Section 3: Math Test — No Calculator

QUESTION 1.

Choice C is correct. Subtracting 6 from each side of $5x + 6 = 10$ yields $5x = 4$. Dividing both sides of $5x = 4$ by 5 yields $x = \frac{4}{5}$. The value of x can now be substituted into the expression $10x + 3$, giving $10\left(\frac{4}{5}\right) + 3 = 11$.

Alternatively, the expression $10x + 3$ can be rewritten as $2(5x + 6) - 9$, and 10 can be substituted for $5x + 6$, giving $2(10) - 9 = 11$.

Choices A, B, and D are incorrect. Each of these choices leads to $5x + 6 \neq 10$, contradicting the given equation, $5x + 6 = 10$. For example, choice A is incorrect because if the value of $10x + 3$ were 4, then it would follow that $x = 0.1$, and the value of $5x + 6$ would be 6.5, not 10.

QUESTION 2.

Choice B is correct. Multiplying each side of $x + y = 0$ by 2 gives $2x + 2y = 0$. Then, adding the corresponding sides of $2x + 2y = 0$ and $3x - 2y = 10$ gives $5x = 10$. Dividing each side of $5x = 10$ by 5 gives $x = 2$. Finally, substituting 2 for x in $x + y = 0$ gives $2 + y = 0$, or $y = -2$. Therefore, the solution to the given system of equations is $(2, -2)$.

Alternatively, the equation $x + y = 0$ can be rewritten as $x = -y$, and substituting x for $-y$ in $3x - 2y = 10$ gives $5x = 10$, or $x = 2$. The value of y can then be found in the same way as before.

Choices A, C, and D are incorrect because when the given values of x and y are substituted into $x + y = 0$ and $3x - 2y = 10$, either one or both of the equations are not true. These answers may result from sign errors or other computational errors.

QUESTION 3.

Choice A is correct. The price of the job, in dollars, is calculated using the expression $60 + 12nh$, where 60 is a fixed price and $12nh$ depends on the number of landscapers, n, working the job and the number of hours, h, the job takes those n landscapers. Since nh is the total number of hours of work done when n landscapers work h hours, the cost of the job increases by \$12 for each hour a landscaper works. Therefore, of the choices given, the best interpretation of the number 12 is that the company charges \$12 per hour for each landscaper.

Choice B is incorrect because the number of landscapers that will work each job is represented by n in the equation, not by the number 12. Choice C is incorrect because the price of the job increases by $12n$ dollars each hour, which will not be equal to 12 dollars unless $n = 1$. Choice D is incorrect because the total number of hours each landscaper works is equal to h. The number of hours each landscaper works in a day is not provided.

QUESTION 4.

Choice A is correct. If a polynomial expression is in the form $(x)^2 + 2(x)(y) + (y)^2$, then it is equivalent to $(x + y)^2$. Because $9a^4 + 12a^2b^2 + 4b^4 = (3a^2)^2 + 2(3a^2)(2b^2) + (2b^2)^2$, it can be rewritten as $(3a^2 + 2b^2)^2$.

Choice B is incorrect. The expression $(3a + 2b)^4$ is equivalent to the product $(3a + 2b)(3a + 2b)(3a + 2b)(3a + 2b)$. This product will contain the term $4(3a)^3\,(2b) = 216a^3b$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $216a^3b$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (3a + 2b)^4$. Choice C is incorrect. The expression $(9a^2 + 4b^2)^2$ is equivalent to the product $(9a^2 + 4b^2)(9a^2 + 4b^2)$. This product will contain the term $(9a^2)(9a^2) = 81a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $81a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a^2 + 4b^2)^2$. Choice D is incorrect. The expression $(9a + 4b)^4$ is equivalent to the product $(9a + 4b)(9a + 4b)(9a + 4b)\,(9a + 4b)$. This product will contain the term $(9a)(9a)(9a)(9a) = 6{,}561a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $6{,}561a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a + 4b)^4$.

QUESTION 5.

