Chapter 4
Oscillations

Worksheet

Worked examples

Practical 1: Simple harmonic motion of a tethered trolley

Practical 2: Investigating resonance

End-of-chapter test

Marking scheme: Worksheet

Marking scheme: End-of-chapter test

Worksheet

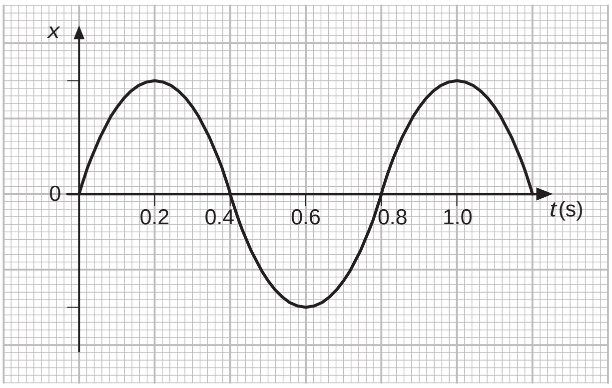
Intermediate level

1 For an oscillating mass, define:

a the period; [1]

b the frequency. [1]

2 The graph of displacement against time for an object executing simple harmonic motion (s.h.m.) is shown here.



a Suggest a time at which the object has maximum speed. Explain your answer. [2]

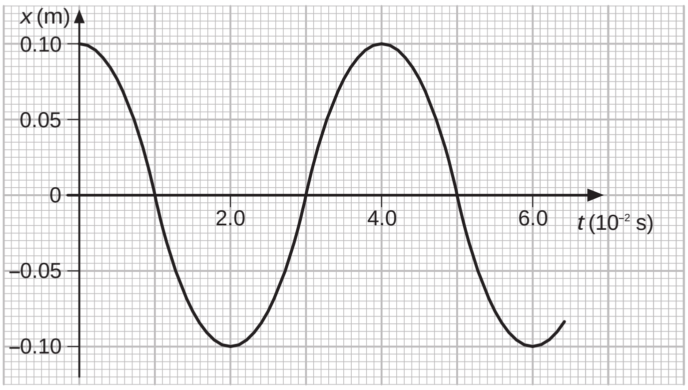
b Suggest a time at which the magnitude of the object's acceleration is a maximum. Explain your answer. [2]

3 An apple is hung vertically from a length of string to form a simple pendulum. The apple is pulled to one side and then released. It executes 12 oscillations in a time of 13.2 s. For this oscillating apple, calculate:

a its period; [2]

b its frequency. [2]

4 The diagram shows the displacement against time graph for an oscillating object.



Use the graph to determine the following:

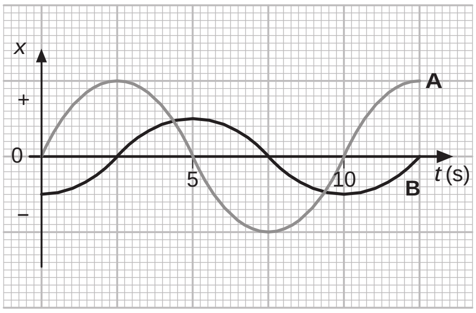
a the amplitude of the oscillation; [1]

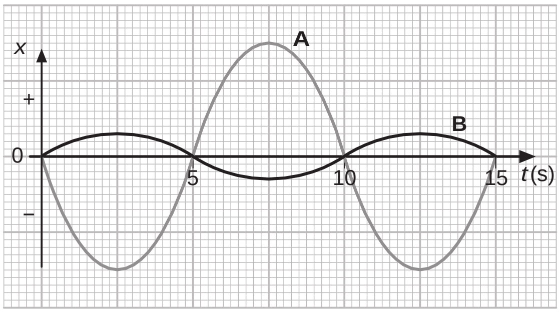
b the period; [1]

c the frequency in hertz (Hz); [2]

d the angular frequency in radians per second (rad s^{-1}). [2]

5 Two objects **A** and **B** have the same period of oscillation. In each case **a** and **b** below, determine the phase difference between the motions of the objects **A** and **B**.





a [2] **b** [2]

6 A mass at the end of a spring oscillates with a period of 2.8 s. The maximum displacement of the mass from its equilibrium position is 16 cm.

a What is the amplitude of the oscillations? [1]

