Chapter 7
Capacitors

Worksheet

Intermediate level

1 A 30 ∝F capacitor is connected to a 9.0 V battery. Calculate:

a the charge on the capacitor; [2]

b the number of excess electrons on the negative plate of the capacitor. [2]

(Elementary charge $e = 1.6 \quad 10^{-19}$ C.)

2 The p.d. across a capacitor is 3.0 V and the charge on the capacitor is 150 nC. Determine the charge on the capacitor when the p.d. is:

a 6.0 V; [2]

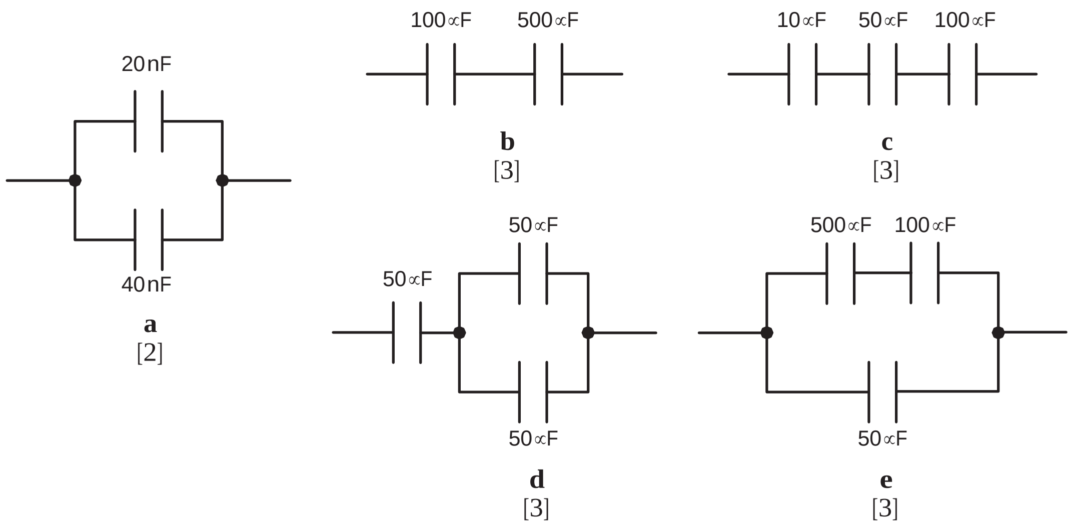
b 9.0 V. [2]

3 A 1000 ∝F capacitor is charged to a potential difference of 9.0 V.

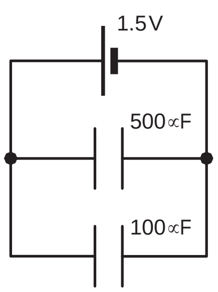
a Calculate the energy stored by the capacitor. [2]

b Determine the energy stored by the capacitor when the p.d. across it is doubled. [2]

4 For each circuit below, determine the total capacitance of the circuit.



5 The diagram shows an electrical circuit.



a Calculate the total capacitance of the two capacitors in parallel. [2]

b What is the potential difference across each capacitor? [1]

c Calculate the total charge stored by the circuit. [2]

d What is the total energy stored by the capacitors? [2]

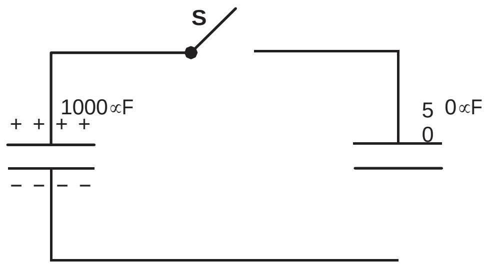
Higher level

6 A 10 000 ∝F capacitor is charged to its maximum operating voltage of 32 V. The charged capacitor is discharged through a filament lamp. The flash of light from the lamp lasts for 300 ms. Calculate:

a the energy stored by the capacitor; [2]

b the average power dissipated in the filament lamp. [2]

