figure above, radii $\overline{AP}$ and $\overline{QB}$ have been drawn. Each of these is perpendicular to line l since line l is tangent to the circle at points P and Q. Notice that the length of $\overline{AB}$ is 8, just the sum of the two radii 2 and 6. $\overline{AT}$ has been drawn perpendicular to radius $\overline{QB}$ to form a rectangle $PATQ$ and a right triangle ATB. The distance, x, you are looking for is the same as side $\overline{AT}$ in the right triangle. So using the Pythagorean theorem, you have:

in right $\triangle ATB$: $x^2 + 4^2 = 8^2$

$x^2 + 16 = 64$

$x^2 + 16 - 16 = 64 - 16$ Subtract 16 from each side of the equation.

$x^2 = 48$

$x = \sqrt{48}$ Take the square root of both sides.

$x = \sqrt{16 \times 3} = 4\sqrt{3}$

20. **B.** To find the times at which the rocket is 80 feet above the ground, you set $h(t) = 80$ and solve for t.

$80 = -16t^2 + 64t + 32$

$80 - 80 = -16t^2 + 64t + 32 - 80$ Subtract 80 from both sides to get 0 one side.

$0 = -16t^2 + 64t - 48$

$0 = -16(t^2 - 4t + 3)$ Take out common factor of –16.

$0 = -16(t - 3)(t - 1)$ Factor $t^2 - 4t + 3$ into $(t - 3)(t - 1)$

$t - 3 = 0\ t - 1 = 0$ Set each factor equal to 0.

$t = 3, \text{t} = 1$

So the rocket is 80 feet above the ground at 1 second, and then again at 3 seconds. The difference between these times is 3 – 1 = 2.

Practice Test IIIB Explanations

1. **C.** $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}}{3} = \dfrac{\frac{3}{6}+\frac{2}{6}+\frac{1}{6}}{3} = \dfrac{\frac{6}{6}}{3} = \dfrac{1}{3}$

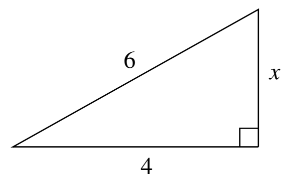
2. **C.** $x = \sqrt{12} + \sqrt{75} = \sqrt{4 \bullet 3} + \sqrt{25 \bullet 3} = 2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

3. **B.** Trying the choices one at a time, with n being even, you have:

(A) $3n + 2 = 3(\text{even}) + 2 = \text{even} + 2 = \text{even}$, so Choice **A** is not correct.

(B) $n^2 + 3 = (\text{even})^2 + 3 = \text{even} + 3 = \text{odd}$, so Choice **B** IS correct.

4. B.



In the right triangle above, the Pythagorean theorem gives us:

$x^2 + 4^2 = 6^2$

$x^2 + 16 = 36$

$x^2 + 16 - 16 = 36 - 16$ Subtract 16 from each side of the equation.

$x^2 = 20$

$x = \sqrt{20}$ Take the square root of each side.

$x = \sqrt{4 \times 5} = 2\sqrt{5}$, or approximately 4.47

5. C. The area of the circle is given as 8π.

Therefore $8\pi = \pi r^2$ Area of a circle is found by the formula πr^2.

$\frac{8\pi}{\pi} = \frac{\pi r^2}{\pi}$ Divide both sides of equation by π.

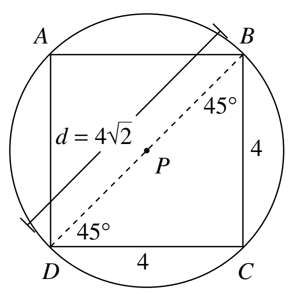
$8 = r^2$

$\sqrt{8} = r$ Take square root of both sides.