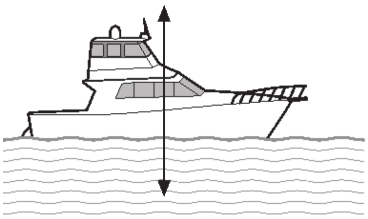
b For this oscillating mass, calculate:

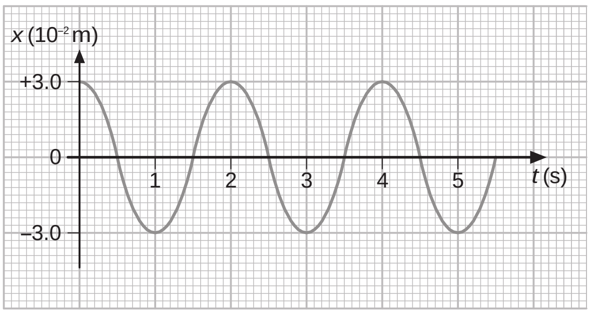
i its angular frequency; [2]

ii its maximum acceleration. [3]

Higher level

7 A small toy boat is floating on the water's surface. It is gently pushed down and then released. The toy executes simple harmonic motion. Its displacement against time graph is shown here.





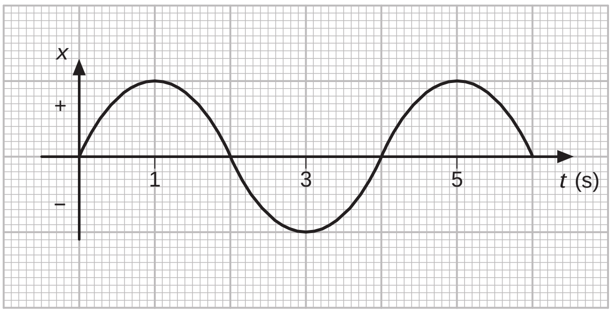
For this oscillating toy boat, calculate:

a its angular frequency; [2]

b its maximum acceleration; [3]

c its displacement after a time of 6.7 s, assuming that the effect of damping on the boat is negligible. [3]

8 The diagram shows the displacement–time graph for a particle executing simple harmonic motion.



Sketch the following graphs for the oscillating particle:

a velocity–time graph; [2]

b acceleration–time graph; [2]

c kinetic energy–time graph; [2]

d potential energy–time graph. [2]

9 A piston in a car engine executes simple harmonic motion. The acceleration a of the piston is related to its displacement x by:

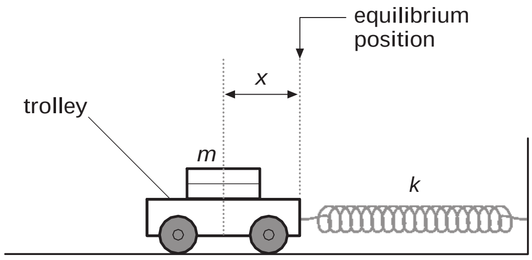
$a = -6.4 \quad 10^5 x$

a Calculate the frequency of the motion. [3]

b The piston has a mass of 700 g and a maximum displacement of 8.0 cm. Calculate the maximum force on the piston. [2]

Extension

10 The diagram shows a trolley of mass m attached to a spring of force constant k. When the trolley is displaced to one side and then released, the trolley executes simple harmonic motion.



a Show that the acceleration a of the trolley is given by:

$$a = -\left(\frac{k}{m}\right)x$$

where x is the displacement of the trolley from its equilibrium position. [3]

b Use the expression in **a** to show that the frequency f of the motion is given by:

$$f = \frac{1}{2}\sqrt{\frac{k}{m}}$$ [2]

c The springs in a car's suspension act in a similar way to the springs on the trolley. For a car of mass 850 kg, the natural frequency of oscillation is 0.40 s. Determine the force constant k of the car's suspension. [3]

Total: $\frac{}{55}$ Score: %

Worked examples

Example 1

A loudspeaker cone oscillates with simple harmonic motion at a frequency of 2.0 kHz. Calculate the period and the angular frequency of these oscillations.

$f = \frac{1}{T}$ or $T = \frac{1}{f}$

$T = \frac{1}{2000} = 5.0 \quad 10^{-4}$ s

Do not forget to convert frequency into hertz.

