# Chapter 3 Moving in a circle 

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## Worksheet

acceleration of free fall $g=9.81 \mathrm{~m} \mathrm{~s}^{-2}$

## Intermediate level

1 Convert the following angles into radians.
a $30^{\circ}$
b $210^{\circ}$
c $0.05^{\circ}$
[3]

2 Convert the following angles from radians into degrees.
a 1.0 rad
b 4.0 rad
c 0.15 rad

3 The planet Mercury takes 88 days to orbit once round the Sun. Calculate its angular displacement in radians during a time interval of:
a 44 days;
b 1 day.
4 In each case below, state what provides the centripetal force on the object:
a a car travelling at a high speed round a sharp corner;
b a planet orbiting the Sun;
c an electron orbiting the positive nucleus of an atom;
d clothes spinning round in the drum of a washing machine.
5 An aeroplane is circling in the sky at a speed of $150 \mathrm{~m} \mathrm{~s}^{-1}$. The aeroplane describes a circle of radius 20 km . For a passenger of mass 80 kg inside this aeroplane, calculate:
a her centripetal acceleration;
b the centripetal force acting on her.
6 The diagram shows a stone tied to the end of a length of string. It is whirled round in a horizontal circle of radius 80 cm .


The stone has a mass of 90 g and it completes 10 revolutions in a time of 8.2 s .
a Calculate:
i the time taken for one revolution;
ii the distance travelled by the stone during one revolution;
iii the speed of the stone as it travels in the circle;
iv the centripetal acceleration of the stone;
$v$ the centripetal force on the stone.
b What provides the centripetal force on the stone?
c What is the angle between the acceleration of the stone and its velocity?

## Higher level

7 A rubber toy of mass 40 g is placed close to the edge of a spinning turntable.


The toy travels in a circle of radius 12 cm . The toy takes 0.85 s to complete one revolution. For this toy, calculate:
a its speed;
b the centripetal force acting on it.
8 The diagram shows a skateboarder of mass 70 kg who drops through a vertical height of 5.2 m .


The dip has a radius of curvature of 16 m .
a Assuming no energy losses due to air resistance or friction, calculate the speed of the skateboarder at the bottom of the dip at point B. You may assume that the speed of the skateboarder at point $\mathbf{A}$ is zero.
b At point $\mathbf{B}$, calculate:
i the centripetal acceleration of the skateboarder;
ii the contact force $R$ acting on the skateboarder.

## Extension

9 A car of mass 820 kg travels at a constant speed of $32 \mathrm{~m} \mathrm{~s}^{-1}$ along a banked track.


The track is banked at an angle of $20^{\circ}$ to the horizontal.
You need to resolve the contact force (vertically).
a The net vertical force on the car is zero. Use this idea to show that the contact force $R$ on the car is 8.56 kN .
b Use the answer from a to calculate the radius of the circle described by the car.
[4]
10 A stone of mass 120 g is fixed to one end of a light rigid rod. The diagram shows the stone whirled at a constant speed of $4.0 \mathrm{~m} \mathrm{~s}^{-1}$ in a vertical circle of radius 80 cm .
Calculate the ratio:
tension in the rod at $\mathbf{A}$ tension in the rod at $\mathbf{B}$


Total: $\overline{53}$ Score: \%

## Worked examples

## Example 1

A stone of mass 60 g is tied to the end of a length of string. The string snaps when the tension in it exceeds 14 N . The stone is whirled in a horizontal circle of radius 50 cm . Calculate the maximum
 speed of the stone such that the string does not snap.

The tension in the string is the centripetal force acting on the stone.
The centripetal force $F$ is given by
$F=\frac{m v^{2}}{r}$
$F=14 \mathrm{~N} \quad m=0.060 \mathrm{~kg} \quad r=0.50 \mathrm{~m}$
$v^{2}=\frac{F r}{m}=\frac{14 \times 0.50}{0.060}=117 \mathrm{~m}^{2} \mathrm{~s}^{-2}$
The stone has maximum speed when the
Therefore the maximum speed, $v$, is: tension is also a maximum.
$v=\sqrt{117} \approx 11 \mathrm{~m} \mathrm{~s}^{-1}$

## Tip

You can also determine the maximum speed of the stone by first determining the centripetal acceleration using:
$F=m a$
followed by:
$a=\frac{v^{2}}{r}$

## Example 2

The planet Mercury has an orbital radius of $5.8 \times 10^{10} \mathrm{~m}$ round the Sun. Mercury takes 88 Earth days to complete one orbit round the Sun. Calculate the speed of Mercury in its orbit and its centripetal acceleration.

