# Chapter 5
Gravitational fields

Worksheet

gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

Intermediate level

1 Define gravitational field strength at a point in space. [1]

2 Show that the gravitational constant G has the unit $\mathrm{N\,m^2\,kg^{-2}}$. [2]

3 The gravitational field strength on the surface of the Moon is $1.6\,\mathrm{N\,kg^{-1}}$. What is the weight of an astronaut of mass $80\,\mathrm{kg}$ standing on the surface of the Moon? [2]

4 Calculate the magnitude of the gravitational force between the objects described below (you may assume that the objects are 'point masses'):
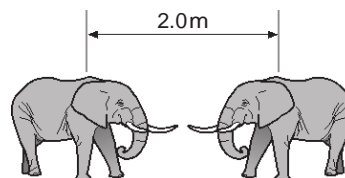
   a two protons separated by a distance of $5.0 \times 10^{-14}\,\mathrm{m}$ (mass of a proton $= 1.7 \times 10^{-27}\,\mathrm{kg}$); [3]
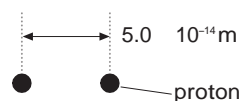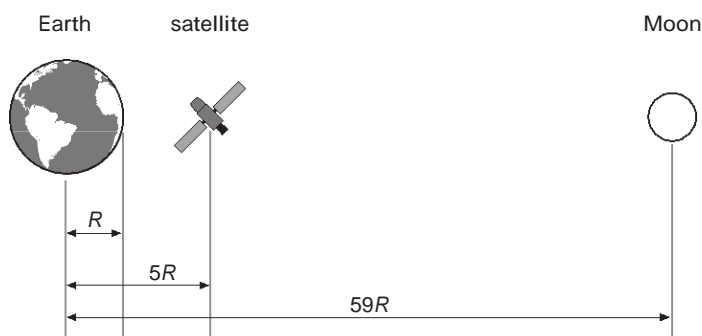

   b two binary stars, each of mass $5.0 \times 10^{28}\,\mathrm{kg}$, with a separation of $8.0 \times 10^{12}\,\mathrm{m}$; [2]
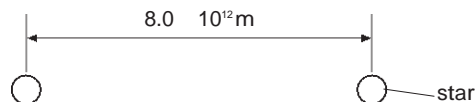
   c two $1500\,\mathrm{kg}$ elephants separated by a distance of $2.0\,\mathrm{m}$. [2]

5 The diagram shows the Moon and an artificial satellite orbiting round the Earth. The radius of the Earth is R.

   a Write an equation for the gravitational field strength g at a distance r from the centre of an isolated object of mass M. [1]

   b By what factor would the gravitational field decrease if the distance from the centre of the mass were doubled? [2]

   c The satellite orbits at a distance of $5R$ from the Earth's centre and the Moon is at a distance of $59R$. Calculate the ratio:

$$\frac{\text{gravitational field strength at position of satellite}}{\text{gravitational field strength at position of Moon}}$$ [3]

Higher level

6 The planet Neptune has a mass of 1.0×10^{26} kg and a radius of 2.2×10^{7} m.
Calculate the surface gravitational field strength of Neptune. [3]

7 Calculate the radius of the planet Pluto given its mass is 5.0×10^{23} kg and its
surface gravitational field strength has been estimated to be 4.0 N kg^{-1}. [3]

8 A space probe of mass 1800 kg is travelling from Earth to the planet Mars. The space
probe is midway between the planets. Use the data given to calculate:

    a the gravitational force on the space probe due to the Earth; [3]

    b the gravitational force on the space probe due to Mars; [2]

    c the acceleration of the probe due to the gravitational force acting on it. [3]

> ### Data
>
> separation between Earth and Mars $= 7.8 \times 10^{10}$ m
> mass of Earth $= 6.0 \times 10^{24}$ kg   mass of Mars $= 6.4 \times 10^{23}$ kg