$\rceil = \frac{2}{T} = 2 \ f$

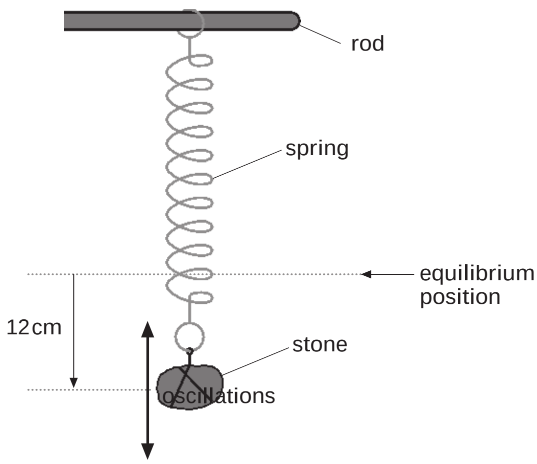
$\rceil = 2 \quad 2000 = 1.26 \quad 10^{4}$ rad s^{-1} $\quad 1.3 \quad 10^{4}$ rad s^{-1}

Tip

It is easy to confuse *frequency* and *angular frequency*. Frequency is measured in hertz and angular frequency is measured in radians per second.

Example 2

A stone is attached to one end of a long spring. The other end of the spring is fixed to a metal rod. The stone is displaced a distance of 12 cm from its equilibrium position and then released (see diagram).



In a time of 45 s, the stone executes 30 oscillations. Calculate the maximum acceleration of the oscillating stone and determine its displacement after a time of 2.0 s.

The period T is given by:

$T = \frac{45}{30} = 1.5$ s

The acceleration of the stone is given by:

$a = -(2 \ f)^2 x$ or $a = -\left(\frac{2}{T}\right)^2 x$

The acceleration is a maximum when the displacement x is equal to the amplitude A of the motion, which is 12 cm. Therefore the magnitude of the maximum acceleration is:

$a = \left(\frac{2}{1.5}\right)^2 \quad 12 \quad 10^{-2} = 2.11$ m s^{-2} $\quad 2.1$ m s^{-2}

The displacement x of the stone after 2.0 s is given by:

$x = A\cos(2 \ ft)$

The stone is released from its maximum displacement at t = 0. Hence the displacement x must be given by the cosine relationship.

$x = 12 \quad 10^{-2} \quad \cos\left(2 \quad \frac{1}{1.5} \quad 2.0\right) = -0.06$ m

After 2 s the stone is displaced a distance of 6 cm **above** the equilibrium position.

Tips

- It is vital that you work in radians when calculating the displacement x.
- Read the question carefully to determine whether the displacement is given by $x = A\cos(2 \ ft)$ or $x = A\sin(2 \ ft)$.

Practical 1

Simple harmonic motion of a tethered trolley

Safety

There are not likely to be any hazards in carrying out this experiment. However, teachers and technicians should always refer to the departmental risk assessment before carrying out any practical work.

Apparatus

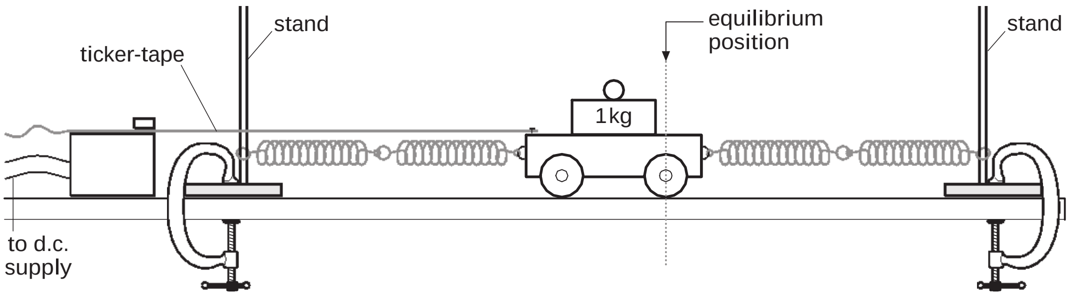
- ticker-timer
- supply for ticker-timer
- trolley
- 1 kg mass
- clamp stands
- four helical springs
- metre rule
- stopwatch
- connecting leads

Introduction

The displacement *x* of a simple harmonic oscillator released from its maximum displacement at time $t = 0$ is given by:

$x = A\cos(2\ ft)$

The equations of s.h.m. are given on page 41 of *Physics 2*. In this experiment you will measure the displacement of a tethered trolley using ticker-tape and compare it with the equation above. The trolley is secured to the springs as shown in the diagram.



