Section 3: Math Test – No Calculator

QUESTION 1

Choice B is correct. Multiplying both sides of the first equation in the system by 2 yields $4x - 2y = 16$. Adding $4x - 2y = 16$ to the second equation in the system yields $5x = 20$. Dividing both sides of $5x = 20$ by 5 yields $x = 4$. Substituting 4 for x in $x + 2y = 4$ yields $4 + 2y = 4$. Subtracting 4 from both sides of $4 + 2y = 4$ yields $2y = 0$. Dividing both sides of this equation by 2 yields $y = 0$. Substituting 4 for x and 0 for y in the expression $x + y$ yields $4 + 0 = 4$.

Choices A, C, and D are incorrect and may result from various computation errors.

QUESTION 2

Choice A is correct. Since $(x^2 - x)$ is a common term in the original expression, like terms can be added: $2(x^2 - x) + 3(x^2 - x) = 5(x^2 - x)$. Distributing the constant term 5 yields $5x^2 - 5x$.

Choice B is incorrect and may result from not distributing the negative signs in the expressions within the parentheses. Choice C is incorrect and may result from not distributing the negative signs in the expressions within the parentheses and from incorrectly eliminating the x^2-term. Choice D is incorrect and may result from incorrectly eliminating the x-term.

QUESTION 3

Choice D is correct. To find the slope and y-intercept, the given equation can be rewritten in slope-intercept form $y = mx + b$, where m represents the slope of the line and b represents the y-intercept. The given equation $2y - 3x = -4$ can be rewritten in slope-intercept form by first adding $3x$ to both sides of the equation, which yields $2y = 3x - 4$. Then, dividing both sides of the equation by 2 results in the equation $y = \frac{3}{2}x - 2$. The coefficient of x, $\frac{3}{2}$, is the slope of the graph and is positive, and the constant term, -2, is the y-intercept of the graph and is negative. Thus, the graph of the equation $2y - 3x = -4$ has a positive slope and a negative y-intercept.

Choice A is incorrect and may result from reversing the values of the slope and the y-intercept. Choices B and C are incorrect and may result from errors in calculation when determining the slope and y-intercept values.

QUESTION 4

Choice A is correct. It's given that the front of the roller-coaster car starts rising when it's 15 feet above the ground. This initial height of 15 feet can be represented by a constant term, 15, in an equation. Each second, the front of the roller-coaster car rises 8 feet, which can be represented by $8s$. Thus, the equation $h = 8s + 15$ gives the height, in feet, of the front of the roller-coaster car s seconds after it starts up the hill.

Choices B and C are incorrect and may result from conceptual errors in creating a linear equation. Choice D is incorrect and may result from switching the rate at which the roller-coaster car rises with its initial height.

QUESTION 5

Choice C is correct. Since the variable h represents the number of hours a job took, the coefficient of h, 75, represents the electrician's charge per hour, in dollars, after an initial fixed charge of \$125. It's given that the electrician worked 2 hours longer on Ms. Sanchez's job than on Mr. Roland's job; therefore, the additional charge for Ms. Sanchez's job is $\$75 \times 2 = \$150$.

Alternate approach: The amounts the electrician charged for Mr. Roland's job and Ms. Sanchez's job can be expressed in terms of t. If Mr. Roland's job took t hours, then it cost $75t + 125$ dollars. Ms. Sanchez's job must then have taken $t + 2$ hours, so it cost $75(t + 2) + 125 = 75t + 275$ dollars. The difference between the two costs is $(75t + 275) - (75t + 125) = \$150$.

Choice A is incorrect. This is the electrician's charge per hour, not the difference between what Ms. Sanchez was charged and what Mr. Roland was charged. Choice B is incorrect. This is the fixed charge for each job, not the difference between the two. Choice D is incorrect and may result from finding the total charge for a 2-hour job.