9 An artificial satellite orbits the Earth at an **altitude** of 400 km above its surface. The
satellite has a mass 5000 kg, the radius of the Earth is 6400 km and the mass of the
Earth is 6.0×10^{24} kg. For this satellite, calculate:

    a the gravitational force experienced; [3]

    b its centripetal acceleration; [2]

    c its orbital speed. [3]

10 The variation of the gravitational field strength g of a planet with the distance r
from its centre is shown below.

    a Explain why the gradient of the graph is equal to GM, where M is the mass of
the planet. [2]

    b Use the information given in **a** and the graph to determine the mass of the
planet. [3]

    c Explain how the graph above would change for a planet of the same density
but twice the radius. [3]

Extension

11 The planets in our Solar System orbit the Sun in almost circular orbits.

    a Show that the orbital speed *v* of a planet at a distance *r* from the centre of the Sun is given by:

$$v = \sqrt{\frac{GM}{r}}$$

                    [4]

    b The mean distance between the Sun and the Earth is 1.5×10^{11} m and the mass of the Sun is 2.0×10^{30} kg. Calculate the orbital speed of the Earth as it travels round the Sun. [2]

12 There is point between the Earth and the Moon where the net gravitational field strength is zero. At this point the Earth's gravitational field strength is equal in magnitude but opposite in direction to the gravitational field strength of the Moon. Given that:

$$\frac{\text{mass of Earth}}{\text{mass of Moon}} = 81$$

calculate how far this point is from the centre of the Moon in terms of *R*, where *R* is the separation between the centres of the Earth and the Moon. [4]

Total: $\overline{58}$ Score: %

Worked examples

Example 1

The Sun has a mean radius of 700 Mm and a surface gravitational field strength of 270 N kg^{-1}. Determine the mass of the Sun and its mean density.
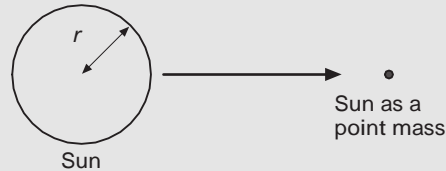(Gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.)

The gravitational field strength g is given by:

$$g = -\frac{GM}{r^2}$$

*We are assuming that the Sun is an isolated body with its entire **mass** concentrated at its **centre**.*

r

Sun

Sun as a point mass

Therefore:

$$M = \frac{gr^2}{G}$$

$$M = \frac{270 \times (700 \times 10^6)^2}{6.67 \times 10^{-11}} = 1.98 \times 10^{30} \text{ kg}$$

$$M \approx 2.0 \times 10^{30} \text{ kg}$$

The density ρ of the Sun is given by:

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi r^3}$$

$$\rho = \frac{1.98 \times 10^{30}}{\frac{4}{3}\pi (700 \times 10^6)^3} \approx 1.4 \times 10^3 \text{ kg m}^{-3}$$

> **Tip**
>
> There is no need to retain the minus sign in the calculation for the mass of the Sun because we are only interested in the magnitude of the field strength. The minus sign signifies an **attractive** gravitational field.

Example 2

The gravitational field strength on the surface of the Earth is 9.81 N kg^{-1}. A satellite in a geostationary orbit round the Earth experiences a gravitational field strength of 0.225 N kg^{-1}. Determine the orbital radius r of the satellite from the centre of the Earth in terms of the radius of the Earth R_E.

The gravitational field strength g is given by:

$$g = -\frac{GM}{r^2}$$

Therefore we have:

$$g \propto \frac{1}{r^2} \quad \text{or} \quad gr^2 = \text{constant}$$

Using the field strengths at distances R_E and r from the centre of the Earth, we have:

$$9.81 \times R_E^2 = 0.225 \times r^2$$

$$\frac{r}{R_E} = \sqrt{\frac{9.81}{0.225}} = 6.60 \approx 6.6$$

The satellite orbits at a distance of 6.6 Earth radii from the centre of the Earth.