Procedure

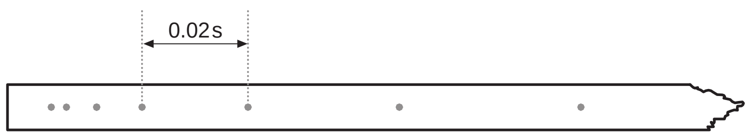
1 Secure a length of ticker-tape to the trolley with some adhesive tape.

2 Place a 1.0 kg mass on the trolley.

3 Pull the trolley to a distance of 15 cm from its equilibrium position.

4 Start the ticker-timer and release the trolley.

5 Use the dots on the ticker-tape to measure the displacement of the trolley at times 0 s, 0.02 s, 0.04 s, 0.06 s, etc. (Remember that the time interval between successive dots on the ticker-tape is 0.02 s.)



6 Record your results for time *t* and the displacement *x* in a table.

7 Use your table to plot a graph of *x* against *t*.

8 Use the graph to determine the time taken for one quarter of a period. Use this to determine the frequency *f* of the trolley.

Hint: $f = \frac{1}{T}$

9 The displacement *x* should obey a cosine rule as given by:

$x = A\cos(2\ \ ft)$

where *A* in this experiment is equal to 15 cm.

10 For the first quarter of the oscillation, does your graph show this to be the case?

Guidance for teachers

For the first quarter of an oscillation, the energy losses are small enough for there to be an agreement within ±10% between the experimental results and the theory.

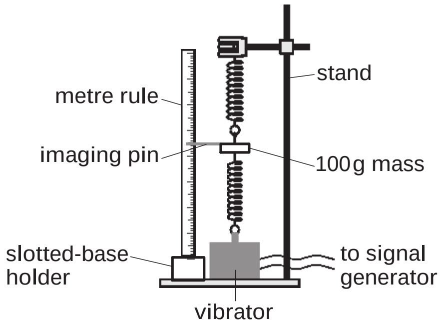
Practical 2

Investigating resonance

Safety

There are not likely to be any hazards in carrying out this experiment. However, teachers and technicians should always refer to the departmental risk assessment before carrying out any practical work.

Apparatus



- two helical springs
- stopwatch
- metre rule
- signal generator
- imaging pin
- 100 g mass
- slotted base holder
- clamp stand
- cardboard disc of radius 15 cm
- connecting leads

Introduction

The phenomenon of resonance is described on pages 45 to 48 of *Physics 2*. In this experiment you will investigate how the amplitude of the motion of a mass attached to some springs is affected by the forcing frequency, and whether a small amount of damping has any significant effect on the resonant frequency.

The experiment is set up as shown in the diagram.

Procedure

1 With the signal generator switched off, displace the 100 g mass and then release it. Measure the time taken for 10 oscillations. Calculate the natural frequency f_0 of the oscillator.

2 Switch on the signal generator. Note the output voltage; you must keep this constant throughout the experiment.

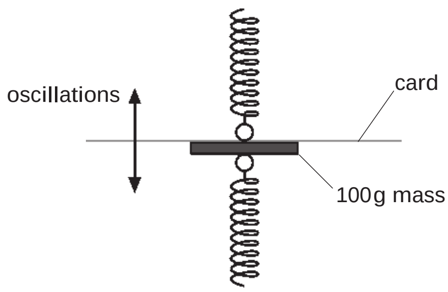
3 Measure the amplitude of the oscillations at a frequency of 0.2 Hz. Repeat for frequencies in steps of 0.2 Hz up to 5.0 Hz.

4 Record your results in a table.

5 Plot a graph of amplitude against forcing frequency.

6 Use your graph to determine the resonant frequency. How does this compare with your value for the natural frequency f_0?

7 Repeat this experiment with the cardboard disc fixed onto the mass. Does the damping introduced by the cardboard affect the resonant frequency of the oscillator?



