SAT II Math Formula Reference

MATH LEVEL IIC　　1.3ver

CHAPTER 1　INTRODUCTION TO FUNCTIONS

(f+g)(x)=f(x)+g(x)

(f • g)(x)=f(x) • g(x)

(f/g)(x)=f(x)/g(x)

$(f \circ g)(x)=f(x) \circ g(x)=f(g(x))$

CHAPTER 2　POLYNOMIAL FUNCTIONS

Linear Functions

Distance= $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Distance= $\dfrac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$

Tan$\theta=\dfrac{m_1-m_2}{1+m_1m_2}$ （**m is the slope of l.**）

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Sum of zeros (roots)= $-\dfrac{b}{a}$

Product of zeros (roots)= $\dfrac{c}{a}$

CHAPTER 3　TRIGONOMETRIC FUNCTIONS

Graphs:

$$y=A\bullet f(Bx+C)$$

$$\begin{cases} |A| \text{ is the amplitude} \\ \dfrac{f}{B} \text{ is the period of the graph} \\ -\dfrac{C}{B} \text{ is the phase shift} \end{cases}$$

$\sin\theta \bullet \csc\theta = 1$

$\cos\theta \bullet \sec\theta = 1$

$\tan\theta \bullet \cot\theta = 1$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Quadrant	**I**	**II**	**III**	**IV**
Function: sin,csc	+	+	-	-
cos,sec	+	-	-	+
tan,cot	+	-	+	-

Arcs and Angles

$s = r\theta$

$A = \dfrac{1}{2} r^2 \theta$

Special Angles

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	**2**
sine	**0**	**1**	**0**	**-1**	**0**
cosine	**1**	**0**	**-1**	**0**	**1**
tangent	**0**	**und**	**0**	**und**	**0**
cotangent	**und**	**0**	**und**	**0**	**und**
secant	**1**	**und**	**-1**	**und**	**1**
cosecant	**und**	**1**	**und**	**-1**	**und**

***und: means that the function is undefined because the definition of the function necessitates division by zero.**

	$\frac{\pi}{6} or 30^{\circ}$	$\frac{\pi}{4} or 45^{\circ}$	$\frac{\pi}{3} or 60^{\circ}$
sine	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cosine	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tangent	$\frac{\sqrt{3}}{3}$	**1**	$\sqrt{3}$
cotangent	$\sqrt{3}$	**1**	$\frac{\sqrt{3}}{2}$

secant	2 3 3	√2	**2** √
cosecant	**2**	2	2 3 3

Formulas:

1. $\sin^2 x + \cos^2 x = 1$

2. $\tan x^2 + 1 = \sec^2 x$

3. $\cot^2 x + 1 = \csc^2 x$

4. $\sin(A+B) = \sin A \bullet \cos B + \cos A \bullet \sin B$

5. $\sin(A-B) = \sin A \bullet \cos B - \cos A \bullet \sin B$

6. $\cos(A+B) = \cos A \bullet \cos B - \sin A \bullet \sin B$

7. $\cos(A-B) = \cos A \bullet \cos B + \sin A \bullet \sin B$

8. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \bullet \tan B}$

9. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \bullet \tan B}$

10. $\sin 2A = 2 \sin A \cos A$

11. $\cos 2A = \cos^2 A - \sin^2 A$

12. $\cos 2A = 2\cos^2 A - 1$

13. $\cos 2A = 1 - 2\sin^2 A$

14. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

15. $\sin \frac{1}{2} A = \pm \sqrt{\dfrac{1-\cos A}{2}}$

16. $\cos \frac{1}{2} A = \pm \sqrt{\dfrac{1+\cos A}{2}}$

17. $\tan \frac{1}{2} A = \pm \sqrt{\dfrac{1-\cos A}{1+\cos A}}$

18. $= \dfrac{1-\cos A}{\sin A}$

19. $= \dfrac{\sin A}{1+\cos A}$

***The correct sign for Formulas 15 through 17is determined by the quadrant in which angle $\frac{1}{2}A$ lies.**

Triangles

Law of sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Law of cosines: $b^2 = a^2 + c^2 - 2ac\cos B$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$Area = \frac{1}{2}bc\sin A$$