Choice C is correct. Since $\sqrt{2k^2 + 17} - x = 0$, and $x = 7$, one can substitute 7 for x, which gives $\sqrt{2k^2 + 17} - 7 = 0$. Adding 7 to each side of $\sqrt{2k^2 + 17} - 7 = 0$ gives $\sqrt{2k^2 + 17} = 7$. Squaring each side of $\sqrt{2k^2 + 17} = 7$ will remove the square root symbol: $\left(\sqrt{2k^2 + 17}\right)^2 = (7)^2$, or $2k^2 + 17 = 49$. Then subtracting 17 from each side of $2k^2 + 17 = 49$ gives $2k^2 = 49 - 17 = 32$, and dividing each side of $2k^2 = 32$ by 2 gives $k^2 = 16$. Finally, taking the square root of each side of $k^2 = 16$ gives $k = \pm 4$, and since the problem states that $k > 0$, it follows that $k = 4$.

Since the sides of an equation were squared while solving $\sqrt{2k^2 + 17} - 7 = 0$, it is possible that an extraneous root was produced. However, substituting 4 for k in $\sqrt{2k^2 + 17} - 7 = 0$ confirms that 4 is a solution for k: $\sqrt{2(4)^2 + 17} - 7 = \sqrt{32 + 17} - 7 = \sqrt{49} - 7 = 7 - 7 = 0$.

Choices A, B, and D are incorrect because substituting any of these values for k in $\sqrt{2k^2 + 17} - 7 = 0$ does not yield a true statement.

QUESTION 6.

Choice D is correct. Since lines ℓ and k are parallel, the lines have the same slope. Line ℓ passes through the points $(-5, 0)$ and $(0, 2)$, so its slope is $\frac{0 - 2}{-5 - 0}$, which is $\frac{2}{5}$. The slope of line k must also be $\frac{2}{5}$. Since line k has slope $\frac{2}{5}$ and passes through the points $(0, -4)$ and $(p, 0)$, it follows that $\frac{-4 - 0}{0 - p} = \frac{2}{5}$, or $\frac{4}{p} = \frac{2}{5}$. Multiplying each side of $\frac{4}{p} = \frac{2}{5}$ by $5p$ gives $20 = 2p$, and therefore, $p = 10$.

Choices A, B, and C are incorrect and may result from conceptual or calculation errors.

QUESTION 7.

Choice A is correct. Since the numerator and denominator of $\frac{x^{a^2}}{x^{b^2}}$ have a common base, it follows by the laws of exponents that this expression can be rewritten as $x^{a^2-b^2}$. Thus, the equation $\frac{x^{a^2}}{x^{b^2}} = 16$ can be rewritten as $x^{a^2-b^2} = x^{16}$. Because the equivalent expressions have the common base x, and $x > 1$, it follows that the exponents of the two expressions must also be equivalent. Hence, the equation $a^2 - b^2 = 16$ must be true. The left-hand side of this new equation is a difference of squares, and so it can be factored: $(a + b)(a - b) = 16$. It is given that $(a + b) = 2$; substituting 2 for the factor $(a + b)$ gives $2(a - b) = 16$. Finally, dividing both sides of $2(a - b) = 16$ by 2 gives $a - b = 8$.

Choices B, C, and D are incorrect and may result from errors in applying the laws of exponents or errors in solving the equation $a^2 - b^2 = 16$.

QUESTION 8.

Choice C is correct. The relationship between n and A is given by the equation $nA = 360$. Since n is the number of sides of a polygon, n must be a positive integer, and so $nA = 360$ can be rewritten as $A = \frac{360}{n}$. If the value of A is greater than 50, it follows that $\frac{360}{n} > 50$ is a true statement. Thus, $50n < 360$, or $n < \frac{360}{50} = 7.2$. Since n must be an integer, the greatest possible value of n is 7.

Choices A and B are incorrect. These are possible values for n, the number of sides of a regular polygon, if $A > 50$, but neither is the greatest possible value of n. Choice D is incorrect. If $A < 50$, then $n = 8$ is the least possible value of n, the number of sides of a regular polygon. However, the question asks for the <u>greatest</u> possible value of n if $A > 50$, which is $n = 7$.