7 The diagram shows a 1000 ∝F capacitor charged to a p.d. of 12 V.



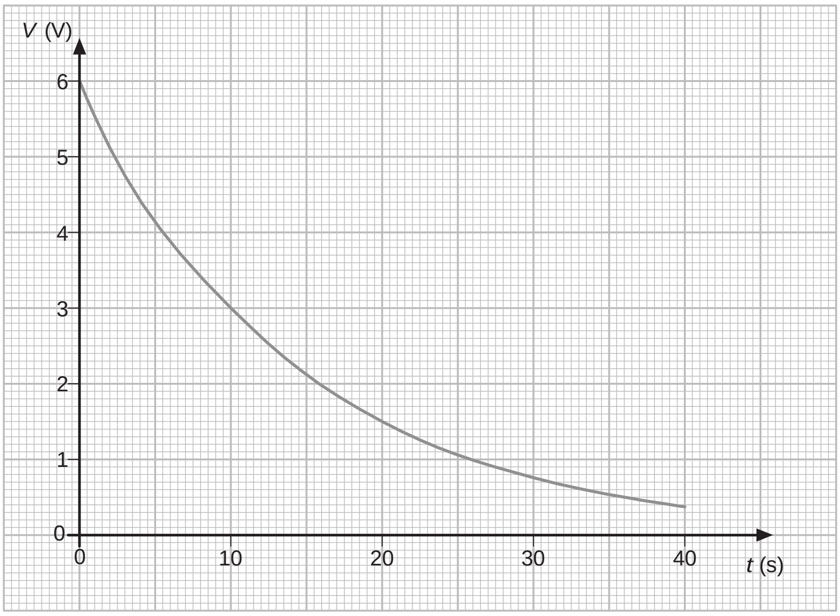
a Calculate the charge on the 1000 ∝F capacitor. [2]

b The 1000 ∝F capacitor is connected across an **uncharged** 500 ∝F capacitor by closing the switch **S**. The charge initially stored by the 1000 ∝F capacitor is now shared with the 500 ∝F capacitor.

i Calculate the total capacitance of the capacitors in parallel. [2]

ii Show that the p.d. across each capacitor is 8.0 V. [2]

8 The diagram shows a voltage–time graph for a capacitor discharging through a 100 k& resistor.



Use the graph to determine:

a the initial current in the circuit; [2]

b the time constant of the circuit; [2]

c the capacitance *C* of the capacitor (use your answer to **b**). [2]

9 A 220 ∝F capacitor is charged to a potential difference of 8.0 V and then discharged through a resistor of resistance 1.2 M&.

a Determine the time constant | of the circuit. [2]

b Calculate:

i the initial current in the circuit; [2]

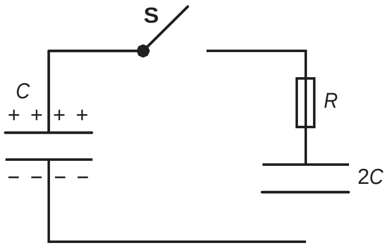
ii the current in the circuit after a time equal to 2 ; [2]

iii the p.d. across the capacitor after a time of 50 s. [3]

Extension

10 A 100 ∝F capacitor is discharged through a resistor of resistance 470 k&. Determine the 'half-life' of this circuit. (The half-life of the circuit is the time taken for the voltage across the capacitor to decrease to 50% of its initial value.) [5]

11 The diagram shows a charged capacitor of capacitance *C*. When the switch **S** is closed, this capacitor is connected across the uncharged capacitor of capacitance 2*C*. Calculate the percentage of energy lost as heat in the resistor and explain why the actual resistance of the resistor is irrelevant. [7]

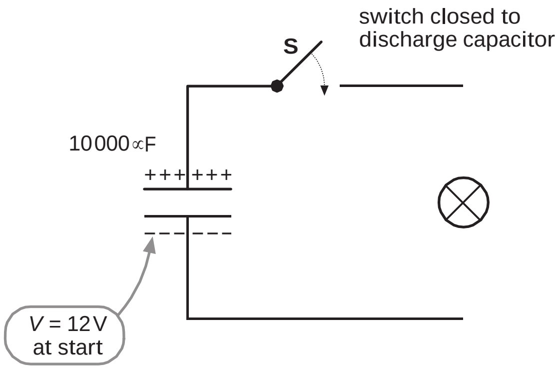


Total: $\frac{\quad}{70}$ Score: %

Worked examples

Example 1

A 10 000 ∝F capacitor is charged to a potential difference of 12 V. The capacitor is then discharged through a filament lamp. Calculate the average power dissipated by the lamp given that the flash of light lasts for 30 ms. You may assume that all the energy stored by the capacitor is transferred to the filament lamp.