$2\sqrt{2} = r$

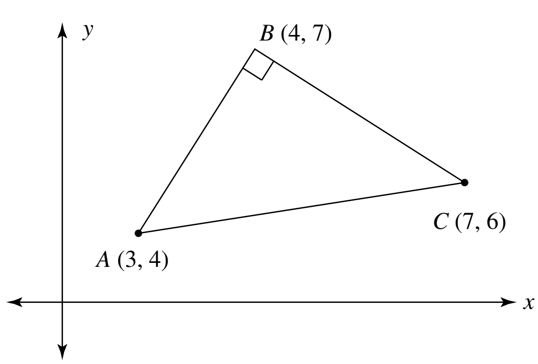
Simplified $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

Since the radius of the circle is $2\sqrt{2}$, its diameter is twice this, or $4\sqrt{2}$.



As you can see in the figure above, $\triangle DBC$ is an isosceles right triangle (45°-45°-90°) with a hypotenuse of $4\sqrt{2}$. So with your $x, x, x\sqrt{2}$ pattern, you know that each side of the square is now just 4. Thus the area of the square is $4 \times 4 = 16$.

6. C.



Since $\overline{AB} \perp \overline{BC}$, you know that the product of their slopes is just –1. So all you have to do is find the slope of $\overline{AC}$, multiply that by –1, and this will give you the product of all three slopes of the sides of $\triangle ABC$.

Slope of $\overline{AC} = \frac{6-4}{7-3} = \frac{2}{4} = \frac{1}{2}$. So the product of all 3 slopes is just $-1 \times \frac{1}{2} = \frac{-1}{2}$.

7. **B.**

i. Longer Method: $(x+m)(x+n) = x^2 - 5x - 36$

$x^2 + nx + mx + mn = x^2 - 5x - 36$	Expand left side (FOIL).
$x^2 + (n+m)x + mn = x^2 - 5x - 36$	Factor 2 middle terms on left.
$x^2 + \underline{(n+m)}\,x + \underline{\underline{mn}} = x^2 \underline{-5x} \underline{\underline{-36}}$	Add underlining for next step.

Since the two polynomial are equal, their respective terms must be equal; specifically, the coefficient of the x terms on the left and right sides of the equation must be equal, and the last constant terms on the left and right sides of the equation must be equal. So you have:

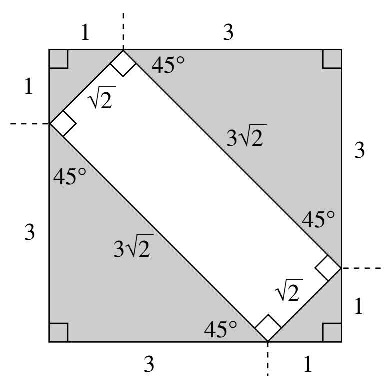
$n + m = -5$ and $mn = -36$

Thus the ratio is $\frac{m+n}{mn} = \frac{-5}{-36} = \frac{5}{36}$.

ii. Shorter Method: If $(x+m)(x+n) = x^2 - 5x - 36$, then you know that $m + n = -5$, and $mn = -36$, since that's how factoring works.

So the ratio is $\frac{m+n}{mn} = \frac{-5}{-36} = \frac{5}{36}$

8. **B.**



In the figure above, each triangle at each corner of the figure is an isosceles right triangle.

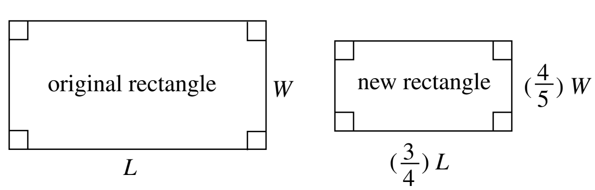
So using your $x,\ x,\ x\sqrt{2}$ pattern, you find the width of the rectangle is $\sqrt{2}$ and the length of the rectangle is $3\sqrt{2}$ as shown in the figure.

Then you have $\text{Area}_{\text{shaded}} = \text{Area}_{\text{square}} - \text{Area}_{\text{rect.}} = 4 \times 4 - \sqrt{2} \times 3\sqrt{2} = 16 - 6 = 10$

9. $\mathbf{\frac{3}{5}}$**.**

Let the length of the original rectangle be L. If you decrease this by 25% (or by $\frac{1}{4}$ of L), you have only $\frac{3}{4}$ remaining.

