Chapter 13
Nuclear physics

Worksheet

speed of light in vacuum, $c = 3.0 \quad 10^{8}\ \mathrm{m\,s^{-1}}$
unified atomic mass unit, u = $1.66 \quad 10^{-27}$ kg
1 eV = $1.6 \quad 10^{-19}$ J

Intermediate level

1 **a** Write Einstein's famous equation relating mass and energy. [1]

b Determine the change in energy equivalent to a change in mass:

i of 1.0 g; [2]

ii equal to that of an electron of $9.1 \quad 10^{-31}$ kg. [2]

2 In nuclear physics, it is common practice to quote the mass of a nuclear particle in terms of the unified atomic mass, u. The unified atomic mass unit u is defined as $\frac{1}{12}$ of the mass of an atom of the isotope of carbon $^{12}_{6}\mathrm{C}$. Experiments show that:

1 u $\quad 1.66 \quad 10^{-27}$ kg

a Determine the mass of each of the following particles in terms of u:

i an -particle of mass $6.65 \quad 10^{-27}$ kg; [1]

ii a carbon-13 atom of mass $2.16 \quad 10^{-26}$ kg. [1]

b Determine the mass of each of the following particles in kg:

i a proton of mass 1.01 u; [1]

ii a uranium-235 nucleus of mass 234.99 u. [1]

3 State three quantities conserved in all nuclear reactions. [3]

4 **a** Explain why external energy is required to 'split' a nucleus. [1]

b Define the binding energy of a nucleus. [1]

c The binding energy of the nuclide $^{16}_{8}\mathrm{O}$ is 128 MeV. Calculate the binding energy per nucleon. [2]

5 For each nuclear reaction below, determine any missing figures.

a $^{228}_{90}\mathrm{Th} \quad ^{4}_{2}\mathrm{He} + ^{?}_{?}\mathrm{Ra}$ [1]

b $^{2}_{1}\mathrm{H} + ^{1}_{1}\mathrm{H} \quad ^{?}_{?}\mathrm{He}$ [1]

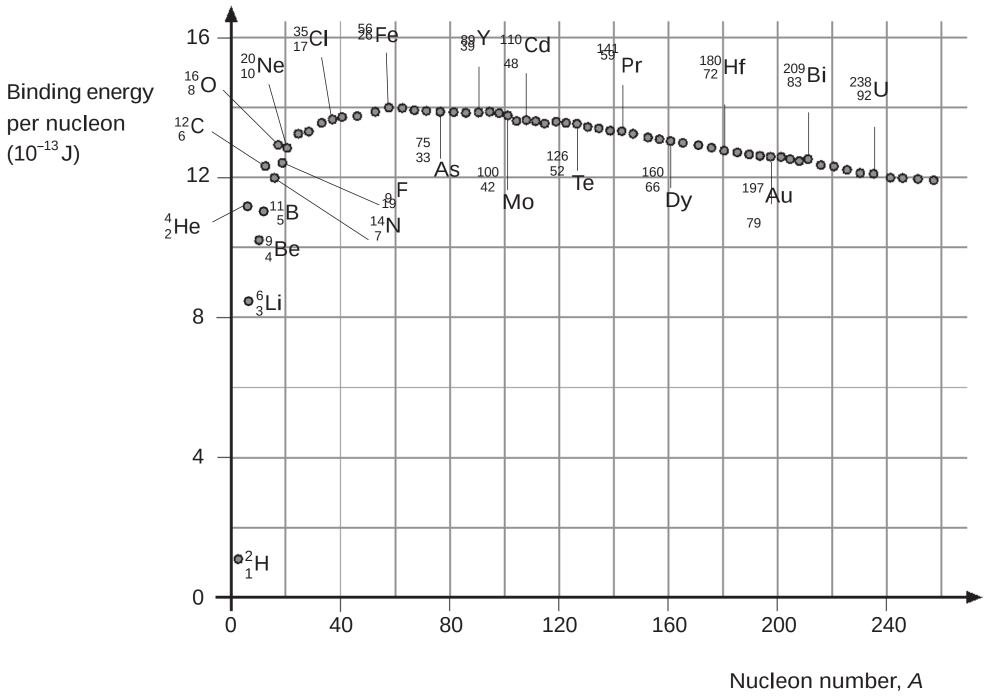
c $^{2}_{1}\mathrm{H} + ^{3}_{1}\mathrm{H} \quad ^{4}_{2}\mathrm{He} + ?\ ^{1}_{0}\mathrm{n}$ [1]