In order to calculate the centripetal acceleration, we need to determine the orbital speed, $v$, of Mercury.
$v=\frac{\text { distance }}{\text { time }}$
$v=\frac{2 \pi r}{T}=\frac{2 \pi \times 5.8 \times 10^{10}}{88 \times 24 \times 3600}=4.794 \times 10^{4} \approx 4.8 \times 10^{4} \mathrm{~m} \mathrm{~s}^{-1}$

In a time of 88 days, the planet travels a distance equal to the circumference of its orbit.

The centripetal acceleration $a$ is given by:
$a=\frac{v^{2}}{r}$
$a=\frac{\left(4.794 \times 10^{4}\right)^{2}}{5.8 \times 10^{10}}=3.9610^{-2} \mathrm{~m} \mathrm{~s}^{-2} \approx 4.0 \times 10^{-2} \mathrm{~m} \mathrm{~s}^{-2}$

## Practical

## Investigating circular motion

## Safety

Wear safety glasses during the experiment and make sure that there is ample space for the whirling bungs. Teachers and technicians should follow their school and departmental safety policies and should ensure that the employer's risk assessment has been carried out before undertaking any practical work.

## Apparatus

- 20 cm long plastic tube
- strong thread or fishing line
- rubber bung
- digital balance
- weights
- stopwatch
- metre rule
- safety glasses


## Introduction

Centripetal acceleration and centripetal force are discussed on page 29 of Physics 2. The centripetal force $F$ is given by:
$F=\frac{m v^{2}}{r}$
where $m$ is the mass of the object describing a circle of radius $r$ and $v$ is the speed of the object. In this experiment you will investigate the relationship between the centripetal force $F$ acting on an object whirled in a horizontal circle and its speed $v$.


## Procedure

1 Tie one end of the thread to a rubber bung and make a mark on the thread at a distance of 30 cm from the bung.
2 Attach a 1.0 N weight at the other end of the thread and whirl the rubber bung in a horizontal circle as shown in the diagram above. (The centripetal force $F$ on the bung is 1.0 N .)

3 Adjust the speed of the bung such that the radius $r$ of the circle is equal to 30 cm and then continue to whirl the bung at a constant speed.
4 Measure the time $t$ for 10 revolutions of the bung.
5 Determine the speed $v$ of the bung using speed $=\frac{\text { distance }}{\text { time }}$ :

$$
v=\frac{10 \times 2 \pi r}{t}=\frac{20 \pi r}{t}
$$

6 Repeat the experiment for different values of centripetal force $F$ and record your results in a table.

7 Plot a graph of force $F$ against $v^{2}$. Draw a straight line of best fit.
8 Explain why the gradient of the graph is given by:
gradient $=\frac{m}{r}$
where $m$ is the mass of the bung and $r$ is the radius of the circle.
9 Determine the mass of the bung from the gradient. How does it compare with the actual mass of the bung when measured using a digital balance?

## Guidance for teachers

The edges of the tube must be rounded and smooth, otherwise there is a serious danger of the thread being cut as it is being whirled. Cheap nylon fishing line is an excellent substitute for thread.

## End-of-chapter test

## Answer all questions.

acceleration of free fall $g=9.81 \mathrm{~m} \mathrm{~s}^{-2}$
1 a Define angular displacement of an object moving in a circle.
b Convert an angle of $64^{\circ}$ into radians.
2 The diagram shows a car of mass 850 kg travelling on a level road in a clockwise direction at a steady speed of $20 \mathrm{~m} \mathrm{~s}^{-1}$ round a bend with radius of curvature 32 m .
a On the diagram, draw an arrow to show the velocity of the car (label this $v$ ) and another arrow to show the acceleration of the car (label this $a$ ).

b Write an equation for the centripetal acceleration $a$ of the car moving on a level road at a speed $v$ round a bend of radius of curvature $r$.
c Calculate the centripetal force acting on the car.
d State what provides the centripetal force in $\mathbf{c}$.
3 The diagram shows the vertical forces acting on a car of mass 800 kg travelling over the top of a curved bridge.