Let the width of the original rectangle be W. If you decrease this by 20% (or by $\frac{1}{5}$ of W), you have only $\frac{4}{5}$ remaining.



Then you have: $\text{Area}_{\text{original}} = LW$ and $AREA_{new} = \left(\frac{3}{4}L\right)\left(\frac{4}{5}W\right) = \frac{3}{5}LW$

So the area of the new rectangle is just $\frac{3}{5}$ of the area of the original rectangle.

10. $\mathbf{\frac{12}{11}}$ **or 1.09**

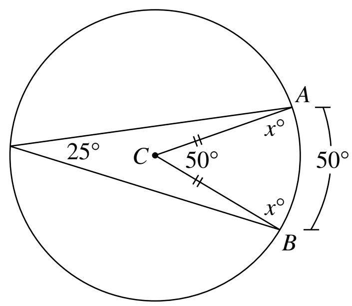
At first glance, it appears that you need to know the value of both x and y to find the value of the expression $\frac{3xy^2}{x^2y^2 - 5y^2}$. But if you do some factoring first, you find that all of the y's will cancel out, as follows:

$$\frac{3xy^2}{x^2y^2 - 5y^2} = \frac{y^2(3x)}{y^2(x^2 - 5)} = \frac{3x}{x^2 - 5}$$

As you can now see, you only need to know that $x = 4$ in order to find the value of this last expression.

$$\frac{3x}{x^2 - 5} = \frac{3 \times 4}{4^2 - 5} = \frac{12}{16 - 5} = \frac{12}{11} \text{ or } 1.09$$

11. 65.



Since the 25° angle is an inscribed angle, its associated arc, $\overset{\frown}{AB}$ is twice that, or 50°.

Since $\angle ACB$ is a central angle, its measure is the same as that of arc $\overset{\frown}{AB}$, which is 50°.

Notice that since C is the center of the circle, segments $\overline{CA}$ and $\overline{CB}$ are both radii of the circle, so their lengths are equal, as marked in the figure. So the angles opposite these two radii are equal, thus the two angles are marked $x°$ in the figure. Then, we have in $\triangle ACB$:

$x + x + 50 = 180$	The sum of the measure of the angles of a triangle is 180.
$2x + 50 = 180$	
$2x + 50 - 50 = 180 - 50$	Subtract 50 from each side of the equation.
$2x = 130$	
$\frac{2x}{2} = \frac{130}{2}$	Divide both sides by 2.
$x = 65$	

12. $\mathbf{\frac{2}{3}}$ **or .666 or .667**

You are asked to find the value of $\frac{m}{x}$ from the following equation:

$\frac{10x+15m}{x+m} = 12$	
$(x+m)\left(\frac{10x+15m}{x+m}\right) = 12(x+m)$	Multiply both sides by $(x + m)$.
$10x + 15m = 12x + 12m$	
$10x + 15m - 10x = 12x + 12m - 10x$	Subtract $10x$ from each side.
$15m = 2x + 12m$	
$15m - 12m = 2x + 12m - 12m$	Subtract $12m$ from both sides.
$3m = 2x$	
$\frac{3m}{x} = \frac{2x}{x}$	Divide both sides by x; we want to get $\frac{m}{x}$ on one side.
$\frac{3m}{x} = 2$	
$\frac{1}{3}\left(\frac{3m}{x}\right) = 2 \times \frac{1}{3}$	Multiply both sides by $\frac{1}{3}$.
$\frac{m}{x} = \frac{2}{3}$	

13. **2.** Your goal is to find the value of $c - r$ from the equation below.

$12(m + n)(c - r) = 72$	
$3 \times 4(m + n)(c - r) = 72$	
$3(m + n) \times 4(c - r) = 72$	Factor the expression on the left this way so you get one factor as $3(m + n)$, whose value you are given.
$9 \times 4(c - r) = 72$	In place of the $3(m + n)$ above, you put a 9.
$36 \times (c - r) = 72$	
$\frac{36 \times (c-r)}{36} = \frac{72}{36}$	Divide both sides by 36.
$c - r = 2$	

14. **50.** Current population is 1,024.

