

Chapter 9

Electromagnetic induction

Worksheet

Worked examples

Practical: Investigating the e.m.f. induced in a coil by a falling magnet

End-of-chapter test

Marking scheme: Worksheet

Marking scheme: End-of-chapter test

Worksheet

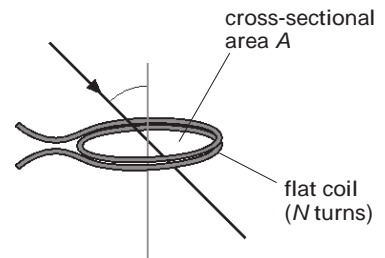
Intermediate level

- 1** A flat coil of N turns and cross-sectional area A is placed in a uniform magnetic field of flux density B . The plane of the coil is normal to the magnetic field.

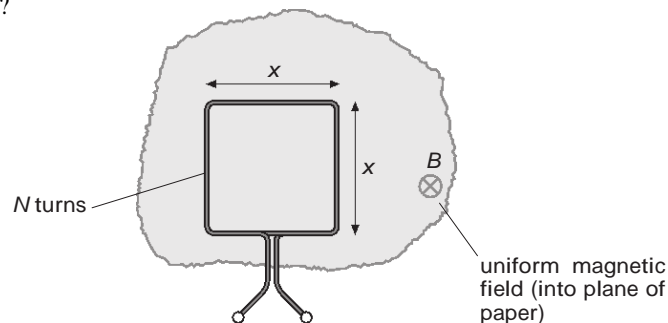
a Write an equation for:

- i** the magnetic flux through the coil; [1]
ii the magnetic flux linkage for the coil. [1]

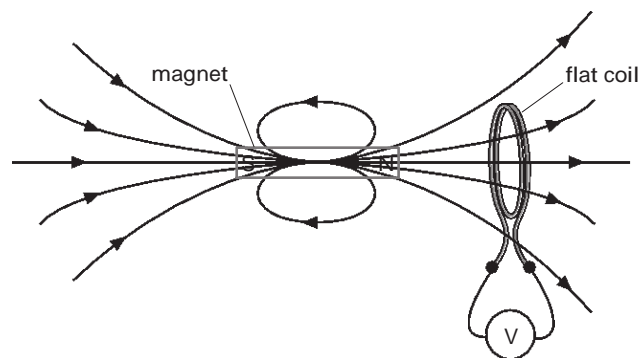
- b** The diagram shows the coil when the magnetic field is at an angle to the normal of the plane of the coil. What is the flux linkage for the coil? [1]



- 2** A square coil of N turns is placed in a uniform magnetic field of magnetic flux density B . Each side of the coil has length x . What is the magnetic flux linkage for this coil? [2]



- 3** The diagram shows a magnet placed close to a flat circular coil.



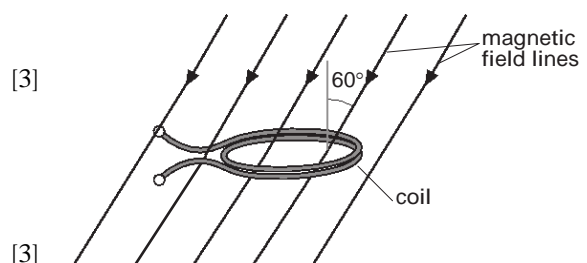
- a** Explain why there is no induced e.m.f. even though there is magnetic flux linking the coil. [1]

- b** Explain why there is an induced e.m.f. when the magnet is pushed towards the coil. [2]

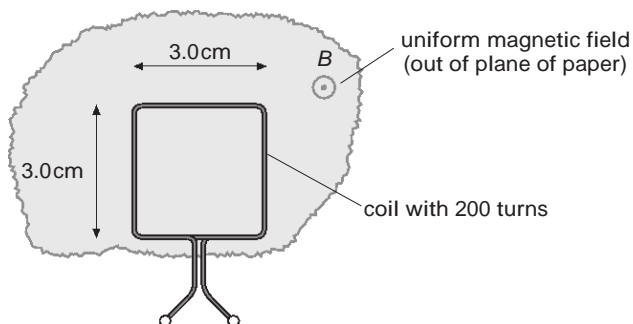
- 4** A coil of cross-sectional area $4.0 \times 10^{-4} \text{ m}^2$ and 70 turns is placed in a uniform magnetic field.