The energy stored by the capacitor is given by:

$$E = \frac{1}{2}V^2C$$

Therefore:

$$E = \frac{1}{2}\quad 12^2\quad 10\,000\quad 10^{-6} = 0.72\,\mathrm{J}$$

It is vital that the capacitance is converted into farads. Remember:
1 ∝F $= 10^{-6}$ F

The power P dissipated by the lamp is given by:

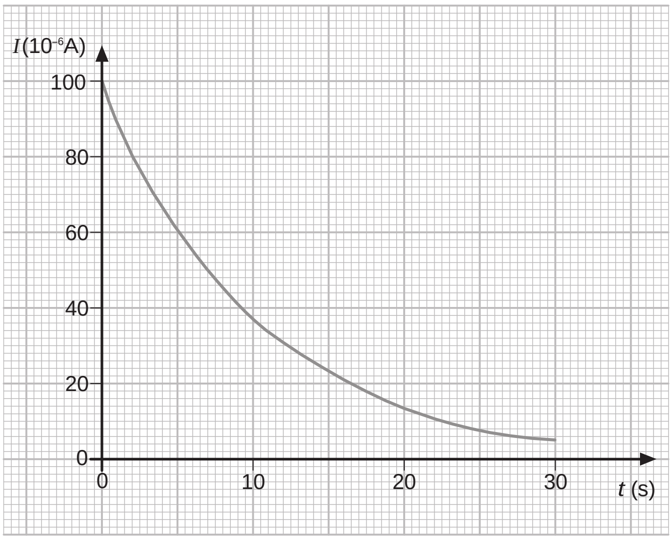
$$P = \frac{E}{t} = \frac{0.72}{30\quad 10^{-3}} = 24\,\mathrm{W}$$

Tip

You can also use the equations $Q = VC$ and $E = \frac{1}{2}QV$ to determine the energy stored by the capacitor.

Example 2

The graph shows the variation of the current with time in a circuit consisting of a discharging capacitor placed across a 120 k& resistor. What was the initial potential difference across the capacitor? Use the graph to determine the time constant of the circuit and hence the value for the capacitance *C* of the capacitor.



The p.d. across the capacitor is a maximum at time $t = 0$. Hence:

$V = IR$

$V = 100 \quad 10^{-6} \quad 120 \quad 10^{3} = 12\,\text{V}$

The p.d. across the capacitor is equal to the p.d. across the resistor because they are connected in parallel.

In a time equal to the time constant | of the circuit, the current in the circuit decreases to 37% of its initial value (37 ∝A). From the graph:

$\quad 10\,\text{s}$

Since $\quad = CR$, we have:

$$C = \frac{}{R} = \frac{10}{120 \quad 10^{3}} \quad 8.3 \quad 10^{-5}\,\text{F} \quad (C \quad 83\,\text{∝F})$$

Tip

You can also determine the capacitance *C* by using $I = I_0\, e^{-t/CR}$ and a particular value from the graph. This requires you to solve the above equation, which can be tricky!

Practical 1

Determining the capacitance of a parallel-plate capacitor

Safety

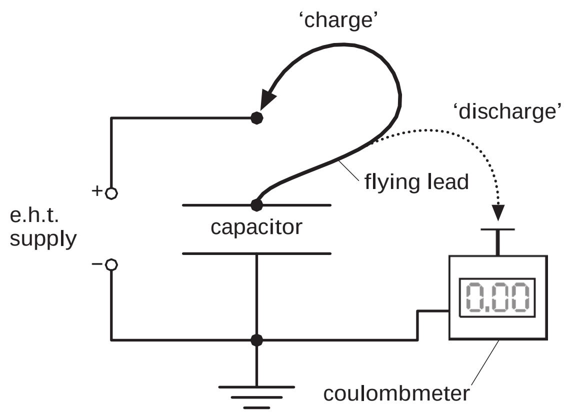
The usual safety rules apply when using an e.h.t. supply. To prevent permanent damage, under no circumstances should the coulombmeter be connected directly to the e.h.t. supply. Teachers and technicians should follow their school and departmental safety policies and should ensure that the employer's risk assessment has been carried out before undertaking any practical work.