10 minutes ago, the population was $\frac{1024}{2} = 512$. Doubles every 10 minutes into the future.

20 minutes ago, the population was $\frac{512}{2} = 256$.

30 minutes ago, the population was $\frac{256}{2} = 128$.

40 minutes ago, the population was $\frac{128}{2} = 64$.

50 minutes ago, the population was $\frac{64}{2} = 32$.

15. **12.** There are a couple of ways to approach this problem—one shorter and one a bit longer.

i. Shorter Method: For any problem with consecutive integers and consecutive odd/even integers, having an odd number of integers (as in this case, seven integers), the median is always the middle number in the list of the consecutive integers. In your case, with 23 being the median, the list of consecutive odd integers is: 17, 19, 21, 23, 25, 27, and 29. So the difference between the largest and smallest of these is 29 – 17 = 12.

ii. Longer Method: Let n = first odd integer; then the next 6 consecutive odd integers after this will be $n + 2$, $n + 4$, $n + 6$, $n + 8$, $n + 10$, and $n + 12$. Since the median of the list of the seven terms is $n + 6$ and you are told that this is 23, you have $n + 6 = 23$, so $n = 17$. Then your seven consecutive odd integers are 17, 19, 21, 23, 25, 27, and 29. The difference between the largest and smallest is $29 - 17 = 12$.

16. **36.** You are using the five digits 2, 3, 4, 5, and 6 to create five-digit numbers beginning and ending with an even digit. So the first and last digits must be with a 2, 4, or a 6. If you use a series of blanks to indicate the choices to be made in creating your five-digit numbers, you have $\frac{3}{\text{even}}$ — — — $\frac{2}{\text{even}}$.

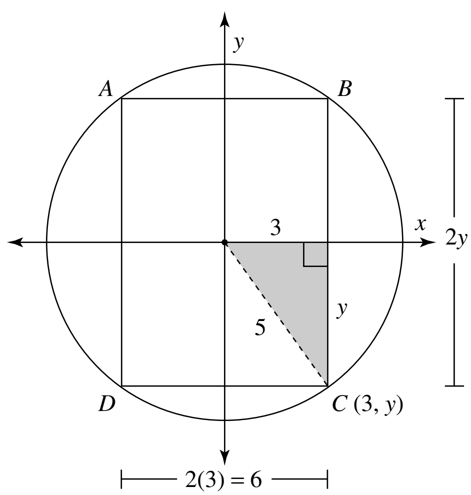
You can choose the first digit in any one of three ways, (2, 4, 6), and once you choose one of these even numbers, you can choose the last digit in only one of two ways—the other two remaining even digits.

The digits in the middle can then be any one of three remaining digits (since you have already picked two), then two of the remaining digits, then the one and only one remaining digit.

$\frac{3}{\text{even}}$ $\underline{3}$ $\underline{2}$ $\underline{1}$ $\frac{2}{\text{even}}$

Our answer is just the product of these numbers in the blanks, so $3 \times 3 \times 2 \times 1 \times 2 = 36$ different five-digit numbers.

17. **48.**



Since the circle has a radius of 5, the length of the hypotenuse of the shaded right triangle in the figure above is labeled as 5. Point C has coordinates $(3, y)$, so the horizontal leg of the right triangle is 3 and its vertical leg y. This is just a 3-4-5 right triangle, so the y must be 4. Notice from the figure that the base of rectangle $ABCD$ is just 6; its height is $2y$, and since $y = 4$, the height is 8. Therefore the area of the rectangle is $6 \times 8 = 48$.