Guidance for teachers

The natural frequency f of oscillations for a mass m oscillating at the end of a spring of force constant k is given by:

$$f=\frac{1}{2}\sqrt{\frac{k}{m}}$$

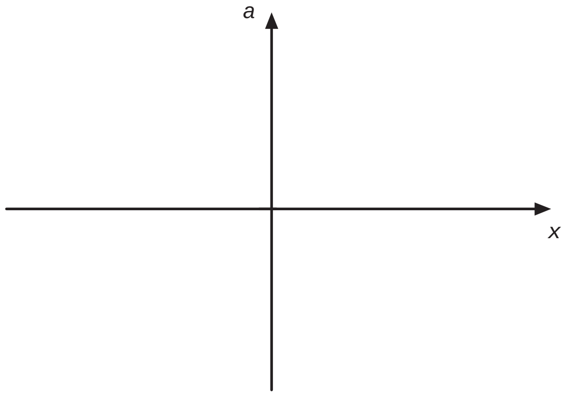
End-of-chapter test

Answer all questions.

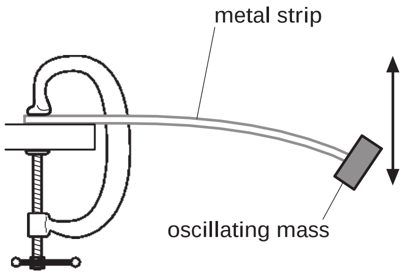
1 **a** Define the amplitude of motion of an oscillator. [1]

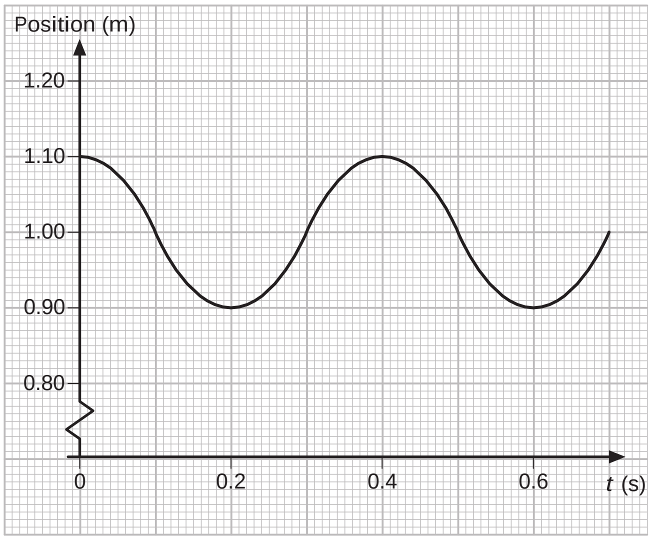
b Define simple harmonic motion. [2]

c On the axes opposite, sketch a graph of acceleration *a* against displacement *x* for an object executing simple harmonic motion. [2]



2 A metal strip is clamped to the edge of a table and a mass is attached to its free end. The mass is gently pushed down and then released. A graph of the position of the mass measured from the floor against time is shown below.





a On the graph above, mark with a letter **X** the point at which the oscillating mass has maximum speed. [1]

b Use the graph of position against time to determine the maximum speed of the mass. [3]

c For the oscillating mass, determine:

i its frequency; [2]

ii its angular frequency. [2]

3 The atoms in a solid may be assumed to vibrate with simple harmonic motion.

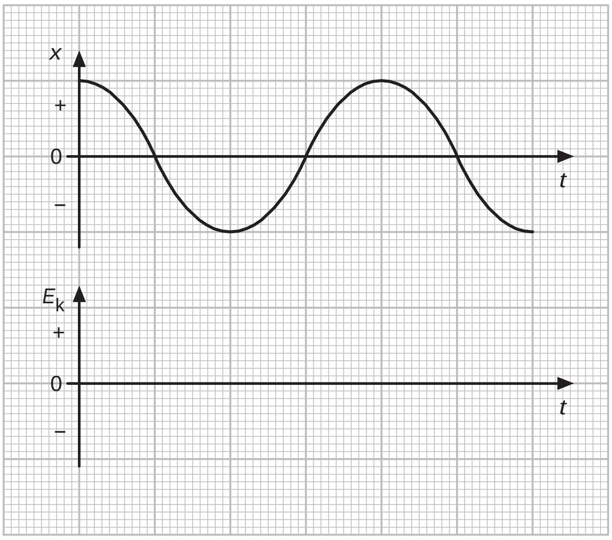
a For a single atom oscillating at a frequency *f*, write an equation for the acceleration *a* in terms of its displacement *x* from its equilibrium position. [1]

b For a single oscillating atom, the amplitude of the motion is 1.2 10^{-11} m and the frequency is 2.0 10^{14} Hz. Calculate:

i the maximum acceleration of the atom; [2]

ii the maximum force acting on the atom given that its mass is 1.1 10^{-25} kg. [2]