- a** The plane of the coil is at right-angles to the magnetic field. Calculate the magnetic flux density when the flux linkage for the coil is $1.4 \times 10^{-4} \text{ Wb}$. [3]

- b** The coil is placed in a magnetic field of flux density 0.50 T . The normal to the coil makes an angle of 60° to the magnetic field, as shown in the diagram. Calculate the flux linkage for the coil. [3]



- 5 A square coil is placed in a uniform magnetic field of flux density 40 mT.

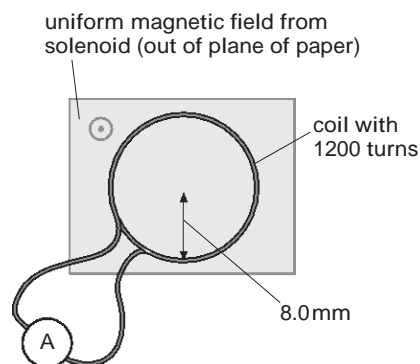


The plane of the coil is normal to the magnetic field. The coil has 200 turns and the length of each side of the coil is 3.0 cm.

- a Calculate:
- the magnetic flux through the coil; [2]
 - the magnetic flux linkage for the coil. [2]
- b The plane of the coil is turned through 90° . What is the change in the magnetic flux linkage for the coil? [2]

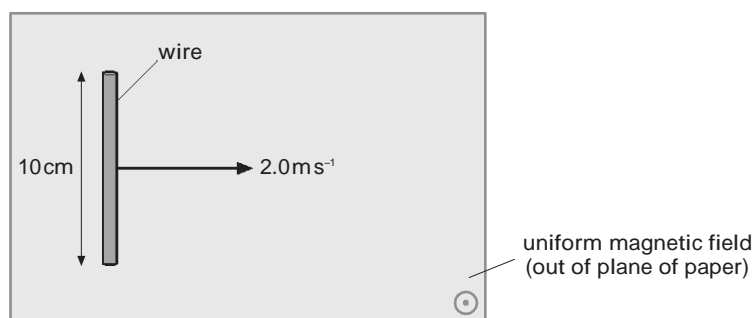
Higher level

- 6 A flat circular coil of 1200 turns and of mean radius 8.0 mm is connected to an ammeter of negligible resistance. The coil has a resistance of 6.3Ω . The plane of the coil is placed at right-angles to a magnetic field of flux density 0.15 T from a solenoid. The current in the solenoid is switched off. It takes 20 ms for the magnetic field to decrease from its maximum value to zero. Calculate:



- the average magnitude of the induced e.m.f. across the ends of the coil; [5]
- the average current measured by the ammeter. [2]

- 7 The diagram shows a straight wire of length 10 cm moved at a constant speed of 2.0 m s^{-1} in a uniform magnetic field of flux density 0.050 T.



For a period of 1 second, calculate:

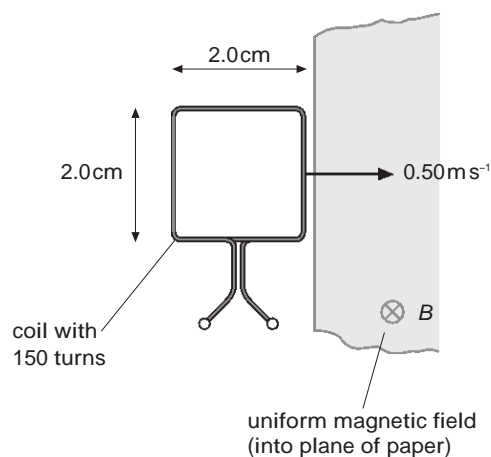
- the distance travelled by the wire; [1]
- the area swept by the wire; [1]
- the change in the magnetic flux for the wire (or the magnetic flux 'cut' by the wire); [2]
- the e.m.f. induced across the ends of the wire using your answer to c; [2]
- the e.m.f. induced across the ends of the wire using $E = Bvl$. [1]

- 8** A circular coil of radius 1.2 cm has 2000 turns. The coil is placed at right-angles to a magnetic field of flux density 60 mT. Calculate the average magnitude of the induced e.m.f. across the ends of the coil when the direction of the magnetic field is **reversed** in a time of 30 ms. [5]

Extension

- 9** The diagram shows a square coil about to enter a region of uniform magnetic field of magnetic flux density 0.30 T.