d $^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \quad ^{?}_{?}\mathrm{Ba} + ^{92}_{36}\mathrm{Kr} + 3\ ^{1}_{0}\mathrm{n}$ [1]

e $^{14}_{?}\mathrm{N} + ^{1}_{0}\mathrm{n} \quad ^{12}_{6}\mathrm{C} + ^{?}_{1}\mathrm{H}$ [1]

Higher level

6 The binding energy per nucleon against nucleon number graph for some common nuclides is shown below.



a Identify the most stable nuclide. Explain your answer. [2]

b Use the graph to estimate the binding energy for the nucleus of $^{12}_{6}$C. [2]

c Use the graph to estimate the energy released in the reaction below. [4]

$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \quad ^{4}_{2}\text{He}$

7 Use the data given below to determine the binding energy and the binding energy per nucleon of the nuclide $^{235}_{92}$U. [7]

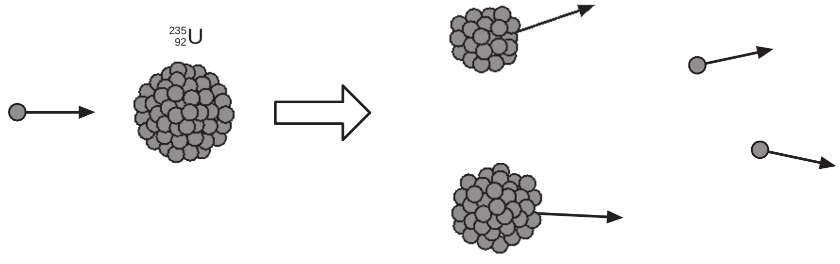
mass of proton = 1.007 u

mass of neutron = 1.009 u   mass of uranium-235 nucleus = 234.992 u

8 **a** Describe the process of fission. [1]

b The diagram shows the fission of uranium-235 in accordance with the nuclear equation:

$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \quad ^{95}_{38}\text{Sr} + ^{139}_{54}\text{Xe} + 2\ ^{1}_{0}\text{n}$



i Copy the diagram, adding labels to identify the neutrons, the strontium nuclide and the xenon nuclide. [1]

ii Explain why energy is released in the reaction above. [2]

iii Use the following data to determine the energy released in a single fission reaction involving $^{235}_{92}$U and $^{1}_{0}$n. [5]

mass of $^{235}_{92}$U = $3.902 \quad 10^{-25}$ kg   mass of $^{95}_{38}$Sr = $1.575 \quad 10^{-25}$ kg

mass of $^{1}_{0}$n = $1.675 \quad 10^{-27}$ kg   mass of $^{139}_{54}$Xe = $2.306 \quad 10^{-25}$ kg

Extension

9 In a process referred to as 'annihilation', a particle interacts with its antiparticle and the entire mass of the combined particles is transformed into energy in the form of photons. The following equation represents the interaction of a proton and its antiparticle, the antiproton.

$^{1}_{1}\mathrm{p} + {}^{1}_{-1}\mathrm{p} \qquad +$

The antiproton has the same mass as a proton – the only difference is that it has a negative charge. Determine the wavelength of each of the two identical photons emitted in the reaction above. (Mass of a proton = 1.7 10^{-27} kg.) [5]

10 Does fusion or fission produce more energy per kilogram of fuel? Answer this question by considering the fusion reaction in **6 c** and the fission reaction in **8 b**.