QUESTION 9.

Choice B is correct. Since the slope of the first line is 2, an equation of this line can be written in the form $y = 2x + c$, where c is the y-intercept of the line. Since the line contains the point (1, 8), one can substitute 1 for x and 8 for y in $y = 2x + c$, which gives $8 = 2(1) + c$, or $c = 6$. Thus, an equation of the first line is $y = 2x + 6$. The slope of the second line is equal to $\frac{1-2}{2-1}$ or -1. Thus, an equation of the second line can be written in the form $y = -x + d$, where d is the y-intercept of the line. Substituting 2 for x and 1 for y gives $1 = -2 + d$, or $d = 3$. Thus, an equation of the second line is $y = -x + 3$.

Since a is the x-coordinate and b is the y-coordinate of the intersection point of the two lines, one can substitute a for x and b for y in the two equations, giving the system $b = 2a + 6$ and $b = -a + 3$. Thus, a can be found by solving the equation $2a + 6 = -a + 3$, which gives $a = -1$. Finally, substituting -1 for a into the equation $b = -a + 3$ gives $b = -(-1) + 3$, or $b = 4$. Therefore, the value of $a + b$ is 3.

Alternatively, since the second line passes through the points (1, 2) and (2, 1), an equation for the second line is $x + y = 3$. Thus, the intersection point of the first line and the second line, (a, b) lies on the line with equation $x + y = 3$. It follows that $a + b = 3$.

Choices A and C are incorrect and may result from finding the value of only a or b, but not calculating the value of $a + b$. Choice D is incorrect and may result from a computation error in finding equations of the two lines or in solving the resulting system of equations.

QUESTION 10.

Choice C is correct. Since the square of any real number is nonnegative, every point on the graph of the quadratic equation $y = (x - 2)^2$ in the xy-plane has a nonnegative y-coordinate. Thus, $y \geq 0$ for every point on the graph. Therefore, the equation $y = (x - 2)^2$ has a graph for which y is always greater than or equal to -1.

Choices A, B, and D are incorrect because the graph of each of these equations in the xy-plane has a y-intercept at $(0, -2)$. Therefore, each of these equations contains at least one point where y is less than -1.

QUESTION 11.

Choice C is correct. To perform the division $\frac{3 - 5i}{8 + 2i}$, multiply the numerator and denominator of $\frac{3 - 5i}{8 + 2i}$ by the conjugate of the denominator, $8 - 2i$. This gives $\frac{(3 - 5i)(8 - 2i)}{(8 + 2i)(8 - 2i)} = \frac{24 - 6i - 40i + (-5i)(-2i)}{8^2 - (2i)^2}$. Since $i^2 = -1$, this can be simplified to $\frac{24 - 6i - 40i - 10}{64 + 4} = \frac{14 - 46i}{68}$, which then simplifies to $\frac{7}{34} - \frac{23i}{34}$.

Choices A and B are incorrect and may result from misconceptions about fractions. For example, $\frac{a + b}{c + d}$ is equal to $\frac{a}{c + d} + \frac{b}{c + d}$, not $\frac{a}{c} + \frac{b}{d}$. Choice D is incorrect and may result from a calculation error.

QUESTION 12.

Choice B is correct. Multiplying each side of $R = \frac{F}{N + F}$ by $N + F$ gives $R(N + F) = F$, which can be rewritten as $RN + RF = F$. Subtracting RF from each side of $RN + RF = F$ gives $RN = F - RF$, which can be factored as $RN = F(1 - R)$. Finally, dividing each side of $RN = F(1 - R)$ by $1 - R$, expresses F in terms of the other variables: $F = \frac{RN}{1 - R}$.

Choices A, C, and D are incorrect and may result from calculation errors when rewriting the given equation.

QUESTION 13.