The magnetic field is at right-angles to the plane of the coil. The coil has 150 turns and each side is 2.0 cm in length. The coil moves at a constant speed of 0.50 m s^{-1} .



- a** Calculate:
- the time taken for the coil to enter completely the region of magnetic field; [1]
 - the magnetic flux linkage through the coil when it is all within the region of magnetic field. [2]
- b** Explain why the induced e.m.f. is a constant value when the coil is entering the magnetic field. [1]
- c** Use your answer to **a** to determine the induced e.m.f. across the ends of the coil. [4]
- d** What is the induced e.m.f. across the ends of the coil when it is completely within the magnetic field? Explain your answer. [2]
- 10** A wire of length l is placed in a uniform magnetic field of flux density B . The wire is moved at a constant velocity v at right-angles to the magnetic field. Use Faraday's law of electromagnetic induction to show that the induced e.m.f. E across the ends of the wire is given by:

$$E = Bvl$$

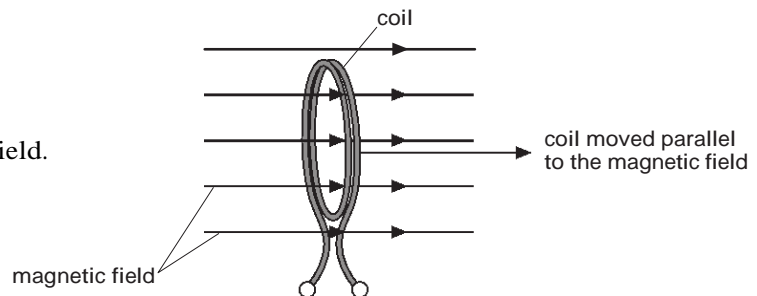
Hence calculate the e.m.f. induced across the ends of a 20 cm long rod rolling along a horizontal table at a speed of 0.30 m s^{-1} . (The vertical component of the Earth's magnetic flux density is about $40 \mu\text{T}$.) [8]

Total: $\frac{\quad}{57}$ Score: %

Worked examples

Example 1

The diagram shows a coil of radius 4.0 cm and 1000 turns placed in a uniform magnetic field of flux density 0.060 T. The plane of the coil is at right-angles to the magnetic field.



Determine the magnetic flux through the coil and the magnetic flux linkage. Explain why there is no induced e.m.f. across the ends of the coil when it is moved in a direction parallel to the magnetic field.

The cross-sectional area A of the coil is:

$$A = r^2 = 0.04^2 = 5.03 \times 10^{-3} \text{ m}^2$$

$$\text{magnetic flux, } \Phi = BA = 0.060 \times 5.03 \times 10^{-3}$$

Therefore:

$$\Phi = 3.02 \times 10^{-4} \text{ Wb} \quad 3.0 \times 10^{-4} \text{ Wb}$$

$$\text{flux linkage} = N \Phi = 1000 \times 3.02 \times 10^{-4} = 0.30 \text{ Wb}$$

Magnetic flux and magnetic flux linkage have the same unit, the weber (Wb).

According to Faraday's law of electromagnetic induction, there can only be an induced e.m.f. when there is **change of magnetic flux linkage**. When the coil is moved along the magnetic field, there is no change in the magnetic flux linkage. Hence there is no induced e.m.f.

Tip

You can answer many electromagnetic induction questions using Faraday's law. In mathematical form this is written as: $E = N \frac{d\Phi}{dt}$. There is an induced e.m.f. in a circuit only when there is **change** in the magnetic flux.