(The molar masses of hydrogen-2 and uranium-235 are 2 g and 235 g, respectively.)

[7]

Total: $\frac{\quad}{57}$ Score: %

Worked examples

Example 1

The binding energy per nucleon of the nuclide $^{56}_{26}Fe$ is 8.8 MeV.

Determine the binding energy of this nuclide in MeV and in joules.

binding energy = binding energy per nucleon   number of nucleons

binding energy (MeV) = 8.8   56 = 492.8 MeV

binding energy   490 MeV

binding energy (J) = 492.8 10^6 1.6 10^{-19}

binding energy   7.9 10^{-11} J

In this module, you are expected to recall that 1 eV = 1.6 10^{-19} J.

Do not forget to include the 10^6 ('mega') factor.

Example 2

The nuclei of thorium-228 naturally decay by emission of   -particles. Use the nuclear equation given below, together with the additional data, to determine the energy released during the decay of a single nuclide of thorium-228.

$^{228}_{90}Th$ $^{4}_{2}He + ^{224}_{88}Ra$

mass of $^{228}_{90}Th$ nucleus = 3.7853 10^{-25} kg

mass of $^{4}_{2}He$ nucleus = 6.6425 10^{-27} kg

mass of $^{224}_{88}Ra$ nucleus = 3.7187 10^{-25} kg

In this reaction the mass decreases, hence according to Einstein's equation, energy will be released.

Initial mass = 3.7853 10^{-25} kg

Final mass = 3.7187 10^{-25} kg + 6.6425 10^{-27} kg = 3.7851 10^{-25} kg

According to Einstein's equation, we have:

$E =\ \ mc^2$

Therefore:

energy released = mc^2 = (3.7853 – 3.7851) 10^{-27} (3.0 $10^8)^2$

energy released   1.8 10^{-14} J   (0.11 MeV)

Tip

Energy is released in this reaction because there is a decrease in mass. This energy is released in the form of **kinetic energy** of the   -particle and the radium nucleus. A   -photon may also be released. It is tempting to say that the energy is released as 'heat'. This is not strictly true!

Practical

Simulation (applet) websites – nuclear physics

Introduction

In the absence of any experimental work, the Internet once again provides free access to a range of material that may be used to enhance your understanding of this chapter.

Create your own binding energy per nucleon against nucleon number graph

http://schools.matter.org.uk/Content/NuclearBindingEnergies/answers.html

- This website gives you access to nuclides and their binding energies, at the click of a button.
- Use this information to calculate the binding energy per nucleon for each nuclide.
- Plot a graph of binding energy per nucleon against nucleon number. Does the graph resemble that shown on page 135 of *Physics 2*?

Class presentations

http://www.chem.ox.ac.uk/vrchemistry/Conservation/page21.htm

- You can use the material on the web page above and subsequent pages to organise a presentation on one of the following:
  – Einstein's mass–energy relation
  – binding energy
  – conservation rules in nuclear reactions
  – fission
  – fusion.

End-of-chapter test

Answer all questions.

speed of light in vacuum $c = 3.0 \quad 10^{8}\,\mathrm{m\,s^{-1}}$
unified atomic mass unit $\mathrm{u} = 1.66 \quad 10^{-27}\,\mathrm{kg}$

1 **a** In particle accelerators, charged particles are accelerated to very high speeds. Using Einstein's mass–energy equation, explain why the mass of an accelerated electron would be greater than when it is at rest. [3]

b **i** Calculate the kinetic energy of an electron travelling at $2.0 \quad 10^{7}\,\mathrm{m\,s^{-1}}$. (Rest mass of electron $= 9.1 \quad 10^{-31}$ kg.) [2]

ii Use your answer to **b i** to estimate the change in the mass of the electron when travelling at $2.0 \quad 10^{7}\,\mathrm{m\,s^{-1}}$. [2]