The downward force acting on the car is its weight $W$ and the upward force acting on the car is the contact force $R$ provided by the road.
The radius of curvature of the bridge is 20 m and the speed of the car is $12 \mathrm{~m} \mathrm{~s}^{-1}$.
a Calculate the centripetal acceleration of the car.
b Use your answer to a to determine the contact force $R$ when the car is at the top of the bridge.
c Show that the car will lose contact with the top of the bridge if its speed exceeds $\sqrt{g r}$, where $g$ is the acceleration of free fall and $r$ is the radius of curvature of the bridge.
d Use your answer to c to determine the minimum speed of the car for which the car loses contact with the top of the bridge.

## Marking scheme

## Worksheet

1 a $\theta=\frac{30}{360} \times 2 \pi=\frac{\pi}{6} \approx 0.52 \mathrm{rad}[1]$
b $\quad \theta=\frac{210}{360} \times 2 \pi \approx 3.7 \mathrm{rad}[1]$
c $\quad \theta=\frac{0.05}{360} \times 2 \pi \approx 8.7 \times 10^{-4} \mathrm{rad}$ [1]
2 a $\theta=\frac{1.0}{2 \pi} \times 360=57.3^{\circ} \approx 57^{\circ}$
b $\quad \theta=\frac{4.0}{2 \pi} \times 360 \approx 230^{\circ} \quad[1$
c $\quad \theta=\frac{0.15}{2 \pi} \times 360 \approx 8.6^{\circ}$
3 a 88 days is equivalent to $2 \pi$ radians.

$$
\theta=\frac{44}{88} \times 2 \pi=\pi \mathrm{rad}[1]
$$

b $\quad \theta=\frac{1}{88} \times 2 \pi \approx 0.071 \mathrm{rad}\left(4.1^{\circ}\right)[1]$
4 a Friction between the tyres and the road. [1]
b Gravitational force acting on the planet due to the Sun. [1]
c Electrical force acting on the electron due to the positive nucleus. [1]
d The (inward) contact force between the clothes and the rotating drum. [1]
5 a $a=\frac{v^{2}}{r}[1] ; \quad a=\frac{150^{2}}{20000}[1] ; \quad a=1.125 \mathrm{~m} \mathrm{~s}^{-2} \approx 1.1 \mathrm{~m} \mathrm{~s}^{-2}[1]$
b $F=m a=80 \times 1.125[1] ; \quad F=90 \mathrm{~N}[1]$
6 a i Time $=\frac{8.2}{10}=0.82 \mathrm{~s} \quad[1]$
ii Distance $=$ circumference of circle distance $=2 \pi r=2 \pi \times 0.80=5.03 \mathrm{~m} \approx 5.0 \mathrm{~m} \quad[1]$
iii Speed $=\frac{\text { distance }}{\text { time }}$

$$
\text { speed, } v=\frac{5.03}{0.82}[1] ; \quad v=6.13 \mathrm{~m} \mathrm{~s}^{-1} \approx 6.1 \mathrm{~m} \mathrm{~s}^{-1}[1]
$$

iv $a={ }^{2} \bar{v}_{r}[1] ; \quad \begin{aligned} & 6=0.13^{2} \\ & 0.80\end{aligned}[1] ; \quad a=47 \mathrm{~m} \mathrm{~s}^{-2}[1]$
v $F=m a=0.090 \times 47[1] ; \quad F \approx 4.2 \mathrm{~N}[1]$
b The tension in the string. [1]
c The stone describes a circle, therefore the angle between the velocity and the acceleration (or centripetal force) must be $90^{\circ}$. [1]

7 a Speed $=\frac{\text { distance }}{\text { time }}$
speed, $v=\frac{2 \pi r}{0.85}=\frac{2 \pi \times 0.12}{0.85} \quad[1] ; \quad v=0.887 \mathrm{~m} \mathrm{~s}^{-1} \approx 0.89 \mathrm{~m} \mathrm{~s}^{-1}[1]$
b $\quad F=m a=m\left(\frac{v^{2}}{r}\right)[1] ; \quad F=\frac{0.040 \times 0.887^{2}}{0.12} \quad[1] ; \quad F \approx 0.26 \mathrm{~N} \quad[1]$
8 a Kinetic energy at $\mathbf{B}=$ loss of gravitational potential energy from $\mathbf{A}$ to $\mathbf{B}$
$\frac{1}{2} m v^{2}=m g h$ or $v=\sqrt{2 g h}[1]$
$v=\sqrt{2 \times 9.81 \times 5.2}=10.1 \mathrm{~m} \mathrm{~s}^{-1} \approx 10 \mathrm{~m} \mathrm{~s}^{-1}[1]$
b i $\quad a=\frac{v^{2}}{r}[1] ; \quad a=\frac{10.1^{2}}{16}[1] ; \quad a=6.38 \mathrm{~m} \mathrm{~s}^{-2} \approx 6.4 \mathrm{~m} \mathrm{~s}^{-2} \quad[1]$
ii Net force = $m a$