Example 2

A rectangular coil of 100 turns has length 2.0 cm and width 1.0 cm. The coil is placed in a uniform magnetic field between the poles of a strong electromagnet. The magnetic field has a magnetic flux density of 0.30 T. The coil is withdrawn from the field in a time of 0.15 s. Determine the magnitude of the average induced e.m.f. across the ends of the coil.

$$\text{initial magnetic flux} = BA = 0.30 \times (0.02 \times 0.01) = 6.0 \times 10^{-5} \text{ Wb}$$

$$\text{final magnetic flux} = 0$$

According to Faraday's law, we have:

$$\text{magnitude of induced e.m.f.} = \text{rate of change of magnetic flux linkage}$$

Therefore:

$$E = 100 \times \frac{0 - 6.0 \times 10^{-5}}{0.15} = 4.0 \times 10^{-2} \text{ V (magnitude only)}$$

$$E = 0.04 \text{ V}$$

$$E = N \frac{d\Phi}{dt}$$

Practical

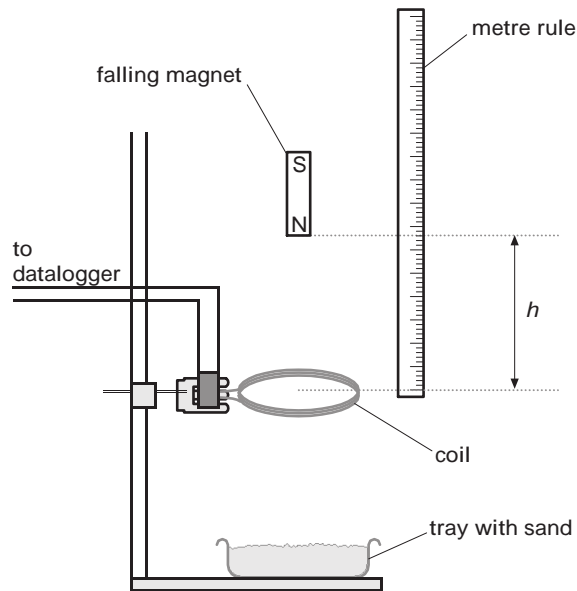
Investigating the e.m.f. induced in a coil by a falling magnet

Safety

There are not likely to be any major hazards in carrying out this experiment. However, teachers and technicians should always refer to the departmental risk assessment before carrying out any practical work.

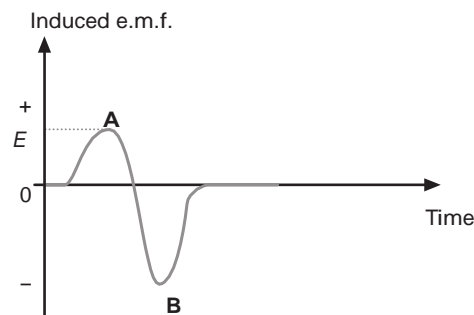
Apparatus

- Helmholtz coil
- bar magnet
- datalogger or storage oscilloscope
- metre rule
- clamp stand
- tray of sand



Introduction

The details of Faraday's law are given on pages 93 and 94 of *Physics 2*. When a bar magnet is dropped through a coil, an e.m.f. is induced across the ends of the coil. This e.m.f. has the form shown in the diagram.



Note: The areas under the peaks are same.

When the leading pole of the magnet enters the plane of the coil, the rate of change of magnetic flux linkage is a maximum (as shown by peak A). The speed of the accelerating magnet is greater when it exits the plane of the coil, resulting in a larger peak (B). (The peak is negative because the pole exiting is the opposite polarity.) What is the relationship between the maximum induced e.m.f. E when the magnet enters the plane of the coil and the instantaneous speed of the falling magnet? This question forms the basis of this experiment.

Procedure

- 1 Drop the magnet from a height h of 20 cm above the plane of the coil.
- 2 Use the trace from the datalogger to measure the maximum induced e.m.f. E when the magnet enters the coil.
- 3 Determine the speed v of the magnet as it just enters the coil. (Use $v^2 = u^2 + 2as$. In this case $u = 0$, $a = 9.81 \text{ m s}^{-2}$ and $s = h$. Therefore $v = \sqrt{2ah}$.)
- 4 Repeat the experiment for different heights.
- 5 Record your results in a table.