Apparatus

- e.h.t. supply
- two large 'capacitor' plates with insulating rods
- coulombmeter
- flying lead on a polythene strip (or plastic ruler)
- clamp stands
- connecting leads

Introduction

The construction of capacitors is described on page 67 of *Physics 2*. The capacitance of a capacitor is defined as the ratio of the charge on the capacitor to the potential difference across it. On page 69, details of an experiment are given where the capacitance of a capacitor is determined by finding the charge on the capacitor from the area under a graph of current against time. The experiment described here uses a coulombmeter to measure the charge. The apparatus is shown in the diagram below.



Procedure

1 Set the separation between the plates to 5.0 cm and the e.h.t. supply to 0.5 kV.

2 Charge the capacitor by momentarily connecting the flying lead to the positive electrode of the e.h.t. supply.

3 Now use the flying lead to discharge the charge stored on the parallel-plate capacitor to the coulombmeter. Measure and record the charge Q measured by the coulombmeter for a p.d. of 0.5 kV.

4 Repeat the experiment for p.d. V in increments of 0.5 kV up to 4.0 kV. Record your results in a table.

V (10^3 V)	Q (10^{-9} C)

5 Plot a graph of potential difference V across the capacitor against the charge Q and draw a straight line of best fit.

6 Explain why the gradient of the graph is equal to the capacitance C of the capacitor. Determine the capacitance of the capacitor. How reliable is your answer?

Guidance for teachers

The capacitance C of the capacitor is given by:

$$C = \frac{\Sigma_0 A}{d}$$

where Σ_0 is the permittivity of free space (8.85 10^{-12} F m^{-1}), d is the separation between the parallel plates, and A is the area of one of the plates. The results from this experiment can be within ±10%.

Practical 2

Determining the capacitance of an unmarked commercial capacitor

Safety

Make sure that when using electrolytic capacitors they are connected with the correct polarity. Teachers and technicians should follow their school and departmental safety policies and should ensure that the employer's risk assessment has been carried out before undertaking any practical work.

Apparatus

- low-voltage power supply (or battery pack)
- digital voltmeter
- commercial capacitor (its rating obscured with tape)
- resistance substitution box
- stopwatch
- connecting leads

Introduction

The importance of time constant is described on pages 78 and 79 of *Physics 2*. In this experiment, the definition for time constant is used to determine the capacitance of an unmarked capacitor.

time constant $= CR$

Hence:

$$C = \frac{}{R}$$

Instead of using a stopwatch, you may determine the time taken for the p.d. across the discharging capacitor to reach 37% of its initial p.d. by using either a datalogger or a storage os cilloscope.

Procedure

1 Set up the apparatus as shown in the diagram.

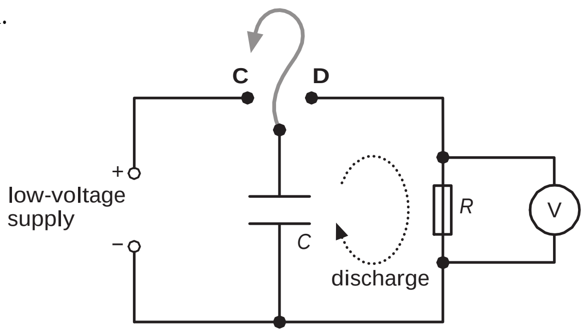
2 Set the low-voltage supply to 10 V.

3 Choose the resistance R of the external resistance to be 220 k&.

4 Connect the flying lead to **C** to charge the capacitor to a p.d. of 10 V.

5 Remove this lead and connect it to **D** and immediately start the stopwatch.

6 Determine the time | taken for the potential difference across the capacitor to decrease to 37% of its initial value. (This time is equal to the time constant of the circuit.)



7 Repeat the experiment for resistances of 100 k&, 47 k&, 22 k& and 10 k& and record your results in a table.

R (&)	**(s)**	**C (∝F)**

8 Determine the value of the capacitance C of the capacitor using $C = \frac{}{R}$. What is your average value for the capacitance, and how reliable is it?

9 You can also determine the value for the capacitance C from the gradient of a graph of against R. Can you explain why?

10 Try different initial p.d.s and investigate whether or not this has any effect on the value of the time constant.

Guidance for teachers

The resistance values given here are for a capacitor of 470 ∝F. For other values of capacitance, the resistance values should be adjusted accordingly.

End-of-chapter test

Answer all questions.