Practical

Data analysis – determining the gravitational constant G from planetary data

Introduction

In the absence of experimental work, you have the opportunity to analyse planetary data in order to determine the gravitational constant G. The data analysis is based on the equation for the gravitational field strength g, which is outlined on pages 53 and 54 of *Physics 2*.

Planetary data for the six closest planets to the Sun

| Planet	Mean radius r (m)	Mass M (kg)	Surface field strength g ($N\,kg^{-1}$)	G ($N\,m^2\,kg^{-2}$)
Mercury	2.42×10^6	3.30×10^{23}	3.76	
Venus	6.09×10^6	4.87×10^{24}	8.77	
Earth	6.38×10^6	5.98×10^{24}	9.81	
Mars	3.38×10^6	6.42×10^{23}	3.80	
Jupiter	7.14×10^7	1.90×10^{27}	24.9	
Saturn	6.04×10^7	5.67×10^{26}	10.4	

Data analysis

1 Copy the table above.

2 Use $g = \dfrac{GM}{r^2}$ to complete the last column of the table.

3 Determine your average value for the gravitational constant G. How does it compare with the accepted value of $6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$?

4 Plot a graph of gravitational field strength g against $\dfrac{M}{r^2}$.

5 Draw a straight line of best fit. The gradient of the graph is equal to G.

6 Determine the value of G from the graph. What is the percentage uncertainty in your value?

7 Next, plot a graph of $\lg(g)$ against $\lg(r)$.

8 Draw a straight line of best fit.

9 Explain why the gradient of the graph is equal to –2.

10 How can you use the graph to determine a value for the gravitational constant G?

End-of-chapter test

Answer all questions.

gravitational constant $G = 6.67 \times 10^{-11} \, \text{N} \, \text{m}^2 \, \text{kg}^{-2}$

1 **a** State Newton's law of gravitation. [3]

    b Suggest why the planets in our Solar System may be regarded as point masses. [1]

    c Determine the force between two identical $6.0 \times 10^8 \, \text{kg}$ asteroids separated by a distance of $8.6 \times 10^6 \, \text{m}$. [3]

2 The graph shows the variation of the gravitational field strength g for a planet with distance r from its centre.
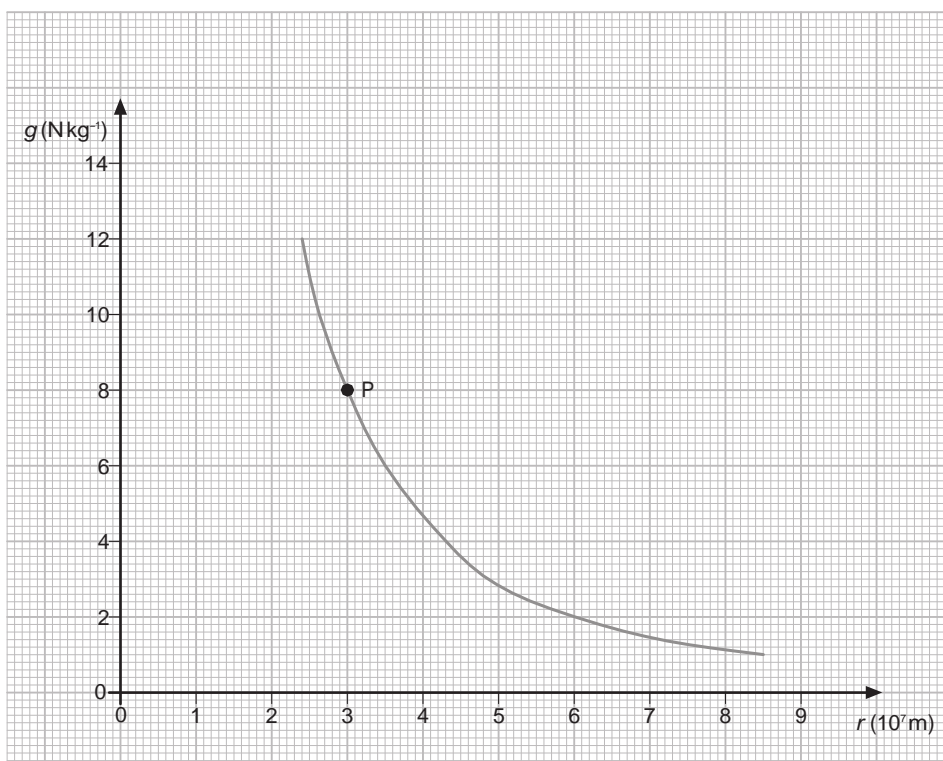
    a State the value of the gravitational field strength at point **P**. [1]