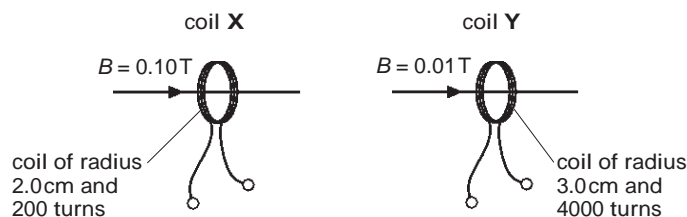
h (m)	v (ms⁻¹)	E (V)

- 6 Plot a graph of induced e.m.f. E against speed v of the falling magnet.
- 7 Draw a straight line of best fit.
- 8 Can you use Faraday's law to explain why the graph is a straight line?
- 9 Repeat the measurements using the results for the second peak **B**. Do you arrive at the same conclusions?

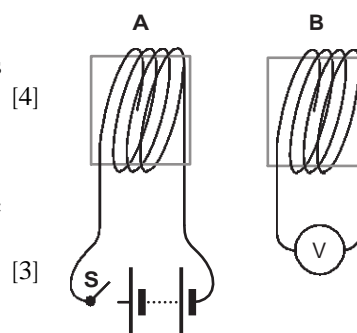
End-of-chapter test

Answer all questions.

- 1 a** State Faraday's law of electromagnetic induction. [1]
- b** Lenz's law expresses an important conservation law. Name this conservation law. [1]
- c i** Define magnetic flux for a coil placed at right-angles to a magnetic field. [1]
- ii** Determine for which of the two coils **X** and **Y**, each placed at right-angles to the magnetic field, is the magnetic flux linkage the greatest. [4]



- 2** The diagram shows two coils **A** and **B** placed close to each other.
- a** The switch **S** is closed. Explain why the voltmeter placed across coil **B** indicates an induced e.m.f. for a short period of time. [4]
- b** The coil **A** has 200 turns and cross-sectional area $9.0 \times 10^{-4}\text{ m}^2$. With the switch **S** closed, the current through the coil **A** produces a uniform magnetic field within the coil of magnetic flux density $2.5 \times 10^{-3}\text{ T}$. Calculate the magnetic flux linkage for this coil. [3]



- 3** A rectangular coil of length 3.0 cm and width 2.0 cm has 100 turns. The coil is placed at right-angles to a uniform magnetic field of magnetic flux density $1.2 \times 10^{-2}\text{ T}$.
- a** The coil is removed from the magnetic field in a time of 50 ms. Calculate the magnitude of the average induced e.m.f. across the ends of the coil. [5]
- b** Explain how your answer to **a** would change if the magnetic field were parallel to the plane of the coil. [2]

Total: $\frac{\quad}{21}$ Score: %

Marking scheme

Worksheet

- 1 a i** Magnetic flux, $\Phi = BA$ [1]
ii Flux linkage $= N\Phi = NBA$ [1]
- b** Flux linkage $= N(B \cos \theta)A = NBA \cos \theta$ [1]
(The component of B normal to the plane of the coil is $B \cos \theta$.)
- 2** Area A of coil $= x^2$ [1]
flux linkage $= NBA = NBx^2$ [1]
- 3 a** There is no change to the magnetic flux linking the coil, hence according to Faraday's law, there is no induced e.m.f. [1]
- b** The magnetic flux density B increases as the magnet moves towards the coil. [1]
There is an increase in the magnetic flux linking the coil, hence an e.m.f. is induced across the ends of the coil. [1]
- 4 a** Flux linkage $= NBA$ so $B = \frac{\text{flux linkage}}{NA}$ [1]
$$B = \frac{1.4 \times 10^{-4}}{70 \times 4.0 \times 10^{-4}}$$
 [1]
$$B = 5.0 \times 10^{-3} \text{ T}$$
 [1]
- b** Flux linkage $= NBA \cos \theta$ [1]; flux linkage $= 70 \times 0.050 \times 4.0 \times 10^{-4} \times \cos 60^\circ$ [1]
flux linkage $= 7.0 \times 10^{-4} \text{ Wb}$ [1]
- 5 a i** Magnetic flux, $\Phi = BA = 40 \times 10^{-3} \times (0.03 \times 0.03)$ [1]; $\Phi = 3.6 \times 10^{-5} \text{ Wb}$ [1]
ii Flux linkage $= N\Phi = 200 \times 3.6 \times 10^{-5}$ [1]; flux linkage $= 7.2 \times 10^{-3} \text{ Wb}$ [1]
- b** Final flux linkage $= 0$, initial flux linkage $= 7.2 \times 10^{-3} \text{ Wb}$ [1]
Hence, change in magnetic flux linkage is $7.2 \times 10^{-3} \text{ Wb}$. [1]
- 6 a** Initial magnetic flux $= BA = 0.15 \times (8.0 \times 10^{-3})^2$ [1]
initial magnetic flux $= 3.02 \times 10^{-5} \text{ Wb}$ [1]
final magnetic flux $= 0$ [1]
average magnitude of induced e.m.f. = rate of change of magnetic flux linkage
$$E = N \frac{d\Phi}{dt}, \text{ so } E = 1200 \times \frac{0 - 3.02 \times 10^{-5}}{0.020}$$
 [1]
$$E = 1.81 \text{ V} \approx 1.8 \text{ V}$$
 (magnitude only) [1]
- b** Average current $= \frac{\text{e.m.f.}}{\text{resistance}}$: $I = \frac{1.81}{6.3}$ [1]
$$I = 0.287 \text{ A} \approx 0.29 \text{ A}$$
 [1]