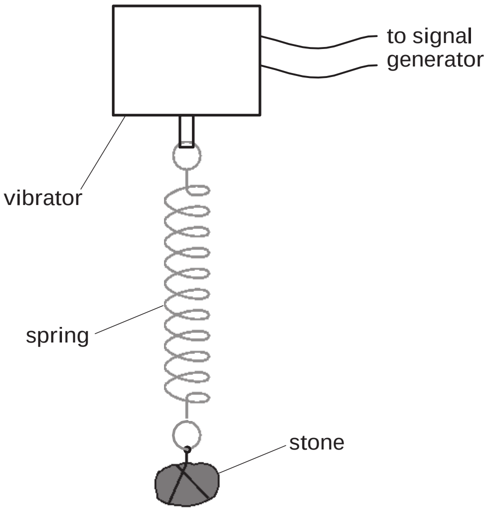
c The displacement–time graph for the atom is shown below.



Show the corresponding variation with time of the kinetic energy E_k of the oscillating atom. [2]

4 A stone is attached to the end of a spring. The other end of the spring is fixed to a mechanical vibrator.

With the mechanical vibrator switched off, the stone is displaced from its equilibrium position and then released. The stone executes 25 oscillations in 1 minute.



The mechanical vibrator is switched on and the amplitude A of the motion of the stone is measured for a range of values of the forcing frequency f.

On the axes above, show how the amplitude A of the motion of the stone is affected by the frequency f of the vibrator. [3]

Total: $\frac{}{23}$ Score: %

Marking scheme

Worksheet

1 **a** The period of an oscillator is the time for one complete oscillation. [1]

b The frequency of an oscillator is the number of oscillations completed per unit time (or per second). [1]

2 **a** The gradient of a displacement–time graph is equal to velocity. [1]

The magnitude of the velocity (speed) is a maximum at 0 s or 0.4 s or 0.8 s. [1]

b For s.h.m., acceleration $\propto$ –displacement.

The magnitude of the acceleration is maximum when the displacement is equal to the amplitude of the motion. [1]

The magnitude of the acceleration is a maximum at 0.2 s or 0.6 s or 1.0 s. [1]

3 **a** $T = \frac{13.2}{12}$ [1]; $T = 1.1$ s [1]

b $f = \frac{1}{T} = \frac{1}{1.1}$ [1]; $f = 0.909$ Hz   0.91 Hz [1]

4 **a** Amplitude = 0.10 m [1]

b Period = 4.0 10^{-2} s [1]

c $f = \frac{1}{T} = \frac{1}{0.04}$ [1]; $f = 25$ Hz [1]

d $\rceil = 2\ \ f = 2\ \ \ \ 25$ [1]; $\rceil = 157$ rad s^{-1} 160 rad s^{-1} [1]

5 **a** Phase difference = $2\ \ \left(\frac{t}{T}\right)$

where T is the period and t is the time lag between the motions of the two objects.

$T = 10$ s and $t = 2.5$ s

phase difference = $2\ \ \left(\frac{t}{T}\right) = 2\ \ \left(\frac{2.5}{10}\right)$ [1]

phase difference = $\frac{\ }{2}$ rad   1.6 rad [1]

b $T = 10$ s and $t = 5.0$ s

phase difference = $2\ \ \left(\frac{t}{T}\right) = 2\ \ \left(\frac{5.0}{10}\right)$ [1]

phase difference =   rad   3.1 rad [1]

6 **a** $A = 16$ cm [1]

b **i** $\rceil = 2\ \ f = \frac{2}{T} = \frac{2}{2.8}$ [1]; $\rceil = 2.24$ rad s^{-1} 2.2 rad s^{-1} [1]

ii $a = (2\ \ f)^2 x$ (magnitude only) [1]

For maximum acceleration, the displacement x must be 16 cm.