    b Use your answer to **a** to determine the mass M of the planet. [3]

    c Show that the field strength obeys an inverse square law with distance. [2]

3 Neutron stars are remnants of collapsed stars in our Universe. Such stars have colossal densities and gravitational field strengths. For one particular neutron star of mass $2.6 \times 10^{28} \, \text{kg}$, the gravitational field strength at its surface is estimated to be $2.3 \times 10^{11} \, \text{N} \, \text{kg}^{-1}$.

    a Determine the radius of the neutron star. [3]

    b What is the **weight** of a mass of $1.0 \, \text{g}$ on the surface of the neutron star? [2]

Total: $\dfrac{}{18}$ Score: %

Marking scheme

Worksheet

1 Gravitational field strength at a point, g, is the force experienced per unit mass at that point. [1]

2 $F = -\dfrac{Gm_1m_2}{r^2}$ [1]

Therefore: $G = \dfrac{Fr^2}{m_1m_2}$ $\left[\dfrac{\text{N m}^2}{\text{kg}^2}\right]$ [N m^2 kg^{-2}] [1]

3 $g = \dfrac{F}{m}$

$F = mg = 80 \times 1.6$ [1]

$F = 128\,\text{N} \approx 130\,\text{N}$ (F is the 'weight' of the astronaut.) [1]

4 a $F = \dfrac{Gm_1m_2}{r^2}$ [1]

$F = \dfrac{6.67 \times 10^{-11} \times 1.7 \times 10^{-27} \times 1.7 \times 10^{-27}}{(5.0 \times 10^{-14})^2}$ [1]; $F \approx 7.7 \times 10^{-38}\,\text{N}$ [1]

b $F = \dfrac{6.67 \times 10^{-11} \times 5.0 \times 10^{28} \times 5.0 \times 10^{28}}{(8.0 \times 10^{12})^2}$ [1]

$F = 2.61 \times 10^{21}\,\text{N} \approx 2.6 \times 10^{21}\,\text{N}$ [1]

c $F = -\dfrac{6.67 \times 10^{-11} \times 1 \times 500^2}{2.0^2}$ [1]

$F = 3.75 \times 10^{-5}\,\text{N} \approx 3.8 \times 10^{-5}\,\text{N}$ [1]

5 a $g = -\dfrac{GM}{r^2}$ [1]

b The field strength obeys an inverse square law with distance ($g \propto \dfrac{1}{r^2}$). [1]

Doubling the distance decreases the field strength by a factor of four. [1]

c Ratio $= \dfrac{GM/(5R)^2}{GM/(59R)^2}$ [1]

ratio $= \dfrac{59^2}{5^2} = \left(\dfrac{59}{5}\right)^2$ [1]; ratio ≈ 140 [1]

6 $g = -\dfrac{GM}{r^2}$ [1]

$g = \dfrac{6.67 \times 10^{-11} \times 1.0 \times 10^{26}}{(2.2 \times 10^7)^2}$ (magnitude only) [1]

$g = 13.8\,\text{N kg}^{-1} \approx 14\,\text{N kg}^{-1}$ [1]

7 $g = -\dfrac{GM}{r^2}$ [1]