Choice D is correct. The problem asks for the sum of the roots of the quadratic equation $2m^2 - 16m + 8 = 0$. Dividing each side of the equation by 2 gives $m^2 - 8m + 4 = 0$. If the roots of $m^2 - 8m + 4 = 0$ are s_1 and s_2, then the equation can be factored as $m^2 - 8m + 4 = (m - s_1)(m - s_2) = 0$. Looking at the coefficient of x on each side of $m^2 - 8m + 4 = (m - s_1)(m - s_2)$ gives $-8 = -s_1 - s_2$, or $s_1 + s_2 = 8$.

Alternatively, one can apply the quadratic formula to either $2m^2 - 16m + 8 = 0$ or $m^2 - 8m + 4 = 0$. The quadratic formula gives two solutions, $4 - 2\sqrt{3}$ and $4 + 2\sqrt{3}$ whose sum is 8.

Choices A, B, and C are incorrect and may result from calculation errors when applying the quadratic formula or a sign error when determining the sum of the roots of a quadratic equation from its coefficients.

QUESTION 14.

Choice A is correct. Each year, the amount of the radioactive substance is reduced by 13 percent from the prior year's amount; that is, each year, 87 percent of the previous year's amount remains. Since the initial amount of the radioactive substance was 325 grams, after 1 year, 325(0.87) grams remains; after 2 years $325(0.87)(0.87) = 325(0.87)^2$ grams remains; and after t years, $325(0.87)^t$ grams remains. Therefore, the function $f(t) = 325(0.87)^t$ models the remaining amount of the substance, in grams, after t years.

Choice B is incorrect and may result from confusing the amount of the substance remaining with the decay rate. Choices C and D are incorrect and may result from confusing the original amount of the substance and the decay rate.

QUESTION 15.

Choice D is correct. Dividing $5x - 2$ by $x + 3$ gives:

$$\begin{array}{r} 5 \\ x + 3\overline{)5x - 2} \\ 5x + 15 \\ \hline -17 \end{array}$$

Therefore, the expression $\frac{5x - 2}{x + 3}$ can be rewritten as $5 - \frac{17}{x + 3}$.

Alternatively, $\frac{5x - 2}{x + 3}$ can be rewritten as

$$\frac{5x - 2}{x + 3} = \frac{(5x + 15) - 15 - 2}{x + 3} = \frac{5(x + 3) - 17}{x + 3} = 5 - \frac{17}{x + 3}.$$

Choices A and B are incorrect and may result from incorrectly canceling out the x in the expression $\frac{5x - 2}{x + 3}$. Choice C is incorrect and may result from finding an incorrect remainder when performing long division.

QUESTION 16.

The correct answer is 3, 6, or 9. Let x be the number of \$250 bonuses awarded, and let y be the number of \$750 bonuses awarded. Since \$3000 in bonuses were awarded, and this included at least one \$250 bonus and one \$750 bonus, it follows that $250x + 750y = 3000$, where x and y are positive integers. Dividing each side of $250x + 750y = 3000$ by 250 gives $x + 3y = 12$, where x and y are positive integers. Since $3y$ and 12 are each divisible by 3, it follows that $x = 12 - 3y$ must also be divisible by 3. If $x = 3$, then $y = 3$; if $x = 6$, then $y = 2$; and if $x = 9$, then $y = 1$. If $x = 12$, then $y = 0$, but this is not possible since there was at least one \$750 bonus awarded. Therefore, the possible numbers of \$250 bonuses awarded are 3, 6, and 9. Any of the numbers 3, 6, or 9 may be gridded as the correct answer.

QUESTION 17.

The correct answer is 19. Since $2x(3x + 5) + 3(3x + 5) = ax^2 + bx + c$ for all values of x, the two sides of the equation are equal, and the value of b can be determined by simplifying the left-hand side of the equation and writing it in the same form as the right-hand side. Using the distributive property, the equation becomes $(6x^2 + 10x) + (9x + 15) = ax^2 + bx + c$. Combining like terms gives $6x^2 + 19x + 15 = ax^2 + bx + c$. The value of b is the coefficient of x, which is 19.