$a = \left(2\ \ \ \frac{1}{2.8}\right)^2\ \ 16\ \ \ 10^{-2}$ [1]

$a = 0.806$ m s^{-2} 0.81 m s^{-2} [1]

7 **a** $\rceil = 2\ \ f = \frac{2}{T} = \frac{2}{2.0}$ [1]; $\rceil = 3.14\ \text{rad s}^{-1}\ \ 3.1\ \ \text{rad s}^{-1}$ [1]

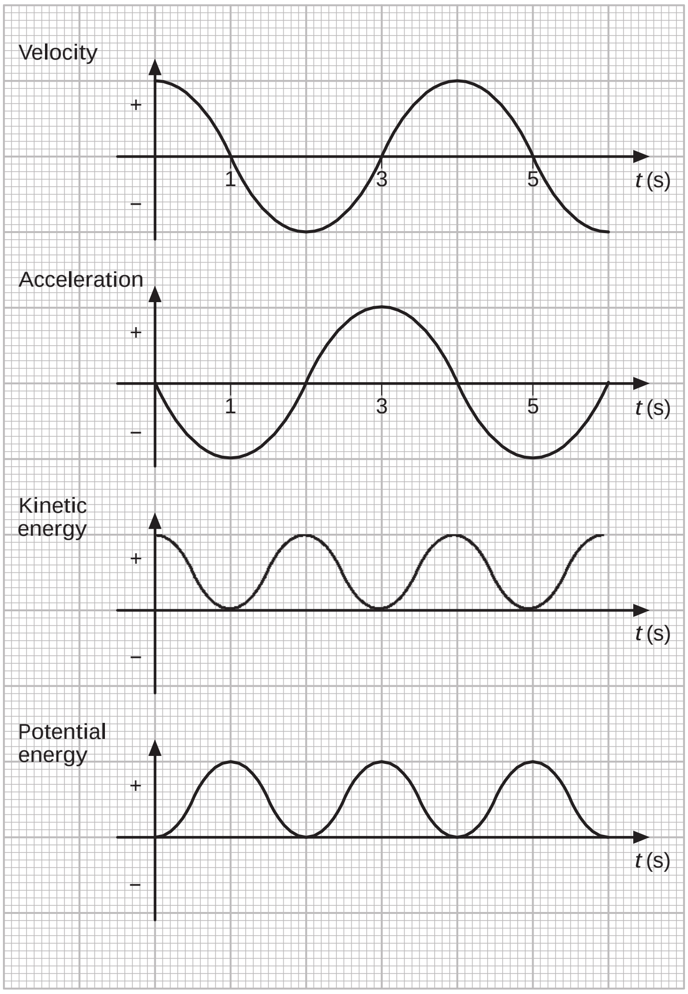
b $a = -(2\ \ f)^2 x$ or $a = -\rceil^2 x$ [1]

$a = 3.14^2\ \ 3.0\ \ 10^{-2}\ \text{m s}^{-2}$ [1]; $a\ \ 0.30\ \text{m s}^{-2}$ [1]

c $x = A\cos(2\ \ ft) = A\cos(\rceil t)$ [1]

$x = 3.0\ \ 10^{-2}\cos(3.14\ \ 6.7)$ [1]; $x\ \ -1.7\ \ 10^{-2}\ \text{m}$ [1]

8



a Gradient from *x*–*t* graph = velocity [2]

b Gradient from *v*–*t* graph = acceleration

Also, for s.h.m. acceleration ∝ – displacement [2]

c Kinetic energy $= \frac{1}{2}mv^2 \propto v^2$ [2]

d Potential energy = total energy – kinetic energy [2]

9 **a** $a = -(2\ \ f)^2 x$ [1]

Therefore $(2\ \ f)^2 = 6.4\ \ 10^5$ [1]

$f = \frac{\sqrt{6.4\ \ 10^5}}{2} = 127\ \text{Hz}\ \ 130\ \text{Hz}$ [1]

b $F = ma$

Acceleration is maximum at maximum displacement, so magnitude of maximum force is given by:

$F = ma = 0.700\ \ (6.4\ \ 10^5\ \ 0.08)$ [1]; $F\ \ 3.6\ \ 10^4\ \text{N}$ [1]

10 **a** According to Hooke's law: $F = -kx$ [1]

(The minus sign shows that the force is directed towards the equilibrium position.)