7 a Distance = speed × time = 2.0 × 1.0 = 2.0 m [1]

b Area swept = length × distance travelled = 0.10 × 2.0 = 0.20 m² [1]

c Change in magnetic flux = area swept × magnetic flux density [1]

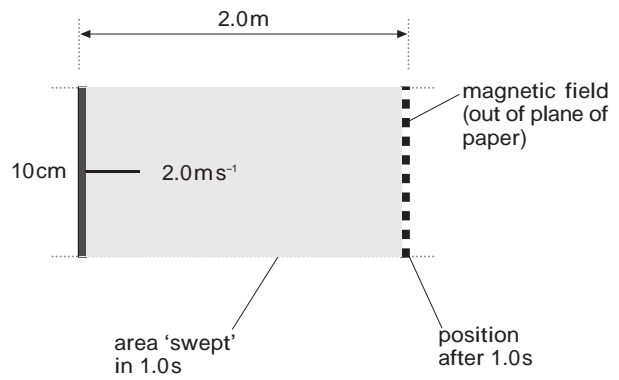
change in magnetic flux = 0.20 × 0.050 = 1.0 × 10⁻² Wb [1]

d Magnitude of e.m.f. = rate of change of magnetic flux linkage [1]

$$E = N \frac{d}{dt} \quad (N = 1)$$

$$E = \frac{1.0 \times 10^{-2}}{1.0} = 1.0 \times 10^{-2} \text{ V (note: } 1 \text{ Wb s}^{-1} = 1 \text{ V)} [1]$$

e $E = Bvl = 0.050 \times 2.0 \times 0.10 = 1.0 \times 10^{-2} \text{ V} [1]$



8 Initial magnetic flux = $BA = 0.060 \times (1.2 \times 10^{-2})^2 [1]$

initial magnetic flux = $2.72 \times 10^{-5} \text{ Wb} [1]$

final magnetic flux = $-2.72 \times 10^{-5} \text{ Wb}$ (since the field is reversed) [1]

average magnitude of induced e.m.f. = rate of change of magnetic flux linkage

$$E = N \frac{d}{dt}, \text{ so } E = 2000 \times \frac{-2.72 \times 10^{-5} - 2.72 \times 10^{-5}}{0.030} [1]$$

$E = 3.62 \text{ V} \approx 3.6 \text{ V}$ (magnitude only) [1]