QUESTION 6

Choice B is correct. The ratio of the lengths of two arcs of a circle is equal to the ratio of the measures of the central angles that subtend the arcs. It's given that arc $\overset{\frown}{ADC}$ is subtended by a central angle with measure 100°. Since the sum of the measures of the angles about a point is 360°, it follows that arc $\overset{\frown}{ABC}$ is subtended by a central angle with measure $360° - 100° = 260°$. If s is the length of arc $\overset{\frown}{ABC}$, then s must satisfy the ratio $\frac{s}{5\pi} = \frac{260}{100}$. Reducing the fraction $\frac{260}{100}$ to its simplest form gives $\frac{13}{5}$. Therefore, $\frac{s}{5\pi} = \frac{13}{5}$. Multiplying both sides of $\frac{s}{5\pi} = \frac{13}{5}$ by 5π yields $s = 13\pi$.

Choice A is incorrect. This is the length of an arc consisting of exactly half of the circle, but arc $\overset{\frown}{ABC}$ is greater than half of the circle. Choice C is incorrect. This is the total circumference of the circle. Choice D is incorrect. This is half the length of arc $\overset{\frown}{ABC}$, not its full length.

QUESTION 7

Choice D is correct. Multiplying both sides of the given equation by x yields $160x = 8$. Dividing both sides of the equation $160x = 8$ by 160 results in $x = \frac{8}{160}$. Reducing $\frac{8}{160}$ to its simplest form gives $x = \frac{1}{20}$, or its decimal equivalent 0.05.

Choice A is incorrect and may result from multiplying, instead of dividing, the left-hand side of the given equation by 160. Choice B is incorrect and may result from a computational error. Choice C is incorrect. This is the value of $\frac{1}{x}$.

QUESTION 8

Choice C is correct. Applying the distributive property of multiplication to the right-hand side of the given equation gives $(3x + 15) + (5x - 5)$, or $8x + 10$. An equation in the form $cx + d = rx + s$ will have no solutions if $c = r$ and $d \neq s$. Therefore, it follows that the equation $2ax - 15 = 8x + 10$ will have no solutions if $2a = 8$, or $a = 4$.

Choice A is incorrect. If $a = 1$, then the given equation could be written as $2x - 15 = 8x + 10$. Since $2 \neq 8$, this equation has exactly one solution. Choice B is incorrect. If $a = 2$, then the given equation could be written as $4x - 15 = 8x + 10$. Since $4 \neq 8$, this equation has exactly one solution. Choice D is incorrect. If $a = 8$, then the given equation could be written as $16x - 15 = 8x + 10$. Since $16 \neq 8$, this equation has exactly one solution.

QUESTION 9

Choice B is correct. A solution to the system of three equations is any ordered pair (x, y) that is a solution to each of the three equations. Such an ordered pair (x, y) must lie on the graph of each equation in the xy-plane; in other words, it must be a point where all three graphs intersect. The graphs of all three equations intersect at exactly one point, $(-1, 3)$. Therefore, the system of equations has one solution.

Choice A is incorrect. A system of equations has no solutions when there is no point at which all the graphs intersect. Because the graphs of all three equations intersect at the point $(-1, 3)$, there is a solution. Choice C is incorrect. The graphs of all three equations intersect at only one point, $(-1, 3)$. Since there is no other such point, there cannot be two solutions. Choice D is incorrect and may result from counting the number of points of intersection of the graphs of any two equations, including the point of intersection of all three equations.

QUESTION 10

Choice C is correct. If the equation is true for all x, then the expressions on both sides of the equation will be equivalent. Multiplying the polynomials on the left-hand side of the equation gives $5ax^3 - abx^2 + 4ax + 15x^2 - 3bx + 12$. On the right-hand side of the equation, the only x^2-term is $-9x^2$. Since the expressions on both sides of the equation are equivalent, it follows that $-abx^2 + 15x^2 = -9x^2$, which can be rewritten as $(-ab + 15)x^2 = -9x^2$. Therefore, $-ab + 15 = -9$, which gives $ab = 24$.

Choice A is incorrect. If $ab = 18$, then the coefficient of x^2 on the left-hand side of the equation would be $-18 + 15 = -3$, which doesn't equal the coefficient of x^2, -9, on the right-hand side. Choice B is incorrect. If $ab = 20$, then the coefficient of x^2 on the left-hand side of the equation would be $-20 + 15 = -5$, which doesn't equal the coefficient of x^2, -9, on the right-hand side. Choice D is incorrect. If $ab = 40$, then the coefficient of x^2 on the left-hand side of the equation would be $-40 + 15 = -25$, which doesn't equal the coefficient of x^2, -9, on the right-hand side.