18. **44.** The average of the five different two-digit positive integers is 20. If your integers are $a, b, c, d,$ and $e,$ then you can write:

$$\frac{a+b+c+d+e}{5} = 20$$

$$5\left(\frac{a+b+c+d+e}{5}\right) = 5 \times 20 \qquad \text{Multiply both sides by 5.}$$

$$a + b + c + d + e = 100$$

If $a = 10$, $b = 11$, $c = 12$, and $d = 13$ were the values of the 4 smallest two-digit positive integers, you would then have:

$$10 + 11 + 12 + 13 + e = 100$$

$$46 + e = 100$$

$$e = 54$$

The difference between the largest and the smallest would be $54 - 10 = 44$

Practice Test IIIC Explanations

1. **C.** You can write the expression $2m - 2n - 7$ as $2(m - n) - 7$ by factoring a 2 out of the first 2 terms.

Then, $2(m - n) - 7$

$= 2(11) - 7$ You were told that $m - n = 11$ in the problem.

$= 22 - 7$

$= 15$

2. **A.** To solve the proportion:

$\frac{2m}{3r} = \frac{8}{5}$

$\frac{3}{2} \times \frac{2m}{3r} = \frac{3}{2} \times \frac{8}{5}$ Multiply by $\frac{3}{2}$ (the reciprocal of $\frac{2}{3}$) to get $\frac{m}{r}$ by itself.

$\frac{m}{r} = \frac{12}{5}$ On the right, reduce the product $\frac{24}{10}$ to $\frac{12}{5}$.

$\frac{r}{m} = \frac{5}{12}$ Take the reciprocal of both sides of the equation.

3. **B.** The perimeter of $\triangle ABC$ is $3 + 6 + 7 = 16$. Since the triangle and square have equal perimeters, the perimeter of square $MNOP$ is also 16. So the length of side $\overline{MP}$ is $\frac{16}{4} = 4$.

4. **C.** To find how much larger one quantity is than another, you just find their difference. So $(5x - 7) - (-10 + 5x) = 5x - 7 + 10 - 5x = 3$.

5. **C.** Since $0 < x < y < 1$, use some convenient fractions for x and y (they are both between 0 and 1). So suppose you let $x = \frac{1}{4}$ and $y = \frac{1}{2}$. Then trying each of the choices, you have:

A. $x - y = \frac{1}{4} - \frac{1}{2}$, which is negative since $\frac{1}{2}$ is greater than $\frac{1}{4}$.

B. $x^2 - y^2 = \left(\frac{1}{4}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{1}{16} - \frac{1}{4}$, which is also negative since $\frac{1}{4}$ greater than $\frac{1}{16}$.

C. $\frac{1}{x} - \frac{1}{y} = \frac{1}{\frac{1}{4}} - \frac{1}{\frac{1}{2}} = 4 - 2 = 2$, which is positive, so Choice **C** is correct.

6. **E.** If you let the number be n, you have:

a given number decreased by 5 $\rightarrow n - 5$

the result is tripled $\rightarrow 3(n - 5)$

the number obtained is 24 $\rightarrow 3(n - 5) = 24$

$3n - 15 = 24$ Distribute 3 on left.

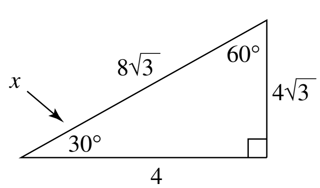
$3n - 15 + 15 = 24 + 15$ Add 15 to each side.

$3n = 39$

$\frac{3n}{3} = \frac{39}{3}$ Divide by 3.

$n = 13$

7. **C.**



Since $\overline{AB} \perp \overline{BC}$ and $x = 30°$, you know that $\triangle ABC$ is a 30°-60°-90° triangle. With $AC = 8\sqrt{3}$ and using your x, $x\sqrt{3}$, $2x$ pattern, you can find that $BC = 4\sqrt{3}$. Then $AB = \left(4\sqrt{3}\right) \times \sqrt{3} = 12$.

The area is $\Delta ABC = \frac{1}{2}bh = \frac{1}{2} \times 12 \times 4\sqrt{3} = 24\sqrt{3}$.