2 One of the many fusion reactions taking place within the interior of stars is:

$${}^{1}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \quad {}^{3}_{2}\mathrm{He}$$

a Name the particle represented by ${}^{1}_{1}\mathrm{H}$. [1]

b In the reaction above, state two quantities that are conserved. [2]

3 Use the data given below to determine the binding energy and the binding energy per nucleon of the nuclide ${}^{6}_{3}\mathrm{Li}$. [7]

mass of proton = 1.007 u

mass of neutron = 1.009u

mass of lithium-6 nucleus = 6.014 u

4 **a** Fusion takes place in the interior of stars. The temperature within the core of a star can be as high as 10^{9} K. Explain why high temperatures are necessary for the fusion of nuclei. [3]

b One of the many fusion reactions that occur in the interior of stars is:

$${}^{3}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \quad {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$$

The table below shows the binding energy per nucleon of the particles involved in this nuclear reaction.

Particle	${}^{2}_{1}\mathrm{H}$	${}^{3}_{1}\mathrm{H}$	${}^{4}_{2}\mathrm{He}$	${}^{1}_{0}\mathrm{n}$
Binding energy per nucleon (10^{-13} J)	1.0	2.911.2	zero	

i Explain why ${}^{1}_{0}\mathrm{n}$ has no binding energy. [1]

ii Determine the energy released in the fusion reaction above. [3]

Total: $\frac{\quad}{24}$ Score: %

Marking scheme

Worksheet

1 **a** Change in energy = change in mass   (speed of light)^2 [$E = \quad mc^2$] [1]

b **i** $E = \quad mc^2$

$E = 0.001 \quad (3.0 \quad 10^8)^2$ [1]

$E = 9.0 \quad 10^{13}$ J [1]

ii $E = \quad mc^2$

$E = 9.1 \quad 10^{-31} \quad (3.0 \quad 10^8)^2 = 8.19 \quad 10^{-14}$ J [1]

$E \approx 8.2 \quad 10^{-14}$ J [1]

2 **a** **i** Mass $= \dfrac{6.65 \quad 10^{-27}}{1.66 \quad 10^{-27}} = 4.01$ u [1]

ii Mass $= \dfrac{2.16 \quad 10^{-26}}{1.66 \quad 10^{-27}} = 13.01$ u [1]

b **i** Mass $= 1.01 \quad 1.66 \quad 10^{-27} = 1.68 \quad 10^{-27}$ kg [1]

ii Mass $= 234.99 \quad 1.66 \quad 10^{-27} = 3.90 \quad 10^{-25}$ kg [1]

3 In all nuclear reactions the following quantities are conserved:

- charge (or proton number)
- nucleon number
- mass–energy
- momentum.

Any **three** of the above. [3]

4 **a** The nucleons within the nucleus are held tightly together by the **strong nuclear force**. [1]

b The binding energy of a nucleus is the **minimum** energy required to completely separate the nucleus into its constituent protons and neutrons. [1]

c Binding energy per nucleon $= \dfrac{\text{binding energy}}{\text{number of nucleons}}$

binding energy per nucleon $= \dfrac{128}{16}$ [1]

binding energy per nucleon = 8.0 MeV [1]

5 **a** ${}^{228}_{90}\mathrm{Th} \qquad {}^{4}_{2}\mathrm{He} + {}^{224}_{88}\mathrm{Ra}$ [1]

b ${}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \qquad {}^{3}_{2}\mathrm{He}$ [1]

c ${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \qquad {}^{4}_{2}\mathrm{He} + 1\,{}^{1}_{0}\mathrm{n}$ [1]

d ${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \qquad {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}$ [1]

e ${}^{14}_{7}\mathrm{N} + {}^{1}_{0}\mathrm{n} \qquad {}^{12}_{6}\mathrm{C} + {}^{3}_{1}\mathrm{H}$ [1]

6 **a** The nuclide ${}^{56}_{26}\mathrm{Fe}$ is the most stable. [1]