$$
\begin{aligned}
& R-m g=m a \\
& R=m g+m a=m(a+g)=70(6.38+9.81) \\
& R \approx 1.1 \times 10^{3} \mathrm{~N}
\end{aligned}
$$



9 a Resolving vertically $\Rightarrow$
vertical component of $R=$ weight of the car
$R \cos \theta=m g \quad[1] ; \quad R=\frac{820 \times 9.81}{\cos 20^{\circ}}=8.56 \times 10^{3} \mathrm{~N} \quad[1]$

b Resolving horizontally $\Rightarrow$
horizontal component of $R=$ centripetal force $=\frac{m v^{2}}{r}[1]$
$R \sin \theta=\frac{m v^{2}}{r}$ [1]
$r=\frac{820 \times 32^{2}}{8.56 \times 10^{3} \times \sin 20^{\circ}} \quad[1] ; \quad r \approx 290 \mathrm{~m} \quad[1]$


10 Net force $=\frac{m v^{2}}{r}$
net force $=\frac{0.120 \times 4.0^{2}}{0.80}=2.4 \mathrm{~N} \quad[2]$
Weight $W$ of stone $=m g=0.120 \times 9.81=1.18 \approx 1.2 \mathrm{~N}[1]$
At the top:
$W+T_{\mathrm{B}}=2.4 \quad$ so $\quad T_{\mathrm{B}}=2.4-1.2=1.2 \mathrm{~N} \quad[1]$


At the bottom: $T_{\mathrm{A}}-W=2.4$ so $T_{\mathrm{A}}=2.4+1.2=3.6 \mathrm{~N}$ [1]
ratio $=\frac{T_{\mathrm{A}}}{T_{\mathrm{B}}}=\frac{3.6}{1.2}=3.0 \quad[1]$

## Marking scheme

## End-of-chapter test

1 a Angular displacement is the angle through which an object has moved in a circular path. (This is often quoted in radians.) [1]
b $\quad \theta=\frac{64}{360} \times 2 \pi=1.12 \mathrm{rad} \approx 1.1 \mathrm{rad}$ (Note: rad is short for radian.) [1]
2 a


Correct arrow for velocity (v). [1]
Correct arrow for acceleration (a). [1]
b $\quad a=\frac{v^{2}}{r}[1]$
c $\quad F=m a=m\left(\frac{v^{2}}{r}\right)[1] ; \quad F=\frac{850 \times 20^{2}}{32}[1] ; \quad F=1.06 \times 10^{4} \mathrm{~N} \approx 1.1 \times 10^{4} \mathrm{~N}[1]$
d Friction between the tyres and the road. [1]
3 a $\quad a=\frac{v^{2}}{T}=\frac{12^{2}}{20}[1] ; \quad a=7.2 \mathrm{~m} \mathrm{~s}^{-2}[1]$
Net force $=m a$
b
$W-R=m a[1] ; \quad m g-R=m a$ so $(800 \times 9.81)-R=800 \times 7.2 \quad[1]$
$R=800(9.81-7.2) \approx 2.1 \times 10^{3} \mathrm{~N}[1]$
c $m g-R=m$ a (from $\mathbf{b}$ )
When the contact force $R$ is zero:
$m a=m g$ or $a=g$ [1]
However:
$a=\frac{v^{2}}{r}$ hence $\frac{v^{2}}{r}=g[1]$ (so $v^{2}=\mathrm{gr}$ or $v=f{ }_{\mathrm{gr}}$ )
d $v=\sqrt{g r} ; v=\sqrt{9.81 \times 20} \approx 14 \mathrm{~m} \mathrm{~s}^{-1} \quad[1]$