QUESTION 11

Choice B is correct. The right-hand side of the given equation, $\frac{2x}{2}$, can be rewritten as x. Multiplying both sides of the equation $\frac{x}{x-3} = x$ by $x - 3$ yields $x = x(x - 3)$. Applying the distributive property of multiplication to the right-hand side of the equation $x = x(x - 3)$ yields $x = x^2 - 3x$. Subtracting x from both sides of this equation yields $0 = x^2 - 4x$. Factoring x from both terms of $x^2 - 4x$ yields $0 = x(x - 4)$. By the zero product property, the solutions to the equation $0 = x(x - 4)$ are $x = 0$ and $x - 4 = 0$, or $x = 4$. Substituting 0 and 4 for x in the given equation yields $0 = 0$ and $4 = 4$, respectively. Since both are true statements, both 0 and 4 are solutions to the given equation.

Choice A is incorrect and may result from a sign error. Choice C is incorrect and may result from an error in factoring. Choice D is incorrect and may result from not considering 0 as a possible solution.

QUESTION 12

Choice D is correct. The original expression can be combined into one rational expression by multiplying the numerator and denominator of the second term by the denominator of the first term: $\frac{1}{2x+1} + 5\left(\frac{2x+1}{2x+1}\right)$, which can be rewritten as $\frac{1}{2x+1} + \frac{10x+5}{2x+1}$. This expression is now the sum of two rational expressions with a common denominator, and it can be rewritten as $\frac{1}{2x+1} + \frac{10x+5}{2x+1} = \frac{10x+6}{2x+1}$.

Choice A is incorrect and may result from a calculation error. Choice B is incorrect and may be the result of adding the denominator of the first term to the second term rather than multiplying the first term by the numerator and denominator of the second term. Choice C is incorrect and may result from not adding the numerator of $\frac{1}{2x+1}$ to the numerator of $\frac{10x+5}{2x+1}$.

QUESTION 13

Choice A is correct. The equation of a parabola in vertex form is $f(x) = a(x - h)^2 + k$, where the point (h, k) is the vertex of the parabola and a is a constant. The graph shows that the coordinates of the vertex are (3, 1), so $h = 3$ and $k = 1$. Therefore, an equation that defines f can be written as $f(x) = a(x - 3)^2 + 1$. To find a, substitute a value for x and its corresponding value for y, or $f(x)$. For example, (4, 5) is a point on the graph of f. So a must satisfy the equation $5 = a(4 - 3)^2 + 1$, which can be rewritten as $4 = a(1)^2$, or $a = 4$. An equation that defines f is therefore $f(x) = 4(x - 3)^2 + 1$.

Choice B is incorrect and may result from a sign error when writing the equation of the parabola in vertex form. Choice C is incorrect and may result from omitting the constant a from the vertex form of the equation of the parabola. Choice D is incorrect and may result from a sign error when writing the equation of the parabola in vertex form as well as by miscalculating the value of a.

QUESTION 14

Choice B is correct. The solutions of the first inequality, $y \geq x + 2$, lie on or above the line $y = x + 2$, which is the line that passes through (−2, 0) and (0, 2). The second inequality can be rewritten in slope-intercept form by dividing the second inequality, $2x + 3y \leq 6$, by 3 on both sides, which yields $\frac{2}{3}x + y \leq 2$, and then subtracting $\frac{2}{3}x$ from both sides, which yields $y \leq -\frac{2}{3}x + 2$. The solutions to this inequality lie on or below the line $y = -\frac{2}{3}x + 2$, which is the line that passes through (0, 2) and (3, 0). The only graph in which the shaded region meets these criteria is choice B.

Choice A is incorrect and may result from reversing the inequality sign in the first inequality. Choice C is incorrect and may result from reversing the inequality sign in the second inequality. Choice D is incorrect and may result from reversing the inequality signs in both inequalities.