$r^2 = \dfrac{GM}{g} = \dfrac{6.67 \times 10^{-11} \times 5.0 \times 10^{23}}{4.0} = 8.34 \times 10^{12}\,\text{m}^2$ [1]

$r = \sqrt{8.34 \times 10^{12}} \approx 2.9 \times 10^6\,\text{m}$ [1]

8 a $F = -\dfrac{Gm_1m_2}{r^2}$ [1]

$F = \dfrac{6.67 \times 10^{-11} \times 1800 \times 6.0 \times 10^{24}}{(3.9 \times 10^{10})^2}$ $\left(r = \dfrac{7.8 \times 10^{10}}{2} = 3.9 \times 10^{10}\,\text{m}\right)$ [1]

$F = 4.74 \times 10^{-4}\,\text{N} \approx 4.7 \times 10^{-4}\,\text{N}$ [1]

b $F = -\dfrac{Gm_1m_2}{r^2}$

$F = \dfrac{6.67 \times 10^{-11} \times 1800 \times 6.4 \times 10^{23}}{(3.9 \times 10^{10})^2}$ [1]

$F = 5.05 \times 10^{-5}\,\text{N} \approx 5.1 \times 10^{-5}\,\text{N}$ [1]

c $a = \dfrac{F}{m}$ (F is the net force.) [1]

$a = \dfrac{4.74 \times 10^{-4} - 5.05 \times 10^{-5}}{1800}$ [1]

$a \approx 2.4 \times 10^{-7}\,\text{m s}^{-2}$ (towards the centre of the Earth) [1]

9 a $F = -\dfrac{Gm_1m_2}{r^2}$ [1]

$F = \dfrac{6.67 \times 10^{-11} \times 5000 \times 6.0 \times 10^{24}}{(6800 \times 10^3)^2}$ $(r = 6400 + 400 = 6800\,\text{km})$ [1]

$F = 4.33 \times 10^4\,\text{N} \approx 4.3 \times 10^4\,\text{N}$ [1]

b $a = \dfrac{F}{m} = \dfrac{4.33 \times 10^4}{5000}$ [1]

$a = 8.66\,\text{m s}^{-2} \approx 8.7\,\text{m s}^{-2}$ [1]

c $a = \dfrac{v^2}{r}$ [1]

$v^2 = ar = 8.66 \times 6800 \times 10^3$ [1]; $v \approx 7.7\,\text{km s}^{-1}$ [1]

10 a $g = \dfrac{GM}{r^2}$ therefore $\underset{y}{\boxed{g}} = \underset{m}{\boxed{GM}} \; \underset{x}{\boxed{\dfrac{1}{r^2}}}$ [1]

Comparing this equation with the equation for a straight line through the origin, we can see that the gradient is equal to *GM*. [1]

b From the graph, gradient $= \dfrac{3.5}{8.0 \times 10^{-14}} = 4.38 \times 10^{13}\,\text{N m}^2\,\text{kg}^{-1} \approx 4.4 \times 10^{13}\,\text{N m}^2\,\text{kg}^{-1}$ [1]

$GM = 4.38 \times 10^{13}$ [1]

$M = \dfrac{4.38 \times 10^{13}}{6.67 \times 10^{-11}} \approx 6.6 \times 10^{23}\,\text{kg}$ [1]

c Mass = density × volume

For a sphere, volume $\propto$ radius^3 $\left(V = \dfrac{4}{3}\pi r^3\right)$ [1]

Since the radius is doubled, the volume of the planet will be eight times as great.