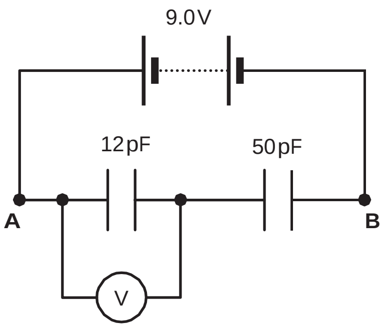
1 **a** Define the capacitance of a capacitor. [1]

b The potential difference across a 120 μF capacitor is 6.0 V. Calculate:

i the charge on the capacitor; [2]

ii the energy stored by the capacitor. [2]

2 The diagram shows an electrical circuit.



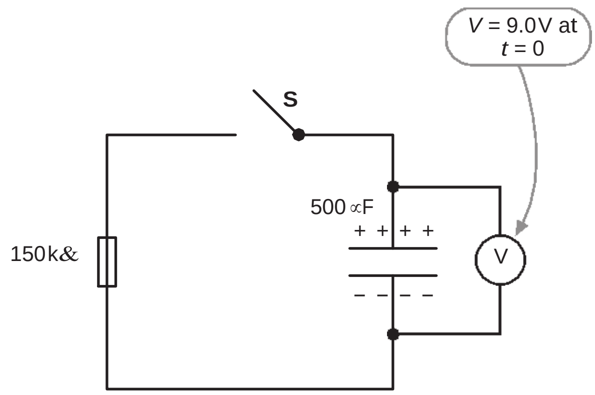
a Determine the total capacitance between points **A** and **B**. [3]

b State the ratio:

$$\frac{\text{charge on the 12 pF capacitor}}{\text{charge on the 50 pF capacitor}}$$ [1]

c Calculate the potential difference across the 12 pF capacitor. [3]

3 The diagram shows a 500 μF capacitor charged to a p.d. of 9.0 V. The switch **S** is then closed.



a Explain why the reading on the high-resistance voltmeter decreases. [2]

b What is the time constant of the circuit? [2]

c Determine the p.d. across the resistor after a time equal to three time constants. [2]

d Calculate the current in the circuit after 20 s. [3]

e Using the axes below, sketch a graph of p.d. V against time t for the discharging capacitor. [2]

Total: $\frac{\quad}{23}$ Score: %

Marking scheme

Worksheet

1 **a** $Q = VC = 9.0 \quad 30 \quad 10^{-6}$ [1]

$Q = 2.7 \quad 10^{-4}\,\mathrm{C}$ (270 $\propto$C) [1]

b Number of excess electrons $= \dfrac{Q}{e} = \dfrac{2.7 \quad 10^{-4}}{1.6 \quad 10^{-19}}$ [1]

number $\quad 1.7 \quad 10^{15}$ [1]

2 **a** The charge is directly proportional to the voltage across the capacitor. Hence doubling the voltage will double the charge. [1]

charge = 2 150 = 300 nC [1]

b Since $Q \propto V$ for a given capacitor, increasing the voltage by a factor of three will increase the charge by the same factor. [1]

charge = 3 150 = 450 nC [1]

3 **a** $E = \dfrac{1}{2}V^2C = \dfrac{1}{2} \quad 9.0^2 \quad 1000 \quad 10^{-6}$ [1]

$E = 4.05 \quad 10^{-2}\,\mathrm{J} \quad 4.1 \quad 10^{-2}\,\mathrm{J}$ [1]

b For a given capacitor, energy stored $\propto$ voltage^2. [1]

energy $= 2^2 \quad 4.05 \quad 10^{-2} \quad 0.16\,\mathrm{J}$ [1]

4 **a** $C_{\mathrm{total}} = C_1 + C_2$ [1]; $C_{\mathrm{total}} = 20 + 40 = 60\,\mathrm{nF}$ [1]

b $\dfrac{1}{C_{\mathrm{total}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ [1]

$\dfrac{1}{C_{\mathrm{total}}} = \dfrac{1}{100} + \dfrac{1}{500} = 1.2 \quad 10^{-2}$ [1]; $C_{\mathrm{total}} = \dfrac{1}{1.2 \quad 10^{-2}} \quad 83\,\propto\mathrm{F}$ [1]

c $\dfrac{1}{C_{\mathrm{total}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$ [1]