8. **E.** Knowing the area, you can find the radius of the circle; from that you can find the circumference.

$\text{Area} = \pi r^2$

$\frac{9\pi}{4} = \pi r^2$

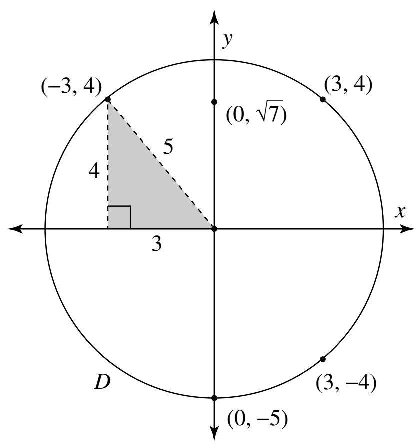
$\frac{1}{\pi}\left(\frac{9\pi}{4}\right) = \frac{1}{\pi}\left(\pi r^2\right)$ Multiply both sides by $\frac{1}{\pi}$.

$\frac{9}{4} = r^2$

$\frac{3}{2} = r$ Find the square root of both sides.

The circumference $= 2\pi r = 2\pi \times \frac{3}{2} = 3\pi$.

9. **D.**



In the figure above, the circle with center at the origin contains the point (–3, 4). Using your 3-4-5 right triangle pattern, you can see that the circle's radius is just 5. From the symmetry of the circle, you can tell that the points (3, –4) and (3, 4) are also on the same circle. Since the circle's radius is 5, point (0, –5) is on the circle also. But point (0, $\sqrt{7}$) is between 2 and 3 units from the circle's center, so this point is not on the circle.

10. **C.** Since $0 < x < 1$, select a convenient value of for x, say $x = \frac{1}{64}$, since you can easily find both its cube and square roots.

Then you have: $\sqrt[3]{x} = \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$

$\sqrt{x} = \sqrt{\frac{1}{64}} = \frac{1}{8}$, and then, $\frac{1}{x} = \frac{1}{\frac{1}{64}} = 64$.

Since $\frac{1}{8} < \frac{1}{4} < 64$, the correct order is Choice **C:** $\sqrt{x} < \sqrt[3]{x} < \frac{1}{x}$

11. **B.** Since the point ($\sqrt{7}$, 5) is on the graph of $y = x^2 + k$, you should get a true statement when substituting the point into the equation:

$y = x^2 + k$

$5 = \left(\sqrt{7}\right)^2 + k$ Replace y with 5 and x with $\sqrt{7}$.

$5 = 7 + k$

$-2 = k$

Therefore, $y = x^2 - 2$

$7 = x^2 - 2$	Replace y with 7.
$9 = x^2$	Add 2 to both sides of the equation.
$3 = x$	Take the square root of both sides.

12. **B.** To solve the equation for x, you have:

$x^{3/4} = y^{3/8}$

$\left(x^{3/4}\right)^{4/3} = \left(y^{3/8}\right)^{4/3}$	Take the $\frac{4}{3}$ power of both sides to get x to just the first power.
$x = y^{1/2} = \sqrt{y}$	Multiply the powers on each side, and then simplify.

13. **C.** If $\sqrt{xy} = 6$, then by squaring each side, you get $xy = 36$.

So you are looking for factors of 36 (their product is 36) whose sum CANNOT be one of the numbers in choices **A** through **E.**

The factor pairs of 36 and their respective sums are as follows:

1, 36: 1 + 36 = 37

2, 18: 2 + 18 = 20

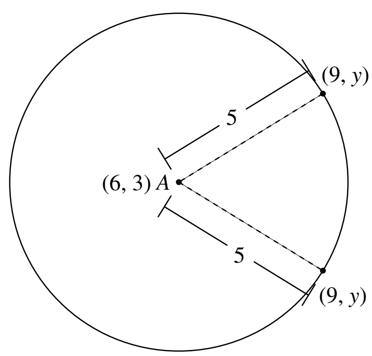
3, 12: 3 + 12 = 15

4, 9: 4 + 9 = 13

6, 6: 6 + 6 = 12

So the sum you CANNOT have is 18, Choice **C.**

14. **B.** In the xy-plane, there are two points that could be 5 units from the point A (6, 3) and have x-coordinate 9.