9 a i Time taken = $\frac{\text{distance}}{\text{speed}} = \frac{0.02}{0.50} = 4.0 \times 10^{-2} \text{ s} [1]$

ii Flux linkage = $NBA = 150 \times 0.30 \times (0.02 \times 0.02) [1]$

flux linkage = $1.8 \times 10^{-2} \text{ Wb} [1]$

b The rate of change of magnetic flux is constant. [1]

c Initial flux linkage = 0 and final flux linkage = $1.8 \times 10^{-2} \text{ Wb} [1]$

magnitude of induced e.m.f. = rate of change of magnetic flux linkage [1]

$$E = \frac{1.8 \times 10^{-2} - 0}{4.0 \times 10^{-2}} [1]; \quad E = 0.45 \text{ V (magnitude only)} [1]$$

d When the coil is completely within the field, the induced e.m.f. is zero. [1]

The reason for this is that there is no change in the magnetic flux linkage. [1]

10 Distance = speed × time = $vt [1]$

area swept = length × distance travelled = $lvt [1]$

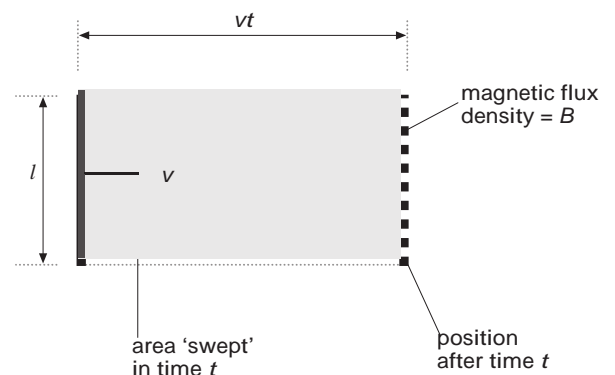
change in magnetic flux = area swept × magnetic flux density [1]

change in magnetic flux = $(lvt) \times B = Blvt [1]$

magnitude of e.m.f. = rate of change of magnetic flux linkage [1]

$$E = \frac{Blvt}{t} = Bvl [1]$$

$E = Bvl = 40 \times 10^{-6} \times 0.30 \times 0.20 [1]; \quad E = 2.4 \times 10^{-6} \text{ V} \quad (2.4 \mu\text{V}) [1]$



Marking scheme

End-of-chapter test

- 1 a** The magnitude of the induced e.m.f. in a circuit is directly proportional to the rate of change of magnetic flux linkage. [1]
- b** Lenz's law expresses the principle of conservation of energy. [1]
- c i** Magnetic flux = magnetic flux density \times cross-sectional area of coil
or $\Phi = BA$ [1]
- ii** Flux linkage = NBA [1]
Coil X: flux linkage = $NBA = 200 \times 0.10 \times (0.02)^2 = 2.5 \times 10^{-2} \text{ Wb}$ [1]
Coil Y: flux linkage = $NBA = 4000 \times 0.01 \times (0.03)^2 = 1.1 \times 10^{-1} \text{ Wb}$ [1]
The coil Y has greater flux linkage. [1]
- 2 a** When the switch S is closed, a current flows in coil A and the magnetic field of coil A increases. [1]
The changing magnetic field of coil A links coil B. [1]
There is a change in the magnetic flux linking coil B, hence according to Faraday's law, an e.m.f. is induced across its ends. [1]
When the current in coil A is constant, then there is no further change in the magnetic flux linking coil B. Therefore there is no induced e.m.f. [1]
- b** Flux linkage = $N\Phi$ [1]
flux linkage = $NBA = 200 \times 2.5 \times 10^{-3} = 9.0 \times 10^{-4}$ [1]; flux linkage = $4.5 \times 10^{-4} \text{ Wb}$ [1]
- 3 a** Initial magnetic flux = $BA = 1.2 \times 10^{-2} \times (0.02 \times 0.03)$ [1]
initial magnetic flux = $7.2 \times 10^{-6} \text{ Wb}$ [1]; final flux = 0 [1]
average magnitude of induced e.m.f. = rate of change of magnetic flux linkage
$$E = N \frac{d\Phi}{dt} \quad \text{so} \quad E = 100 \times \frac{(0 - 7.2 \times 10^{-6})}{0.050}$$
 [1]
 $E = 1.44 \times 10^{-2} \text{ V} = 1.4 \times 10^{-2} \text{ V}$ (magnitude only) [1]
- b** The induced e.m.f. is zero. [1]
With the plane of the coil parallel to the magnetic field, there is no magnetic flux linking the coil. [1]