$\dfrac{1}{C_{\mathrm{total}}} = \dfrac{1}{10} + \dfrac{1}{50} + \dfrac{1}{100} = 1.3 \quad 10^{-1}$ [1]; $C_{\mathrm{total}} = \dfrac{1}{1.3 \quad 10^{-1}} \quad 7.7\,\propto\mathrm{F}$ [1]

d Total capacitance of the two capacitors in parallel = 50 + 50 = 100 $\propto$F. [1]

$\dfrac{1}{C_{\mathrm{total}}} = \dfrac{1}{50} + \dfrac{1}{100} = 3.0 \quad 10^{-2}$ [1]; $C_{\mathrm{total}} = \dfrac{1}{3.0 \quad 10^{-2}} \quad 33\,\propto\mathrm{F}$ [1]

e Total capacitance of the two capacitors in series is 83 $\propto$F (from **b**). [2]

$C_{\mathrm{total}} = 83 + 50 = 1\,33\,\propto\mathrm{F} \quad 130\,\propto\mathrm{F}$ [1]

5 **a** $C_{\mathrm{total}} = C_1 + C_2$ [1]; $C_{\mathrm{total}} = 100 + 500 = 600\,\propto\mathrm{F}$ [1]

b The potential difference across parallel components is the same and equal to 1.5 V. [1]

c $Q = VC = 1.5 \quad 600 \quad 10^{-6}$ [1]; $Q = 9.0 \quad 10^{-4}\,\mathrm{C}$ (900 $\propto$C) [1]

d $E = \dfrac{1}{2}QV = \dfrac{1}{2} \quad 9.0 \quad 10^{-4} \quad 1.5$ [1]

$E = 6.75 \quad 10^{-4}\,\mathrm{J} \quad 6.8 \quad 10^{-4}\,\mathrm{J}$ [1]

6 **a** $E = \dfrac{1}{2}V^2C = \dfrac{1}{2} \quad 32^2 \quad 10\,000 \quad 10^{-6}$ [1]

$E = 5.12\,\mathrm{J} \quad 5.1\,\mathrm{J}$ [1]

b $P = \frac{E}{t} = \frac{5.12}{0.300}$ [1]; $P \quad 17\,\text{W}$ [1]

7 **a** $Q = VC = 12 \quad 1000 \quad 10^{-6}$ [1]; $Q = 1.2 \quad 10^{-2}\,\text{C}$ (12 mC) [1]

b **i** $C_{\text{total}} = C_1 + C_2$ [1]; $C_{\text{total}} = 1000 + 500 = 1500\,\propto\text{F}$ [1]

ii $V = \frac{Q}{C}$ (The charge Q is conserved and C is the total capacitance.) [1]

$V = \frac{1.2 \quad 10^{-2}}{1500 \quad 10^{-6}} = 8.0\ \text{V}$ [1]

8 **a** $I = \frac{V}{R} = \frac{6.0}{100 \quad 10^{3}}$ [1]; $I = 6.0 \quad 10^{-5}\,\text{A}$ (60 ∝A) [1]

b After a time equal to one time constant, , the current will be $\frac{1}{\text{e}}$ (37%) of its initial value. The voltage after time will be:

0.37 6.0 2.2 V [1]

Therefore 15 s (allow ±2 s) [1]

c $= CR$ [1]; $C = \frac{}{R} = \frac{15}{100 \quad 10^{3}} = 1.5 \quad 10^{-4}\,\text{F}$ (150 ∝F) [1]

9 **a** $= CR = 220 \quad 10^{-6} \quad 1.2 \quad 10^{6}$ [1]; $= 264\,\text{s} \quad 260\,\text{s}$ [1]

b **i** $I = \frac{V}{R} = \frac{8.0}{1.2 \quad 10^{6}}$ [1]; $I = 6.67 \quad 10^{-6}\,\text{A} \quad 6.7\,\propto\text{A}$ [1]

ii After a time equal to two time constants, the current will be:

$\left(\frac{1}{\text{e}}\right)^2 \quad 6.67 \quad 10^{-6} = (0.37)^2 \quad 6.67 \quad 10^{-6}$ [1]

current 9.1 10^{-7} A (0.91 ∝A) [1]

iii $V = V_0\,\text{e}^{-t/CR}$ [1]; $V = 8.0 \quad \text{e}^{-(50/264)}$ [1]; $V = 6.62\,V \quad 6.6\,\text{V}$ [1]