It has the maximum value for the binding energy per nucleon. [1]

b Binding energy = binding energy per nucleon   number of nucleons

binding energy $12.3 \quad 10^{-13} \quad 12$ [1]

binding energy $1.5 \quad 10^{-11}$ J [1]

c From the graph, the binding energies per nucleon of ${}^2_1H$ and ${}^4_2He$ are approximately $1.0 \quad 10^{-13}$ J and $11.2 \quad 10^{-13}$ J. [1]

energy released
= difference in binding energy per nucleon   number of nucleons [1]

energy released = $[11.2 \quad 10^{-13} - 1.0 \quad 10^{-13}] \quad 4$ [1]

energy released = $4.08 \quad 10^{-12}$ J $4.1 \quad 10^{-12}$ J [1]

7 $92\ {}^1_1$proton + $143\ {}^1_0$neutron ${}^{235}_{92}$uranium [1]

mass defect = [(143   1.009) + (92   1.007)]u – (234.992)u [1]

mass defect = 1.939 u = $1.939 \quad 1.66 \quad 10^{-27}$ kg [1]

mass defect = $3.219 \quad 10^{-27}$ kg [1]

binding energy = mass defect   (speed of light)^2 [1]

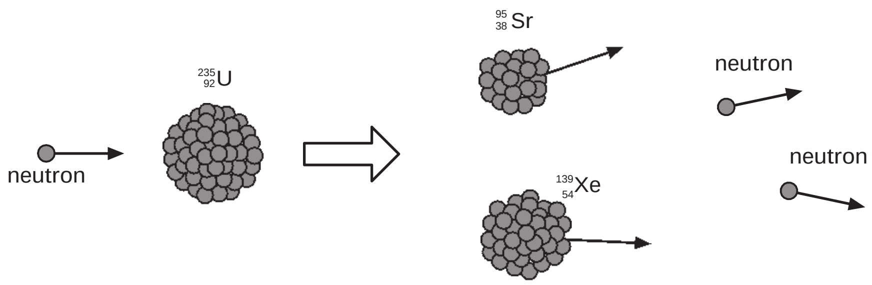
binding energy = $3.219 \quad 10^{-27} \quad (3.0 \quad 10^8)^2 = 2.897 \quad 10^{-10}$ J [1]

binding energy per nucleon = $\dfrac{\text{binding energy}}{\text{number of nucleons}}$

binding energy per nucleon = $\dfrac{2.897 \quad 10^{-10}}{235} = 1.233 \quad 10^{-12} \quad 1.2 \quad 10^{-12}$ J [1]

8 **a** Fission is the splitting of a heavy nucleus like ${}^{235}_{92}U$ into two approximately equal fragments. The splitting occurs when the heavy nucleus absorbs a neutron. [1]

b **i** All particles identified on the diagram. [1]



ii In the reaction above, there is a decrease in the mass of the particles. [1]

According to $E = \ mc^2$, a **decrease** in mass implies that energy is **released** in the process. [1]

iii change in mass
= $[1.575 \quad 10^{-25} + 2.306 \quad 10^{-25} + 2(1.675 \quad 10^{-27})]$
$-[3.902 \quad 10^{-25} + 1.675 \quad 10^{-27}]$ [1]

change in mass = $-4.250 \quad 10^{-28}$ kg [1]

(The minus sign means a decrease in mass and hence energy is released in this reaction.)

$E = \ mc^2$ [1]

$E = 4.250 \quad 10^{-28} \quad (3.0 \quad 10^8)^2$ [1]

$E = 3.83 \quad 10^{-11}$ J $3.8 \quad 10^{-11}$ J [1]

9 According to Einstein's equation: $E = \; mc^2$ [1]

In this case, E is the energy of two photons and m is the mass of two protons. [1]