QUESTION 18.

The correct answer is 12. Angles ABE and DBC are vertical angles and thus have the same measure. Since segment AE is parallel to segment CD, angles A and D are of the same measure by the alternate interior angle theorem. Thus, by the angle-angle theorem, triangle ABE is similar to triangle DBC, with vertices A, B, and E corresponding to vertices D, B, and C, respectively. Thus, $\frac{AB}{DB} = \frac{EB}{CB}$ or $\frac{10}{5} = \frac{8}{CB}$. It follows that $CB = 4$, and so $CE = CB + BE = 4 + 8 = 12$.

QUESTION 19.

The correct answer is 6. By the distance formula, the length of radius OA is $\sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = 2$. Thus, $\sin(\angle AOB) = \frac{1}{2}$. Therefore, the measure of $\angle AOB$ is 30°, which is equal to $30\left(\frac{\pi}{180}\right) = \frac{\pi}{6}$ radians. Hence, the value of a is 6.

QUESTION 20.

The correct answer is $\frac{1}{4}$ or .25. In order for a system of two linear equations to have infinitely many solutions, the two equations must be equivalent.

Thus, the equation $ax + by = 12$ must be equivalent to the equation $2x + 8y = 60$. Multiplying each side of $ax + by = 12$ by 5 gives $5ax + 5by = 60$, which must be equivalent to $2x + 8y = 60$. Since the right-hand sides of $5ax + 5by = 60$ and $2x + 8y = 60$ are the same, equating coefficients gives $5a = 2$, or $a = \frac{2}{5}$, and $5b = 8$, or $b = \frac{8}{5}$. Therefore, the value of $\frac{a}{b} = \left(\frac{2}{5}\right) \div \left(\frac{8}{5}\right)$, which is equal to $\frac{1}{4}$. Either the fraction $\frac{1}{4}$ or its equivalent decimal, .25, may be gridded asthe correct answer.

Alternatively, since $ax + by = 12$ is equivalent to $2x + 8y = 60$, the equation $ax + by = 12$ is equal to $2x + 8y = 60$ multiplied on each side by the same constant. Since multiplying $2x + 8y = 60$ by a constant does not change the ratio of the coefficient of x to the coefficient of y, it follows that $\frac{a}{b} = \frac{2}{8} = \frac{1}{4}$.

Section 4: Math Test — Calculator

QUESTION 1.

Choice C is correct. Since the musician earns \$0.09 for each download, the musician earns $0.09d$ dollars when the song is downloaded d times. Similarly, since the musician earns \$0.002 each time the song is streamed, the musician earns $0.002s$ dollars when the song is streamed s times. Therefore, the musician earns a total of $0.09d + 0.002s$ dollars when the song is downloaded d times and streamed s times.

Choice A is incorrect because the earnings for each download and the earnings for time streamed are interchanged in the expression. Choices B and D are incorrect because in both answer choices, the musician will lose money when a song is either downloaded or streamed. However, the musician only earns money, not loses money, when the song is downloaded or streamed.

QUESTION 2.

Choice B is correct. The quality control manager selects 7 lightbulbs at random for inspection out of every 400 lightbulbs produced. A quantity of 20,000 lightbulbs is equal to $\frac{20{,}000}{400} = 50$ batches of 400 lightbulbs. Therefore, at the rate of 7 lightbulbs per 400 lightbulbs produced, the quality control manager will inspect a total of $50 \times 7 = 350$ lightbulbs.

Choices A, C, and D are incorrect and may result from calculation errors or misunderstanding of the proportional relationship.

QUESTION 3.

Choice A is correct. The value of m when ℓ is 73 can be found by substituting the 73 for ℓ in $\ell = 24 + 3.5m$ and then solving for m. The resulting equation is $73 = 24 + 3.5m$; subtracting 24 from each side gives $49 = 3.5m$. Then, dividing each side of $49 = 3.5m$ by 3.5 gives $14 = m$. Therefore, when ℓ is 73, m is 14.