The mass *M* will therefore be eight times as great. [1]

The gradient of the graph is equal to *GM*. Therefore the gradient will increase by a factor of eight. [1]

11 a Gravitational force on planet $= \dfrac{GMm}{r^2}$ [1]

Centripetal force $= \dfrac{mv^2}{r}$ [1]
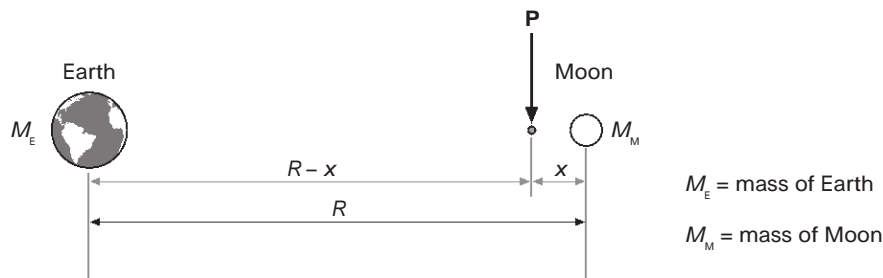
Equating these two forces, we have: $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$ [1]

Therefore: $v^2 = \dfrac{GM}{r}$ or $v = \sqrt{\dfrac{GM}{r}}$ [1]

b $v = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{6.67 \times 10^{-11} \times 2.0 \times 10^{30}}{1.5 \times 10^{11}}}$ [1]

$v = 2.98 \times 10^4 \, \text{m s}^{-1} \approx 30 \, \text{km s}^{-1}$ [1]

12 The field strengths are the same at point **P**.

M_E = mass of Earth

M_M = mass of Moon

$\dfrac{GM_M}{x^2} = \dfrac{GM_E}{(R-x)^2}$ [1]

$R - x = x \sqrt{\dfrac{M_E}{M_M}}$ [1]

$R - x = x \sqrt{81}$ so $R - x = 9x$ [1]

$10x = R$ so $x = \dfrac{R}{10}$ [1]

Marking scheme

End-of-chapter test

1 **a** The **attractive** force between two **point** masses is directly proportional to the product of their masses and inversely proportional to the square of their separation.

One mark for the idea that $F \propto m_1 m_2$. [1]

One mark for the idea that $F \propto \dfrac{1}{r^2}$. [1]

One mark for reference to either attractive or point masses. [1]

b The size of the planets is very small compared with the distances between the planets and the Sun. [1]

c $F = -\dfrac{G m_1 m_2}{r^2}$ [1]

$F = \dfrac{6.67 \times 10^{-11} \times 6.0 \times 10^8 \times 6.0 \times 10^8}{(8.6 \times 10^6)^2}$ [1]

$F \approx 3.2 \times 10^{-7}\,\text{N}$ [1]

2 **a** $8.0\,\text{N}\,\text{kg}^{-1}$ [1]

b $g = -\dfrac{GM}{r^2}$ [1]

$M = \dfrac{g r^2}{G} = \dfrac{8.0 \times (3.0 \times 10^7)^2}{6.67 \times 10^{-11}}$ [1]; $M \approx 1.1 \times 10^{26}\,\text{kg}$ [1]

c $g \propto \dfrac{1}{r^2}$ or $g r^2 = \text{constant}$ [1]

Taking values for g and r from the graph:

$12.0 \times (2.4 \times 10^7)^2 \approx 6.9 \times 10^{15}$

$8.0 \times (3.0 \times 10^7)^2 \approx 7.2 \times 10^{15}$

$4.0 \times (4.3 \times 10^7)^2 \approx 7.1 \times 10^{15}$

$2.0 \times (6.0 \times 10^7)^2 \approx 7.2 \times 10^{15}$

These products $g r^2$ are almost the same (to 1 s.f. they are all 7×10^{15}). [1]

3 **a** $g = -\dfrac{GM}{r^2}$ [1]

$r^2 = \dfrac{GM}{g} = \dfrac{6.67 \times 10^{-11} \times 2.6 \times 10^{28}}{2.3 \times 10^{11}} = 7.54 \times 10^6\,\text{m}^2$ [1]

$r = \sqrt{7.54 \times 10^6} \approx 2.7 \times 10^3\,\text{m}$ [1]

b $F = mg = 0.001 \times 2.3 \times 10^{11}$ [1]

$F = 2.3 \times 10^8\,\text{N}$ [1]