Hence:

$$2 \quad \frac{hc}{\;} = (2 \quad m_p)c^2 \text{ [1]}$$

$$= \frac{hc}{m_p c^2} = \frac{h}{m_p c} = \frac{6.63 \quad 10^{-34}}{1.7 \quad 10^{-27} \quad 3.0 \quad 10^8} \text{ [1]}$$

$$= 1.3 \quad 10^{-15}\,\text{m} \text{ [1]}$$

10 For **fusion**, we have:

energy released per kg = number of 'pairs' of ^2_1H in 1 kg $\quad 4.08 \quad 10^{-12}$ J (from **6 c**) [1]

$$\text{energy per kg} = \left(\frac{1}{2} \quad \frac{1000}{2} \quad 6.02 \quad 10^{23}\right) \quad 4.08 \quad 10^{-12} \text{ [1]}$$

energy per kg = $6.14 \quad 10^{14}$ J $\quad 6.1 \quad 10^{14}$ J [1]

For **fission**, we have:

energy released per kg = number of nuclei in 1 kg $\quad 3.83 \quad 10^{-11}$ J (from **8 b**) [1]

$$\text{energy per kg} = \left(\frac{1000}{235} \quad 6.02 \quad 10^{23}\right) \quad 3.83 \quad 10^{-11} \text{ [1]}$$

energy per kg = $9.8 \quad 10^{13}$ J [1]

There is less energy released per fusion than per fission. However, there are many more nuclei per kg for fusion. Hence fusion produces more energy per kg than fission. [1]

Marking scheme

End-of-chapter test

1 **a** The energy of an electron moving is greater because it has kinetic energy. [1]

According to Einstein's equation: $E = \quad mc^2$ [1]

An **increase** in energy implies **greater** mass. [1]

Hence, the mass of the moving electron is greater than its 'rest' mass.

b **i** $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \quad 9.1 \quad 10^{-31} \quad (2.0 \quad 10^7)^2$ [1]

$E_k = 1.82 \quad 10^{-16}\,\text{J} \quad 1.8 \quad 10^{-16}\,\text{J}$ [1]

ii $m = \frac{E}{c^2} = \frac{1.82 \quad 10^{-16}}{(3.0 \quad 10^8)^2}$ [1]

$m = 2.02 \quad 10^{-33}\,\text{kg} \quad 2.0 \quad 10^{-33}\,\text{kg}$ [1]

2 **a** It is a proton. [1]

b Any **two** from: [2]

- charge (or proton number)
- nucleon number
- mass–energy
- momentum.

3 $3\,{}^1_1\text{proton} + 3\,{}^1_0\text{neutron} \quad {}^6_3\text{lithium}$ [1]

mass defect = $[(3 \quad 1.009) + (3 \quad 1.007)]\text{u} - (6.014)\text{u}$ [1]

mass defect = $0.034\,\text{u} = 0.034 \quad 1.66 \quad 10^{-27}\,\text{kg}$ [1]

mass defect = $5.644 \quad 10^{-29}\,\text{kg}$ [1]

binding energy = mass defect (speed of light)^2 [1]

binding energy = $5.644 \quad 10^{-29} \quad (3.0 \quad 10^8)^2 = 5.080 \quad 10^{-12}\,\text{J}$ [1]

binding energy per nucleon = $\frac{\text{binding energy}}{\text{number of nucleons}}$

binding energy per nucleon = $\frac{5.080 \quad 10^{-12}}{6} = 8.466 \quad 10^{-13}\,\text{J} \quad 8.5 \quad 10^{-13}\,\text{J}$ [1]

4 **a** The positive nuclei repel each other. [1]

At higher temperatures the nuclei move faster [1]

and have a greater chance of approaching close enough so that they combine with each other due to the strong nuclear force. [1]

b **i** A neutron ${}^1_0\text{n}$ is a lone particle (there are no other nucleons). [1]

ii Energy released = difference in binding energy [1]

energy released = $(11.2 \quad 10^{-13} \quad 4) - [(1.0 \quad 10^{-13} \quad 2) + (2.9 \quad 10^{-13} \quad 3)]$ [1]

energy released = $3.41 \quad 10^{-12}\,\text{J} \quad 3.4 \quad 10^{-12}\,\text{J}$ [1]