Area of a Δ : $Area = \frac{1}{2}ac\sin B$

$$Area = \frac{1}{2}ab\sin C$$

CHAPTER 4 MISCELLANEOUS RELATIONS AND FUNCTIONS

The general quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

If $B^2 - 4AC < 0$ **and** $A = C$ **, the graph is a circle.**

If $B^2 - 4AC < 0$ **and** $A \neq C$ **, the graph is an ellipse.**

If $B^2 - 4AC = 0$ **, the graph is a parabola.**

If $B^2 - 4AC > 0$ **, the graph is a hyperbola.**

Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Ellipse:

if C>A, $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ **, transverse axis horizontal**

if C<A, $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ **, transverse axis vertical, where** $a^2 = b^2 + c^2$

Vertices: $\pm a$ **units along major axis from center**

Foci: $\pm c$ **units along major axis from center**

Length=2b

Eccentricity= $\frac{c}{a}$ **<1**

Length of latus rectum= $\frac{2b^2}{a}$

Parabola:

if C=0, $(x-h)^2 = 4p(y-k)$ **opens up and down---axis of symmetry is vertical**

if A=0, $(y-k)^2 = 4p(x-h)$ **opens to the side---axis of symmetry is horizontal**

Equation of axis of symmetry:
x=h if vertical
y=k if horizontal
Focus: p units along the axis of symmetry from vertex
Equation of directrix:
y=-p if axis of symmetry is vertical
x=-p if axis of symmetry is horizontal

Eccentricity=$\frac{c}{a}$ **=1**

Length of latus rectum=4p
Hyperbola:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$ **, transverse axis horizontal**

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$ **, transverse axis vertical, where** $c^2 = a^2 + b^2$

Vertices: $\pm a$ **units along the transverse axis from center**
Foci: $\pm c$ **units along the transverse from center**

Length of latus rectum=$\frac{2b^2}{a}$

Eccentricity=$\frac{c}{a}$ **>1**

the slopes of the asymptotes are $\pm\frac{a}{b}$ **(vertical)or** $\pm\frac{b}{a}$**(horizontal).**

Exponential and Logarithmic Functions

$x^a \bullet x^b = x^{a+b}$

$x^0 = 1$

$\frac{x^a}{x^b} = x^{a-b}$

$x^{-a} = \frac{1}{x^a}$

$(x^a)^b = x^{ab}$

$x^a \bullet y^a = (xy)^a$

$\log_b(p \bullet q) = \log_b p + \log_b q$

$\log_b 1 = 0$

$b^{\log_b p} = p$

$\log_b\left(\frac{p}{q}\right) = \log_b p - \log_b q$

$\log_b b = 1$

$\log_b(p^x) = x \bullet \log_b p$

$\log_b p = \frac{\log_a p}{\log_a b}$

Greatest Integer Functions:

$[x] = i,$ where i is an interger and $i \le x < i + 1$

Polar Coordinates:

x

$= r \bullet \cos\theta \; y$

$= r \bullet \sin\theta \; x$

$^2 + y^2 = r^2$

De Moivre's Throrem:

$z_1 = x_1 + y_1 i = r_1(\cos\theta_1 + i \bullet \sin\theta_1) = r_1 cis\theta_1$

and

$z_2 = x_2 + y_2 i = r_2(\cos\theta_2 + i \bullet \sin\theta_2) = r_2 cis\theta_2$:

$1. z_1 \bullet z_2 = r_1 \bullet r_2[\cos(\theta_1 + \theta_2) + i \bullet \sin(\theta_1 + \theta_2)]$

If $2. \frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i \bullet \sin(\theta_1 - \theta_2)]$

$3. z^n = r^n(\cos n\theta_2 + i \bullet \sin n\theta_2) = r^n cisn\theta$

$4. z^{1/n} = r^{1/n}(\cos\frac{\theta + 2\pi k}{n} + i \sin\frac{\theta + 2\pi k}{n}) = r^{1/n} cis\frac{\theta + 2\pi k}{n}$

where k is an integer taking on values from 0 to n-1.