If you let your desired point be P, it would then have coordinates (9, y). Using the distance formula, you have:

$AP = 5 = \sqrt{(9-6)^2 + (y-3)^2}$	Find the distance between (9, y) and (6, 3).
$5 = \sqrt{9 + (y-3)^2}$	
$25 = 9 + (y - 3)^2$	Square both sides of the previous equation.
$25 = 9 + y^2 - 6y + 9$	Expand $(y - 3)^2 = (y - 3)(y - 3)$, and then use FOIL.
$25 = y^2 - 6y + 18$	Add the 9 + 9 on the right.
$25 - 25 = y^2 - 6y + 18 - 25$	Subtract 25 from both sides to set the equation equal to 0.
$0 = y^2 - 6y - 7$	
$0 = (y - 7)(y + 1)$	Factor the right-hand side.
$y - 7 = 0,\ y + 1 = 0$	Set each factor equal to 0.
$y = 7,\ y = -1$	

Therefore, the points are (9, 7) and (9, –1).

15. C. Using the given equation, substitute values of t until you arrive at the desired amount of money.

$A(t) = 1{,}000(1.06)^t$

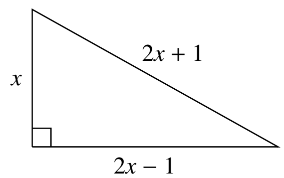
A. $t = 1$: $A(1) = 1{,}000(1.06) = 1{,}060$

B. $t = 2$: $A(1) = 1{,}000(1.06)^2 = 1{,}123.60$

C. $t = 3$: $A(1) = 1{,}000(1.06)^3 = 1{,}191.02$, so Choice **C** is correct.

NOTE: This is one of the few problems where you really need to have a calculator to do the problem quickly.

16. D. With x = length of short leg of right triangle, then $2x - 1$ = length of longer leg and $2x + 1$ = length of hypotenuse.



Referring to the figure above, you can then use the Pythagorean theorem to create the equation:

$(\text{short leg})^2 + (\text{long leg})^2 = (\text{hypotenuse})^2$

Substituting the appropriate pieces in terms of the variable x, you have:

$x^2 + (2x - 1)^2 = (2x + 1)^2$, which is Choice **D.**

SAT I Score Range Approximator

The following charts are designed to give you only a very approximate score range, not an exact score. When you take the actual new SAT I, you will see questions similar to those in this book; however, some questions may be slightly easier or more difficult. Needless to say, this may affect your scoring range.

How to Approximate Your Score in Mathematics

1. Add the total number of correct responses for the three Mathematics sections.
2. Add the total number of incorrect responses for the multiple-choice questions only.
3. The total number of incorrect responses for the multiple-choice questions should be divided by 4, giving you an adjustment factor (round off to the nearest whole number).
4. Subtract this adjustment factor from the total number of correct responses to obtain a raw score.
5. This raw score is then scaled to a range from 200 to 800.

   Example:

   If the total number of correct answers is 30 out of a possible 45
   and 16 multiple-choice problems were attempted but missed,
   dividing 16 by 4 gives an adjustment factor of 4.
   Subtracting this adjustment factor of 4 from the original 30 correct gives a raw score of 24.
   This raw score is then scaled to a range from 200 to 800.

   Note: No deduction is made for incorrect grid-in responses.
6. Using your scores:

   ________________ – ______________________________ = ____________________

   total correct answers — wrong answers on multiple choice ÷ 4 — raw score
7. Use the following table to match your raw score for Mathematics and the corresponding approximate score range:

Raw Score	Approximate Score Range
49–55	710–800
41–49	640–700
26–40	500–630
11–25	380–490
5–10	310–370
1–4	240–300
4–0	200–230

Keep in mind that this is only an *approximate* score range.