10 $CR = 100 \quad 10^{-6} \quad 470 \quad 10^{3}$ [1]; $CR = 47\,\text{s}$ [1]

$V = V_0\,\text{e}^{-t/CR}$ [1]

$\frac{V}{V_0} = 0.5$, therefore: $0.5 = \text{e}^{-t/47}$ [1]

$\ln(0.5) = -\frac{t}{47}$ so $t = -\ln(0.5) \quad 47 = 32.6\,\text{s} \quad 33\,\text{s}$ [1]

11 The capacitors are in series so the total capacitance = 3C. [1]

The total charge Q remains constant. [1]

The energy stored by a capacitor is given by: $E = \frac{1}{2}\frac{Q^2}{C}$ [1]

$E_{\text{initial}} = \frac{1}{2}\frac{Q^2}{C}$ and $E_{\text{final}} = \frac{1}{2}\frac{Q^2}{3C}$ [1]

Fraction of energy left $= \frac{E_{\text{final}}}{E_{\text{initial}}} = \frac{Q^2/2(3C)}{Q^2/2C} = \frac{1}{3}$ [1]

Fraction of energy 'lost' as heat in resistor $= 1 - \frac{1}{3} = \frac{2}{3}$(67% of the initial energy is lost as heat in the resistor) [1]

The resistance governs how long it takes for the capacitor to discharge. The final voltage across each capacitor is independent of the resistance. Hence, the energy lost as heat is independent of the actual resistance of the resistor. [1]

Marking scheme

End-of-chapter test

1 **a** The capacitance of a capacitor is the ratio of the charge stored to the voltage across the capacitor. [1]

b **i** $Q = VC = 6.0 \quad 120 \quad 10^{-6}$ [1]; $Q = 7.2 \quad 10^{-4}$ C (720 ∝C) [1]

ii $E = \frac{1}{2}QV = \frac{1}{2} \quad 7.2 \quad 10^{-4} \quad 6.0$ [1]

$E = 2.16 \quad 10^{-3}$ J $\quad 2.2 \quad 10^{-3}$ J [1]

2 **a** $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$ [1]

$\frac{1}{C_{total}} = \frac{1}{12} + \frac{1}{50} = 1.033 \quad 10^{-1}$ [1]; $C_{total} = \frac{1}{1.033 \quad 10^{-1}} = 9.68$ pF $\quad 9.7$ pF [1]

b The capacitors are connected in series, therefore the charge on each capacitor is the same. Hence:

ratio = 1 [1]

c Considering the whole circuit:

$Q = VC = 9.0 \quad 9.68 \quad 10^{-12}$ [1]; $Q = 8.7 \quad 10^{-11}$ C [1]

Now considering the 12 pF capacitor:

$V = \frac{Q}{C} = \frac{8.7 \quad 10^{-11}}{12 \quad 10^{-12}} \quad 7.3$ V [1]

3 **a** The capacitor discharges through the resistor and therefore the charge decreases exponentially. [1]

The voltage across the capacitor is directly proportional to the charge ($Q = VC$), hence the voltage across the capacitor also decreases. [1]

b $= CR = 500 \quad 10^{-6} \quad 150 \; 1 \;\; 0^{3}$ [1]; $= 75$ s [1]

c After a time equal to three time constants, the voltage will be:

$\left(\frac{1}{e}\right)^{3} \quad 9.0 = (0.37)^{3} \quad 9.0$ [1]

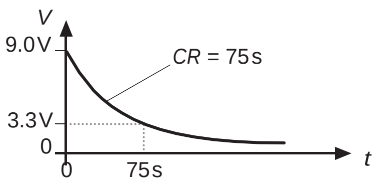
voltage $\quad 0.46$ V [1]

d $I_0 = \frac{V}{R} = \frac{9.0}{150 \quad 10^{3}} = 6.0 \quad 10^{-5}$ A (This is the maximum current at $t = 0$.) [1]

$I = I_0 e^{-t/CR} = 6.0 \quad 10^{-5} \quad e^{-20/75}$ [1]

$I \quad 4.6 \quad 10^{-5}$ A [1]

e



A smooth curve starting at 9.0 V. [1]

At $t = 75$ s, voltage is about 37% of initial value (3.3 V). [1]