CHAPTER 5 MISCELLANEOUS TOPICS

$n! = n(n-1)(n-2)...3 \bullet 2 \bullet 1$

Permutations:

Circular permutation (e.g., around a table) of n elements= $(n-1)!$

Circular permutation (e.g., beads on a bracelet) of n elements= $\frac{(n-1)!}{2}$

Permutations of n elements with a repetitions and with b repetitions= $\frac{n!}{a!b!}$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = \frac{{}_nP_r}{r!} = \frac{\text{the product of the largest r factors of n!}}{r!}$$

The number of combinations of n things taken r at a time is denoted by ${}_nC_r$ **or C(n,r) or** $\binom{n}{r}$**.**

$$\binom{n}{r} = \binom{n}{n-r}$$

Binomial Theorem:

$$T_{r+1} = {}_nC_r a^{n-r} b^r$$

Probability:

Independent events: $P(A \cap B) = P(A) \bullet P(B)$

Mutually exclusive events: $P(A \cap B) = 0$ $and\ P(A \cup B) = P(A) + P(B)$

Sequences and Series

In general, an arithmetic sequence is denoted by

$$t_1, t_1 + d, t_1 + 2d, t_1 + 3d ......t_1 + (n-1)d$$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

or

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$

In general, a geometric sequence is denoted by

$$t_1, t_1 r, t_1 r^2, t_1 r^3, ..., t_1 r^{n-1}$$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$\lim_{n\to\infty} S_n = \frac{t_1}{1-r}$$

Geometry and Vectors

If $\vec{V}(v_1, v_2)$ **and** $\vec{U}(u_1, u_2)$ **,**

$$\vec{U} + \vec{V} = (u_1 + v_1, u_2 + v_2)$$

$$\left\|\vec{V}\right\| = \sqrt{(v_1)^2 + (v_2)^2}$$

$$\vec{V} \bullet \vec{U} = v_1 u_1 + v_2 u_2$$

Two vectors are perpendicular if and only if $\vec{V} \bullet \vec{U} = 0$

Logic:

$conjunction = (A \cap B)$

$disjunction = (A \cup B)$

$implication = (A \to B), negation, A \cap B'$

If $A \to B$ is true, then $B' \to A'$ is also true.

Determinates:

$$\begin{vmatrix} a & c \\ b & d \end{vmatrix} = ad - bc$$

$$If \begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

$$x = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

Geometry:

Distance between two points with coordinates

$(x_1, y_1, z_1) and \left(x_2, y_2, z_2\right) =$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

The distance between a point and a plane:

Distance= $\dfrac{\left|Ax_1 + By_1 + Cz_1 + D\right|}{\sqrt{A^2 + B^2 + C^2}}$

Triange:

Heron's formular:

A= $\sqrt{s(s-a)(s-b)(s-c)}$; a, b, c are the three sides of the triangle,

S= $\dfrac{1}{2}(a + b + c)$

Rhombus:

Area=bh=$\frac{1}{2}d_1d_2; b = base,\ h = height,\ d = diagonal$

Cylinder

Volume=$\pi r^2 h$

Later surface area= $2\pi rh$

Total surface area= $2\pi rh + 2\pi r^2$

Cone:

The volume of the cone:

$V = \frac{1}{3}\pi r^2 h$

Later surface area $= \pi r\sqrt{r^2 + h^2} = \frac{1}{2}cl$

Total surface area $= \pi r\sqrt{r^2 + h^2} + \pi r^2$

Sphere

Volume=$\frac{4}{3}\pi r^3$

Surface area= $4\pi r^2$