QUESTION 15

Choice B is correct. Squaring both sides of the given equation yields $x + 2 = x^2$. Subtracting x and 2 from both sides of $x + 2 = x^2$ yields $x^2 - x - 2 = 0$. Factoring the left-hand side of this equation yields $(x - 2)(x + 1) = 0$. Applying the zero product property, the solutions to $(x - 2)(x + 1) = 0$ are $x - 2 = 0$, or $x = 2$ and $x + 1 = 0$, or $x = -1$. Substituting $x = 2$ in the given equation gives $\sqrt{4} = -2$, which is false because $\sqrt{4} = 2$ by the definition of a principal square root. So, $x = 2$ isn't a solution. Substituting $x = -1$ into the given equation gives $\sqrt{1} = -(-1)$, which is true because $-(-1) = 1$. So $x = -1$ is the only solution.

Choices A and C are incorrect. The square root symbol represents the principal, or nonnegative, square root. Therefore, in the equation $\sqrt{x + 2} = -x$, the value of $-x$ must be zero or positive. If $x = 2$, then $-x = -2$, which is negative, so 2 can't be in the set of solutions. Choice D is incorrect and may result from incorrectly reasoning that $-x$ always has a negative value and therefore can't be equal to a value of a principal square root, which cannot be negative.

QUESTION 16

The correct answer is 360. The volume of a right rectangular prism is calculated by multiplying its dimensions: length, width, and height. Multiplying the values given for these dimensions yields a volume of $(4)(9)(10) = 360$ cubic centimeters.

QUESTION 17

The correct answer is 2. The left-hand side of the given equation contains a common factor of 2 and can be rewritten as $2(2x + 1)$. Dividing both sides of this equation by 2 yields $2x + 1 = 2$. Therefore, the value of $2x + 1$ is 2.

Alternate approach: Subtracting 2 from both sides of the given equation yields $4x = 2$. Dividing both sides of this equation by 4 yields $x = \frac{1}{2}$. Substituting $\frac{1}{2}$ for x in the expression $2x + 1$ yields $2\left(\frac{1}{2}\right) + 1 = 2$.

QUESTION 18

The correct answer is 8. The graph shows that the maximum value of $f(x)$ is 2. Since $g(x) = f(x) + 6$, the graph of g is the graph of f shifted up by 6 units. Therefore, the maximum value of $g(x)$ is $2 + 6 = 8$.

QUESTION 19

The correct answer is $\frac{3}{4}$, or .75. By definition of the sine ratio, since $\sin R = \frac{4}{5}$, $\frac{PQ}{PR} = \frac{4}{5}$. Therefore, if $PQ = 4n$, then $PR = 5n$, where n is a positive constant. Then $QR = kn$, where k is another positive constant. Applying the Pythagorean theorem, the following relationship holds: $(kn)^2 + (4n)^2 = (5n)^2$, or $k^2n^2 + 16n^2 = 25n^2$. Subtracting $16n^2$ from both sides of this equation yields $k^2n^2 = 9n^2$. Taking the square root of both sides of $k^2n^2 = 9n^2$ yields $kn = 3n$. It follows that $k = 3$. Therefore, if $PQ = 4n$ and $PR = 5n$, then $QR = 3n$, and by definition of the tangent ratio, $\tan P = \frac{3n}{4n}$, or $\frac{3}{4}$. Either 3/4 or .75 may be entered as the correct answer.

QUESTION 20

The correct answer is 2.5. The graph of the linear function f passes through the points (0, 3) and (1, 1). The slope of the graph of the function f is therefore $\frac{1 - 3}{1 - 0} = -2$. It's given that the graph of the linear function g is perpendicular to the graph of the function f. Therefore, the slope of the graph of the function g is the negative reciprocal of -2, which is $-\frac{1}{-2} = \frac{1}{2}$, and an equation that defines the function g is $g(x) = \frac{1}{2}x + b$, where b is a constant. Since it's given that the graph of the function g passes through the point (1, 3), the value of b can be found using the equation $3 = \frac{1}{2}(1) + b$. Solving this equation for b yields $b = \frac{5}{2}$, so an equation that defines the function g is $g(x) = \frac{1}{2}x + \frac{5}{2}$. Finding the value of $g(0)$ by substituting 0 for x into this equation yields $g(0) = \frac{1}{2}(0) + \frac{5}{2}$, or $\frac{5}{2}$. Either 2.5 or 5/2 may be entered as the correct answer.