From Newton's second law: $F = ma$ [1]

Equating, we have: $ma = -kx$ [1]

Hence: $a = -\left(\frac{k}{m}\right)x$

b For s.h.m. we have: $a = -(2\ \ f)^2 x$ [1]

Hence: $(2\ \ f)^2 = \frac{k}{m}$ [1]

Therefore: $f = \frac{1}{2\ \ }\sqrt{\frac{k}{m}}$

c $f = \frac{1}{T} = \frac{1}{0.4} = 2.5\,\text{Hz}$ [1]

$2.5 = \frac{1}{2\ \ }\sqrt{\frac{k}{850}}$ [1]

$k = (2\ \ 2\ \ .5)^2\ \ 850\ \ 2.1\ \ 10^5\ \text{N m}^{-1}$ [1]

Marking scheme

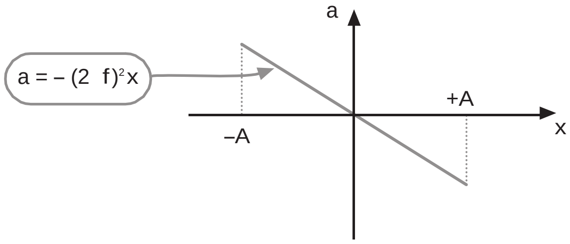
End-of-chapter test

1 **a** The amplitude of the motion of an oscillator is the maximum displacement from its equilibrium position. [1]

b For simple harmonic motion, the acceleration of the oscillator is directly proportional to its displacement from a fixed point [1]

and is directed towards this point. [1]

c



A straight line through the origin. [1]

The line has a negative slope (because $a \propto -x$). [1]

2 **a** An **X** marked where the gradient of the graph is a maximum (0.1 s or 0.3 s or 0.5 s or 0.7 s). [1]

b Maximum speed = maximum gradient from position against time graph [1]

A tangent drawn and numbers substituted to determine the gradient. [1]

maximum speed = 1.6 m s^{-1} (Allow a tolerance of ±0.2 m s^{-1}.) [1]

c **i** $f = \frac{1}{T} = \frac{1}{0.4}$ [1]; $f = 2.5$ Hz [1]

ii ⌉ = 2 $f = \frac{2}{T} = \frac{2}{0.4}$ [1]; ⌉ = 15.7 rad s^{-1} 16 rad s^{-1} [1]

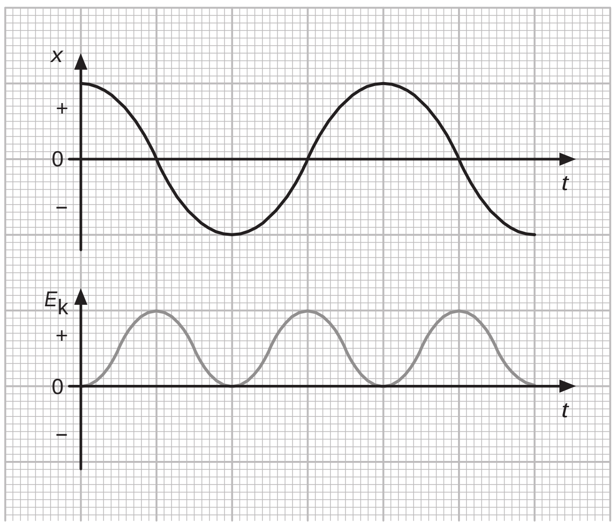
3 **a** $a = -(2\ \ f)^2 x$ [1]

b **i** $a = (2\ \ \ 2.0\ \ 10^{14})^2\ \ 1.2\ \ 10^{-11}$ [1]

$a = 1.9\ \ 10^{19}$ m s^{-2} [1]

ii $F = ma = 1.1\ \ 10^{-25}\ \ 1.9\ \ 10^{19}$ [1]; $F\ \ 2.1\ \ 10^{-6}$ N [1]

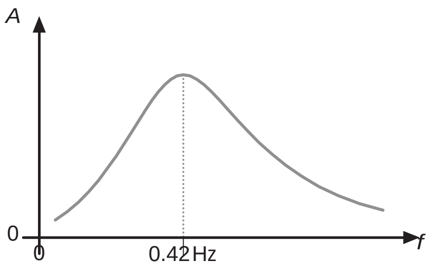
c



The kinetic energy is shown to have only positive values ($E_k \propto v^2$). [1]

Correct shape and phase for the kinetic energy against time graph. [1]

4



Correct shape for the graph. [2]

Resonant frequency = natural frequency

($f_0 = \frac{25}{60}$ 0.42 Hz) [1]